

# Suggested Solutions of Assignment 10:

11

(1) Helicoid:  $X_u = (-a \sinh v \sin u, a \sinh v \cos u, a)$

$$X_v = (a \cosh v \cos u, a \cosh v \sin u, 0)$$

$$\Rightarrow \langle X_u, X_v \rangle = 0, \quad |X_u| = |X_v| \quad (\text{isothermal})$$

By Prop. 1 in handout-11, to see  $X$  is a minimal surf

We only need to show  $X_{uu} + X_{vv} = \vec{0}$ . (\*)

$$X_{uu} = (-a \sinh v \cos u, -a \sinh v \sin u, 0)$$

$$X_{vv} = (a \sinh v \cos u, a \sinh v \sin u, 0)$$

$$\Rightarrow (*)$$

Enneper's surface:

$$X_u = (1 - u^2 + v^2, 2uv, 2u)$$

$$X_v = (2uv, 1 - v^2 + u^2, -2v)$$

$$\Rightarrow \langle X_u, X_v \rangle = 0, \quad |X_u| = |X_v| \quad (\text{isothermal})$$

$$X_{uu} = (-2u, 2v, 2), \quad X_{vv} = (2u, -2v, -2)$$

$$\Rightarrow X_{uu} + X_{vv} = 0 \Rightarrow X \text{ is a minimal surface.}$$

#

(2)  $X(u, v) = (\phi(u) \cos u, \phi(u) \sin u, v)$

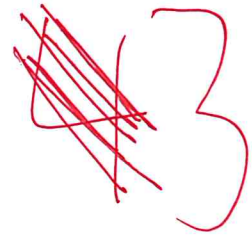
$X_u = (-\phi \sin u, \phi \cos u, 0)$  ,  $X_v = (\phi' \cos u, \phi' \sin u, 1)$

$X_{uu} = (-\phi \cos u, -\phi \sin u, 0)$  ,  $X_{uv} = (-\phi' \sin u, \phi' \cos u, 0)$

$X_{vv} = (\phi'' \cos u, \phi'' \sin u, 0)$  ,  $\lceil \rceil \doteq \frac{X_u \times X_v}{|X_u \times X_v|} = \frac{(\cos u, \sin u, -\phi')}{\sqrt{1+\phi'^2}}$

$E = \phi^2$  ,  $F = 0$  ,  $G = \phi'^2 + 1$

$e = \frac{-\phi}{\sqrt{1+\phi'^2}}$  ,  $f = 0$  ,  $g = \frac{\phi''}{\sqrt{1+\phi'^2}}$



$H = \frac{1}{2} \frac{eG - 2fg + gE}{EG - F^2} = \frac{1}{2} \cdot \frac{-\phi(\phi'^2 + 1)^{\frac{1}{2}} + \phi^2 \phi'' \frac{1}{\sqrt{1+\phi'^2}}}{\phi^2 \cdot (\phi'^2 + 1)}$

$= - \frac{1 + (\phi')^2 - \phi\phi''}{2\phi(1 + (\phi')^2)^{\frac{3}{2}}}$  . (Note that  $\phi \neq 0$  otherwise  $(\phi(u), 0, v)$  touch z-axis.)

#

(3)  $M$  is minimal  $\Rightarrow$  <sup>by Prop 1 in handout 11</sup>  $X_{uu} + X_{vv} = 0$ .

$\Rightarrow X_{uu}^i + X_{vv}^i = 0 \quad \forall i = 1, 2, 3$ .

$\Rightarrow \frac{\partial \xi_i}{\partial u} = - \frac{\partial \eta_i}{\partial v}$  2

#

("  $\frac{\partial \xi_i}{\partial v} = \frac{\partial \eta_i}{\partial u}$  " follows from the fact  $\chi_i$  is  $C^\infty \forall i=1,2,3$ )