Tutorial: Regular surface.

① Let $f(x,y,z) = (x+y+z-1)^2$

(a) Locate the critical points and critical values of f.

(b) For what values of c is the set $f(x,y,z)=c$ a regular surface?

pf: (a) $\nabla f = 2(x+y+z-1)\cdot(1,1,1)$

$\therefore \nabla f = (0,0,0)$

$\Leftrightarrow \quad 2(x+y+z-1)=0$

$\Leftrightarrow \quad x+y+z=1$

(b) when $x+y+z\neq 1$, $\nabla f \neq (0,0,0)$

$\therefore$ $f(x,y,z)=c$ is a regular surface when $c>0$

when $x+y+z=1$, $c=0$

$f(x,y,z)=0$ is the set $\{(x,y,z) \mid x+y+z=1\}$

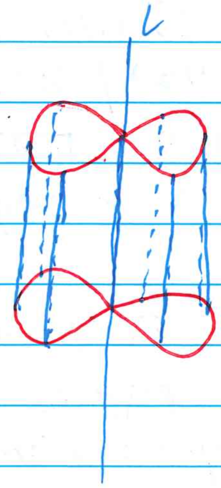
this is a plane in R^3

$\therefore$ for $c\geq 0$, $f(x,y,z)=c$ is a regular surface.

② Let C be a figure "8" in the xy plane. let S be the cylindrical surface over C, that is

$$S=\{(x,y,z)\in R^3 \mid (x,y)\in C\}$$

Is the set S a regular surface?



pf: Suppose $L=\{(0,0,z) \mid z\in R\}$

and S near L is

$$\{(x,y,z) \mid |x|=|y|\}$$

1°, S is not one-to-one on the xy plane

2°, on the yz plane

$h(y,z)=x=\pm y$ is not a function near $(0,0)$

3°, on the xz plane

$g(x,z)=y=\pm x$ is not a function near $(0,0)$

$\therefore$ S is not a regular surface.

3, Show that the one-sheeted cone S, given by

$$z = \sqrt{x^2+y^2}, \quad (x,y) \in R^2$$

is not a regular surface.

pf: 1°, obviously,

$z = f(x,y) = \sqrt{x^2+y^2}$ is not a differentiable function.

2°, on the yz plane

$g(y,z) = x = \pm\sqrt{z^2-y^2}$ where $z \geq 0$

is not one-to-one

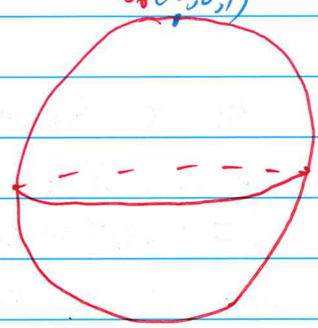
3°, on the xz plane

$h(x,z) = y = \pm\sqrt{z^2-x^2}$ where $z \geq 0$

is not one-to-one

$\therefore$ S is not a regular surface.

4, Show that the sphere is a surface.



pf: $\because$ S is rotationally symmetric

$\therefore$ we only need to show that

S is a regular surface near $(0,0,1)$

$\because$ near $(0,0,1)$, S is

$\{(x,y,z) \mid z = \sqrt{1-x^2-y^2}\}$

$\because$ $h(x,y) = \sqrt{1-x^2-y^2}$ is a smooth function near $(0,0)$

$\therefore$ S is a regular surface.

• Remark: the projection of S on the xy plane is NOT 1-1

But the projection of the part of S near x_0 is 1-1

$\therefore$ S is a regular surface means S is regular "locally".