

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH2050 (First Term, 2012-2013)
Mathematical Analysis I
Homework I

Questions with * will be marked. Deadline for Homework I: 28th Sept 5pm.

1. Let $a, b \in \mathbb{R}$. Show that

(a)* $a \cdot 0 = 0$;

(b)* $-a = (-1)a$;

(c) $-(-a) = a$;

(d) $(-a)(-b) = ab$;

(e) $a^2 \geq 0$;

(f) If $c < 0$ and $a > b$ then $ac < bc$;

(g) If $a, b \geq 0$ then

$$a < b \Leftrightarrow a^2 < b^2 \Leftrightarrow \sqrt{a} < \sqrt{b},$$

where \sqrt{a} denotes the positive real number such that $(\sqrt{a})^2 = a$; the existence of the square root is assumed and will be discussed later.

2. (a)* Show that $|x - a| < \varepsilon$ iff $a - \varepsilon < x < a + \varepsilon$.

(b) Find all $x \in \mathbb{R}$ satisfying $|x - 1| > |x + 1|$.

3. Let A be a nonempty subset of \mathbb{R} and $\ell \in \mathbb{R}$. Give the definition for each of the following and the corresponding negation:

(a)* ℓ is a lower bound of A ;

(b) A is bounded below.

4. Let $(x_n), (y_n)$ be sequences converge to x, y respectively. Show that

(a) There exist $X, Y \in \mathbb{R}$ such that $|x_n| \leq X$ and $|y_n| \leq Y$ for all $n \in \mathbb{N}$;

(b)* $\lim_{n \rightarrow \infty} (x_n + y_n) = x + y$;

(c) $\lim_{n \rightarrow \infty} (x_n y_n) = xy$.