MATH 4030 Differential Geometry
Homework 10
due 17/11/2015 (Tue) at 5PM

Problems

You can directly quote results from previous Homeworks.

1. (do Carmo P.237 Q.4) Show that no neighborhood of a point in a sphere may be isometrically mapped into a plane.

2. (do Carmo P.237 Q.6) Show that there exists no surfaces such that $(g_{ij}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $(h_{ij}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

3. (do Carmo P.237 Q.7) Does there exist a surface $f(u, v)$ with $(g_{ij}) = \begin{pmatrix} 1 & 0 \\ 0 & \cos^2 u \end{pmatrix}$ and $(h_{ij}) = \begin{pmatrix} \cos^2 u & 0 \\ 0 & 1 \end{pmatrix}$?

4. (do Carmo P.237 Q.8) Compute the Christoffel symbols for an open set of the plane in (a) cartesian coordinates and (b) polar coordinates. Use the Gauss equation to compute K in both cases.

5. (do Carmo P.237 Q.9) Justify why the surfaces below are not pairwise locally isometric: (a) sphere, (b) cylinder, (c) saddle $z = x^2 - y^2$.

Suggested Exercises

1. (do Carmo P.237 Q.3) Verify that the surfaces

$$f(u, v) := (u \cos v, u \sin v, \log u),$$

$$\tilde{f}(u, v) := (u \cos v, u \sin v, v),$$

have equal Gauss curvature at the points $f(u, v)$ and $\tilde{f}(u, v)$ but that the mapping $\tilde{f} \circ f^{-1}$ is not an isometry. This shows that the "converse" of the Gauss Theorem is not true, i.e. equal Gauss curvature does not imply that the two surfaces are isometric.

2. (Kühnel Ch.4 Q.8) Show that for a Tchebychev grid (c.f. Suggested Exercise Q.1 in HW 4), the Gauss curvature is given by

$$K = -\frac{1}{\sin \vartheta} \frac{\partial^2 \vartheta}{\partial u_1 \partial u_2}.$$