THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Fall MATH2230
Homework Set 5(Due on)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P.76-77

1. Apply the theorem in Sec. 23 (Cauchy Riemann equation) to verify that each of these functions is entire:
(a) $f(z) = 3x + y + i(3y - x)$; (b) $f(z) = \cosh x \cos y + i \sinh x \sin y$;
(c) $f(z) = e^{-y} \sin x - ie^{-y} \cos x$; (d) $f(z) = (z^2 - 2)e^{-x-iy}$.

7. Let a function f be analytic everywhere in a domain D. Prove that if $f(z)$ is real-valued for all z in D. then $f(z)$ must be constant throughout D.

P.119

2. Evaluate the following integrals:
(a) $\int_0^1 (1+it)^2 dt$; (b) $\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$; (c) $\int_0^{\pi/6} e^{i2t} dt$; (d) $\int_0^\infty e^{-zt} dt$ $(\text{Re}(z) > 0)$.

3. Show that if m and n are integers,

$$\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta = \begin{cases} 0 & \text{when } m \neq n \ , \\ 2\pi, & \text{when } m = n. \end{cases}$$

4. According 10 definition (2). Sec. 42. of definite integrals of complex-valued functions of a real variable,

$$\int_0^\pi e^{(1+i)x} dx = \int_0^\pi e^x \cos x dx + i \int_0^\pi e^x \sin x dx.$$

Evaluate the two integrals on the right here by evaluating the single integral on the left and then using the real and imaginary parts of the value found.