MATH2230A Homework 6
(due on Oct. 20).

Sect. 33, No. 1, 2;
Sect. 36, No. 1;
Sect. 108, No. 8.

Sect. 33, No. 1

Show that

(a) $\mathrm{Log}(-ei) = 1 - \frac{\pi}{2}i$; (b) $\mathrm{Log}(1-i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}i$.

No. 2 Show that

(a) $\log e = 1 + 2n\pi i \quad (n = 0, \pm 1, \pm 2, \cdots)$;

(b) $\log i = (2n + \frac{1}{2})\pi i \quad (n = 0, \pm 1, \pm 2, \cdots)$;

(c) $\log(-1+\sqrt{3}i) = \ln 2 + 2(n + \frac{1}{3})\pi i \quad (n = 0, \pm 1, \pm 2, \cdots)$.

Sect. 36, No. 1 Show that

(a) $(1+i)^i = \exp(-\frac{\pi}{4} + 2n\pi)\exp(i\frac{\ln 2}{2}) \quad (n = 0, \pm 1, \pm 2, \cdots)$;

(b) $\frac{1}{i^{2i}} = \exp[(4n+1)\pi] \quad (n = 0, \pm 1, \pm 2, \cdots)$.

Sect. 108, No. 8

Determine the image of the domain $r > 0$, $-\pi < \theta < \pi$ in the z plane under each of the transformation $w = F_k(z)$ $(k = 0, 1, 2, 3)$, where $F_k(z)$ are the four branches of $z^{1/4}$ given by equation (7), Sec. 108, when $n = 4$. Use these branches to determine the fourth roots of i.