

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2050B Mathematical Analysis I
Tutorial 8 (October 31)

The following were discussed in the tutorial this week:

1. Recall the definitions of one-sided limits and limits at infinity.
2. Evaluate the following limits (if exist) by definition.

(a) $\lim_{x \rightarrow 1^+} \frac{x}{x-1}$

(b) $\lim_{x \rightarrow \infty} \frac{\sqrt{x} - x}{\sqrt{x} + x}$,

3. Let $f : (0, \infty) \rightarrow \mathbb{R}$, $L \in \mathbb{R}$. Prove that $\lim_{x \rightarrow \infty} f(x) = L$ if and only if $\lim_{x \rightarrow 0^+} f(1/x) = L$.
4. Recall the definition of continuity, sequential criterion for continuity, and discontinuity criterion.
5. Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous on \mathbb{R} and that $f(r) = 0$ for every rational number r . Show that $f(x) = 0$ for all $x \in \mathbb{R}$.
6. Let $[\cdot]$ be the greatest integer function defined by

$$[x] := \sup\{n \in \mathbb{Z} : n \leq x\}.$$

Determine the points of continuity of the function $f(x) := [1/x]$.