**THE CHINESE UNIVERSITY OF HONG KONG**
**DEPARTMENT OF MATHEMATICS**
**MATH2050 (First Term, 2015-16 )**
**Mathematical Analysis I**
**Homework I**

**Questions with * will be marked.**

1. Let $a$, $b \in \mathbb{R}$. Show that

   (a)* $a \cdot 0 = 0$;

   (b)* $-a = (-1)a$;

   (c) $-(-a) = a$;

   (d) $(-a)(-b) = ab$;

   (e) $a^2 \geq 0$;

   (f) If $c < 0$ and $a > b$ then $ac < bc$;

   (g) If $a$, $b \geq 0$ then
   $$a < b \Leftrightarrow a^2 < b^2 \Leftrightarrow \sqrt{a} < \sqrt{b},$$
   where $\sqrt{a}$ denotes the positive real number such that $(\sqrt{a})^2 = a$; the exisitence of the square root is assumed and will be discussed later.

2. (a)* Show that $|x - a| < \varepsilon$ iff $a - \varepsilon < x < a + \varepsilon$.

   (b) Find all $x \in \mathbb{R}$ satisfying $|x - 1| > |x + 1|$.

3. Let $A$ be a nonempty subset of $\mathbb{R}$ and $\ell \in \mathbb{R}$. Give the definition for each of the following and the corresponding negation:

   (a)* $\ell$ is a lower bound of $A$;

   (b) $A$ is bounded below.

4. Let $(x_n)$, $(y_n)$ be sequences converge to $x$, $y$ respectively. Show that

   (a) There exist $X$, $Y \in \mathbb{R}$ such that $|x_n| \leq X$ and $|y_n| \leq Y$ for all $n \in \mathbb{N}$;

   (b)* $\lim_{n\to\infty} (x_n + y_n) = x + y$;

   (c) $\lim_{n\to\infty} (x_n y_n) = xy$.