Suggested Solutions of Assignment 7

(1) By Theorema Egregium,

$$K = \frac{1}{\det(g_{ij})}\, g_{l2}\left(\frac{\partial \Gamma^l_{11}}{\partial u^2} - \frac{\partial \Gamma^l_{12}}{\partial u^1} + \Gamma^s_{11}\Gamma^l_{s2} - \Gamma^s_{12}\Gamma^l_{s1}\right).$$

$$F=0 \Rightarrow \Gamma^1_{11} = \frac{1}{2g_{11}} g_{11,1},\quad \Gamma^2_{11} = -\frac{1}{2g_{22}} g_{11,2},\quad \Gamma^1_{21} = \Gamma^1_{12} = \frac{1}{2g_{11}} g_{11,2},$$

$$\Gamma^2_{21} = \Gamma^2_{12} = \frac{1}{2g_{22}} g_{22,1},\quad \Gamma^1_{22} = -\frac{1}{2g_{11}} g_{22,1},\quad \Gamma^2_{22} = \frac{1}{2g_{22}} g_{22,2}$$

$$K = \frac{1}{g_{11}}\left(\underline{\Gamma^2_{11,2} - \Gamma^2_{12,1}} + \underline{\Gamma^1_{11}\Gamma^2_{12} - \Gamma^1_{12}\Gamma^2_{11} + \Gamma^2_{11}\Gamma^2_{22} - \Gamma^2_{12}\Gamma^2_{21}}\right) \qquad (1)$$

The terms without derivatives give

$$\frac{1}{g_{11}}(\sim\sim) = \underbrace{\frac{E_uG_u}{4E^2G} + \frac{E_v^2}{4E^2G} - \frac{E_vG_v}{4EG^2} - \frac{G_u^2}{4EG^2}}_{\triangle\triangle\triangle\triangle\triangle\triangle\triangle\triangle\triangle\triangle\triangle\triangle\triangle\triangle} \qquad (2)$$

The terms with derivatives give

$$\frac{1}{g_{11}}(\text{---}) = -\frac{1}{2\sqrt{EG}}\left[\left(\frac{E_v}{\sqrt{EG}}\right)_v + \left(\frac{G_u}{\sqrt{EG}}\right)_u\right] - (\triangle\triangle\triangle\triangle) \qquad (3)$$

By ①②③, we get what we want.

In addition, if $E=G$, then

$$K = -\frac{1}{2E}\left[\left(\frac{E_v}{E}\right)_v + \left(\frac{E_u}{E}\right)_u\right]$$

$$\downarrow$$

$$= -\frac{1}{2e^{2f}}\left[\qquad\qquad\right] \qquad (4)$$

On the other hand, $\Delta f = \frac{1}{2}\left(\frac{E_u}{E}\right)_u + \frac{1}{2}\left(\frac{E_v}{E}\right)_v$ (5)

By ④, ⑤, we have $K = -e^{-2f}\Delta f$. #

(2) See Tutorial 8.

(3) See Tutorial 9.