THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Fall MATH2230
Homework Set 3 (Due on)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P.54

5. Show that the function has the value 1 al all nonzero points on the real and imaginary axes, where $z = (x, 0)$ and $z = (0, y)$, respectively, but that it has the value -1 at all nonzero points on the line $y = x$, where $z = (x, x)$. Thus show that the limit of $f(z)$ as z tends to 0 does not exist.

P.61-62

1. Use definition $\dfrac{dw}{dz} = \lim\limits_{\Delta z \to 0} \dfrac{\Delta w}{\Delta z}$ to give a direct proof that $\dfrac{dw}{dz} = 2z$ when $w = z^2$.

3. Using results in Sec. 20. show that
(a) a polynomial $P(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n$ $(a_n \neq 0)$ of degree $n(n \geq 1)$ is differentiable everywhere with derivative $P'(z) = a_1 + 2a_2 z + ... + na_n z^{n-1}$;
(b) the coefficients in the polynomial $P(z)$ in part (a) can be written

$$a_0 = P(0) \;\;, a_1 = \frac{P'(0)}{1!} \;\;, a_2 = \frac{P''(0)}{2!}, ..., a_n = \frac{P^{(n)}(0)}{n!}$$

8. Use the method in Example 2. Sec. 19. to show that $f'(z)$ does not exist at any point z when
(a) $f =$Re z, (b) $f =$Im z.

9. Let f denote 1he function whose values are

$$f(z) = \begin{cases} \dfrac{\overline{z}^2}{z}, & \text{when } z \neq 0\ , \\ 0, & \text{when } z = 0. \end{cases}$$

Show that if $z = 0$, then $\dfrac{\Delta w}{\Delta z} = 1$ at each nonzero point on the real and imaginary axes in the Δz, or $\Delta x \Delta y$ plane. Then show that $\dfrac{\Delta w}{\Delta z} = -1$ at each nonzero point $(\Delta x, \Delta x)$ on the line $\Delta y = \Delta x$ in that plane. Conclude from these observations that $f'(0)$ does not exist. Note that to obtain this result, it is not sufficient to consider only horizontal and vertical approaches to the origin in the Δz plane.

P.70

1. Use the theorem in Sec. 21 (Cauchy Riemann equations) to show that $f'(z)$ does

not exist at any point if
(a) $f(z) = \overline{z}$; (b) $f(z) = z - \overline{z}$; (c) $f(z) = 2x + ixy^2$; (d)$f(z) = e^x e^{-iy}$.