THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Fall MATH2230
Homework Set 8 (Due on)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P170

1 Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of these integrals:
(a) $\displaystyle\int_C \frac{e^{-z}dz}{z-(\pi i/2)}$; (b) $\displaystyle\int_C \frac{\cos z dz}{z(z^2+8)}$; (c) $\displaystyle\int_C \frac{zdz}{2z+1}$;
(d) $\displaystyle\int_C \frac{\cosh z dz}{z^4}$; (e) $\displaystyle\int_C \frac{\tan(z/2)dz}{(z-x_0)}$ $(-2 < x_0 < 2)$.

2 Find the value of the integral of $g(z)$ around the circle $|z-i| = 2$ in the positive sense when
(a) $g(z) = \dfrac{1}{z^2+4}$; (b) $g(z) = \dfrac{1}{(z^2+4)^2}$.

3 Let C be the circle $|z| = 3$, described in the positive sense. Show that if

$$g(z) = \int_C \frac{2s^2 - s - 2}{s-z} ds \ \ (|z| \neq 3) .$$

then $g(2) = 8\pi i$. What is the value of $g(z)$ when $|z| > 3$?

4 Let C be any simple closed contour, described in the positive sense in the z plane and write

$$g(z) = \int_C \frac{s^3+2s}{(s-z)^3} ds.$$

Show that $g(z) = 6\pi i z$ when z is inside C and that $g(z) = 0$ when z is outside.

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10 Let f be an entire function such that $|f(z)| \leq A|z|$ for all z, where A is a fixed positive number. Show that $f(z) = a_1 z$, where a_1 is a complex constant.
Suggestion: Use Cauchy's inequality (Sec. 57) to show that the second derivative $f''(z)$ is zero everywhere in the plane. Note that the constant M_R in Cauchy's inequality is less than or equal to $A(|z_0| + R)$.

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4 Let R region $0 \leq x \leq \pi$, $0 \leq y \leq 1$. Show that the modulus of the entire function $f(z) = \sin z$ has a maximum value in R at the boundary point $z = (\pi/2) + i$. Suggestion: Write $|f(z)|^2 = \sin^2 x + \sinh^2 y$ and locate points in R al which $\sin^2 x$ and $\sinh^2 y$ are the largest.

6 Let f be the function $f(z) = e^z$ and R the rectangular region $0 \leq x \leq 1$, $0 \leq y \leq \pi$. Illustrate results in Sec. 59 and Exercise 5 by finding points in R where the component function $u(x, y) =$Re$[f(z)]$ reaches its maximum and minimum values.