MATH1010A-H Assignment 5 Due date 3-12-2015

1. Use a suitable substitution to evaluate the following integrals.

   (a) $\int \frac{x}{\sqrt{1-x^2}}\,dx$

   (b) $\int \frac{xdx}{(1+x^2)^2}$

   (c) $\int \frac{e^x dx}{2+e^x}$

   (d) $\int x\sqrt{x-1}dx$

2. Use integration by parts to evaluate the following integrals.

   (a) $\int x\cos(5-x)dx$

   (b) $\int x^2 e^{-2x}dx$

3. Evaluate the following trigonometric integrals.

   (a) $\int \sin^5 x\cos x\,dx$

   (b) $\int \sin 3x\sin 5x\,dx$

   (c) $\int \sin^4 x\,dx$

   (d) $\int \cos^5 x\sin^4 x\,dx$

   (e) $\int \sin^2 x\cos^4 x\,dx$

   (f) $\int \tan^5 x\,dx$

4. Evaluate the following integrals by trigonometric substitution.

   (a) $\int \frac{dx}{(1-x^2)^{\frac{3}{2}}}$

   (b) $\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$

   (c) $\int \frac{x^2 dx}{\sqrt{9-x^2}}$

   (d) $\int \frac{dx}{x^2\sqrt{x^2+4}}$

5. Evaluate the following integrals of rational functions.

   (a) $\int \frac{x^3}{3+x}\,dx$

   (b) $\int \frac{(1+x)^2}{1+x^2}\,dx$

   (c) $\int \frac{dx}{x^2+2x-3}$

   (d) $\int \frac{4-2x}{(x^2+1)(x-1)^2}\,dx$

6. Find $\int f(x)dx$ for the following functions $f(x)$.

   (a) $f(x)=\begin{cases}4x-1, & x<1\\ \frac{3}{\sqrt{x}}, & x\geq 1\end{cases}$

   (b) $f(x)=|x-3|$

   (c) $f(x)=\begin{cases}e^{2x}, & x<0\\ \cos^2 x, & x\geq 0\end{cases}$

   (d) $f(x)=\frac{1}{1+|x|}$

7. Prove the following reduction formulas.

   (a) $I_n=\int \frac{x^n dx}{\sqrt{x+a}}$; $I_n=\frac{2x^n\sqrt{x+a}}{2n+1}-\frac{2an}{2n+1}I_{n-1}$, $n\geq 1$

   (b) $I_n=\int \frac{1}{\sin^n x}\,dx$; $I_n=-\frac{\cos x}{(n-1)\sin^{n-1}x}+\frac{n-2}{n-1}I_{n-2}$, $n\geq 2$

   (c) $I_n=\int \cos^n x dx$; $I_n=\frac{\sin x\cos^{n-1}x}{n}+\frac{n-1}{n}I_{n-2}$, $n\geq 2$

   (d) $I_n=\int_0^1 x^n\sqrt{1-x}dx$; $I_n=\frac{2n}{2n+3}I_{n-1}$, $n\geq 2$.

8. Use parts (c) and (d) of the Question 7 to evaluate the following definite integrals:

(a) $\int_0^{\pi} \cos^6 x dx$

(b) $\int_0^1 x^4 \sqrt{1-x} dx$

9. Evaluate the following integrals.

(a) $\int x^2 \sqrt{16-x^2} dx$

(b) $\int \tan^{-1} x \, dx$ ($\tan^{-1}$ stands for arctan)

(c) $\int \frac{dx}{e^x + e^{-x}}$

(d) $\int \sin 5x \cos x dx$

(e) $\int x^2 \ln x dx$

(f) $\int \frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3} dx$

(g) $\int \frac{e^{3x} + 1}{e^x + 1} dx$

(h) $\int \frac{\sin x \cos^3 x}{1 + \cos^2 x} dx$

(i) $\int \frac{x^2}{(x^2 - 3x + 2)^2} dx$

(j) $\int \frac{x^2}{\sqrt{9 - x^3}} dx$

(k) $\int x \sec^2(x) dx$

(l) $\int \frac{dx}{\cos x \sin^2 x}$

(m) $\int \ln(x + \sqrt{1 + x^2}) dx$

(n) $\int \frac{1}{x^2} \sin \frac{1}{x} dx$

(o) $\int \frac{dx}{(4x^2 + 1)^{3/2}}$

(p) $\int \frac{x dx}{\sqrt{x + 9}}$

(q) $\int e^{2x} \cos(3x) dx$

(r) $\int \frac{dx}{\sqrt{x}(1 + x)}$

(s) $\int x \sin^2 x dx$

(t) $\int \cos x \cos 2x \cos 3x dx$

(u) $\int x^3 (1 + 3x^2)^{\frac{1}{2}} dx$

(v) $\int \frac{x^2 + 5x + 4}{x^4 + 5x^2 + 4} dx$

(w) $\int \ln^2(x) dx$

(x) $\int \frac{dx}{1 - \cos x}$