Math 2230A, Complex Variables with Applications

1. Use an antiderivative to show that for every contour C extending from a point z_1 to a point z_2,

$$\int_C z^n dz = \frac{1}{n+1}\left(z_2^{n+1} - z_1^{n+1}\right) \quad (n = 0, 1, 2, \ldots)$$

.

2. By finding an antiderivative, evaluate each of these integrals, where the path is any contour between the indicated limits of integration:

$$\text{(a) } \int_0^{1+i} z^2 dz \quad \text{(b) } \int_0^{\pi+2i} \cos\left(\tfrac{z}{2}\right) dz; \quad \text{(c) } \int_1^3 (z-2)^3 dz$$

3. Use the theorem in Sec.48 to show that

$$\int_{C_0} (z - z_0)^{n-1}\, dz = 0 \quad (n = \pm 1, \pm 2, \ldots)$$

when C_0 is any closed contour which does not pass through the point z_0. (Compare with Exercise 13, Sec. 46.)

4. Find an antiderivative $F_2(z)$ of the branch $f_2(z)$ of $z^{1/2}$ in example 4, Sec.48, to show that integral (3) there has value $2\sqrt{3}(-1+i)$. Note that the value of the integral of the function (2) around the closed contour $C_2 - C_1$ in that example is, therefore, $-4\sqrt{3}$.

5. Show that

$$\int_{-1}^{1} z^i dz = \frac{1+e^{-\pi}}{2}(1-i),$$

where the integrand denotes the principal branch

$$z^i = \exp(i\text{Log} z) \quad (|z| > 0, -\pi < \text{Arg}\, z < \pi)$$

of z^i and where the path of integration is any contour from $z = -1$ to $z = 1$ that, except for its end points, lies above the real axis. (Compare with Exercise 6, Sec. 46.) *Suggestion:* Use an antiderivative of the branch

$$z^i = \exp(i \log z) \quad \left(|z| > 0, -\frac{\pi}{2} < \arg z < \frac{3\pi}{2}\right)$$

of the same power function.