(1a) $$\frac{1}{z^2+2z+2} = \left(\frac{1}{z+(1-i)}\right)\left(\frac{1}{z+(1+i)}\right)$$

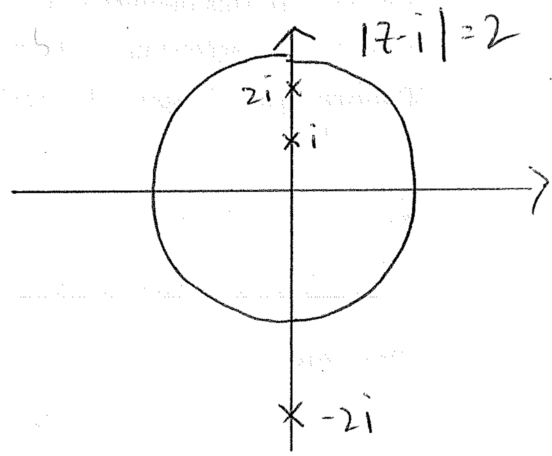
since $(z+(1-i))$ and $(z+(1+i))$ are not zero on $C$, so $\frac{1}{z^2+2z+2}$ is analytic on $C$ and inside $C$.

(1b) Since $\text{Log}\, z$ is not analytic on $\left\{\begin{matrix} Re(z) \le 0 \\ Im(z) = 0 \end{matrix}\right\}$.

But $z+2 = e^{i\theta}+2 = \cos\theta + 2 + i\sin\theta$ on $C$,

$Re(z+2) \geq 1$, thus $\text{Log}(z+2)$ is analytic on $C$.

(2) $$\frac{1}{z^2+4} = \left(\frac{1}{z+2i}\right)\left(\frac{1}{z-2i}\right)$$




We see that $z=2i$ is inside the circle $|z-i|=2$, thus,

$$\int_C \frac{1}{z^2+4}\,dz = \int_C \underbrace{\left(\frac{1}{z+2i}\right)}_{=f(z)}\underbrace{\left(\frac{1}{z-2i}\right)}_{=\frac{1}{z-z_0}} dz$$

$$= 2\pi i\left(\frac{1}{4i}\right) = \frac{\pi}{2}$$

③ $$\frac{\cos z}{z(z^2+2)} = \cos z\left(\frac{1}{z}\right)\left(\frac{1}{z-\sqrt{2}i}\right)\left(\frac{1}{z+\sqrt{2}i}\right)$$

We see that $z=0$, $z=\pm\sqrt{2}i$ are inside the square.

We apply theorem 2 to obtain that

$$\int_C \frac{\cos z}{z(z^2+2)}\,dz = \int_{C_1} + \int_{C_2} + \int_{C_3} \frac{\cos z}{z(z^2+2)}\,dz$$

$$= 2\pi i\left(\frac{\cos 0}{2} + \frac{\cos\sqrt{2}i}{\sqrt{2}i\,(2\sqrt{2}i)} + \frac{\cos(-\sqrt{2}i)}{\sqrt{2}i\,(2\sqrt{2}i)}\right)$$

$$= 2\pi i\left(\frac{1}{2} + \frac{2\cos\sqrt{2}i}{-4}\right)$$

$$= \pi i\left(1 - \frac{\cos\sqrt{2}i}{2}\right)$$

$$= \pi i\left(1 - \frac{e^{\sqrt{2}} + e^{-\sqrt{2}}}{4}\right)$$