THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050B Mathematical Analysis I

Extra Tutorial 4 (November 23)

Exercise 1. *Let* $\{a_n\}$ *be defined by*

$$a_1 = 1 \quad \text{and} \quad a_{n+1} = \frac{a_n + 2}{a_n + 1} \quad \text{for } n \geq 1.$$

Determine if the sequence $\{a_n\}$ *is convergent. If yes, find its limit.*

Exercise 2. *Let* $\{r_j\}$ *be the set of all rational numbers in* $\mathbb{R}$*. Define the function* φ *to be*

$$\varphi(x) = \sum_{r_j < x} \frac{1}{2^j}, \quad x \in \mathbb{R}.$$

Show that φ *is an increasing function which is continuous at every irrational numbers, but discontinuous at every irrational numbers.*

The following theorem was proved in Extra Tutorial 1.

Theorem (Heine-Borel Theorem)**.** *Every closed and bounded interval* $[a, b]$ *is compact, i.e. any open interval covers* $\mathcal{C}$ *of* $[a, b]$ *has a finite subcover.*

Exercise 3. *Use Heine-Borel Theorem to show that a function continuous on* $[a, b]$ *is uniformly continuous on* $[a, b]$*.*