1. $z = 6x - 8y - x^2 - y^2$, $(x,y) \in \mathbb{R}^2$.

$x \to \infty$, $y \to \infty$. $z \to -\infty$. $\Rightarrow z$ doesn't have minmum. set $z = f(x,y)$

$\cancel{z = f(x,y)}$. $f_x = 6 - 2x = 0 \Rightarrow x = 3$

$f_y = -8 - 2y = 0 \Rightarrow y = -4$

$\Rightarrow$ unique critical point $(3,-4)$

$\partial \mathbb{R}^2 = \phi$ then we have $\max f(x,y)$

$= f(3,-4) = 25$

method 2: $f(x,y) = -(x-3)^2 - (y+4)^2 + 25 \leq \underline{25}$

2. $z = f(x,y) = \exp(2x - 4y - x^2 - y^2)$

$\partial \mathbb{R}^2 = \phi$, ① $x \to \infty$, $y \to \infty$

$\exp(2x - 4y - x^2 - y^2) \to 0$. $(2x - 4y - x^2 - y^2$

$= -(x-1)^2 - (y+2)^2 + 5)$.

but we can ~~attane~~ assign a value on (x,y) such that $f(x,y) = 0$. but by the definition of ~~the~~ $\lim_{\substack{x \to \infty \\ y \to \infty}} f(x,y) = 0$.

given a number $A > 0$ we can find an (x, y), $|x|, |y|$ sufficient big such that $0 \leq f(x,y) < A$. this imply that $\min f(x,y)$ doesn't exist.

② critical point: set $f_x = \exp(2x-4y-x^2-y^2) \cdot (2-2x) = 0$

$f_y = \exp(2x-4y-x^2-y^2)(-4-2y) = 0$

$\Rightarrow (1, -2)$ is the unique critical point

as $\partial R^2 = \phi$. we get

$\max f(x,y) = f(1,-2) = \exp(5)$

3. $f(x,y)=2xy$, in the region $x^2+y^2\leq 1$

$\begin{cases} f_x=0 \\ f_y=0 \end{cases} \Rightarrow (x,y)=(0,0)$ is the unique critical point in the interior of $x^2+y^2\leq 1$; but once we choose

1. $(x,y)=\left(\frac{1}{2},\frac{1}{2}\right)$ we see $f\left(\frac{1}{2},\frac{1}{2}\right)>f(0,0)$

2. choose $(x,y)=\left(\frac{1}{2},-\frac{1}{2}\right)$, $f\left(\frac{1}{2},-\frac{1}{2}\right)<f(0,0)$

$\Rightarrow (x,y)=(0,0)$ is not the max/min point. So the max/min point are all on the boundary $x^2+y^2=1$. we use polar Coodinate transform: set $\begin{cases} x=r\cos\theta \\ y=r\sin\theta \end{cases}$

in ~~thus~~ this case. $r=1 \Rightarrow \begin{cases} x=\cos\theta \\ y=\sin\theta \end{cases}$ $0\leq\theta\leq 2\pi$

$$f(x,y) = f(\cos\theta, \sin\theta) = 2\cos\theta \cdot \sin\theta$$
$$= \sin 2\theta.$$

we can see, $f(x,y) = 1$ when $\theta = \frac{\pi}{4}, \frac{5}{4}\pi$

$f(x,y) = -1$ when $\theta = \frac{3}{4}\pi, \frac{7}{4}\pi$

maximum point $(x,y) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

minmum point $(x,y) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$