MATH6032 ASSIGNMENT 2

DUE MARCH 31, 2021

1. Let K be a field with valuation and k be its residue field. Show that if K is algebraically closed, then k is algebraically closed. Give an example to show that the converse is not true.

2. Let K be an algebraically closed field with valuation. Let $f, g \in K[x_1^{\pm 1}, \dots, x_n^{\pm 1}]$. Show that for any $w \in \mathbb{R}^n$, $in_w(fg) = in_w(f)in_w(g)$.

3. Give an ideal I of $\mathbb{Q}[x_1^{\pm 1}, x_2^{\pm 1}, x_3^{\pm 1}]$ such that $I_{aff} \subset \mathbb{Q}[x_1, x_2, x_3]$ requires more generators than I, and $I_{proj} \subset \mathbb{Q}[x_0, x_1, x_2, x_3]$ requires more generators than I_{aff}.

4. Let I be the homogeneous ideal in $\mathbb{Q}[x, y, z]$ generated by the set

$$\mathcal{G} = \{x + y + z, x^2y + xy^2, x^2z + xz^2, y^2z + yz^2\}.$$

Show that $\mathcal{G}$ is a universal Gröbner basis and show that $\mathcal{G}$ is NOT a tropical basis.

5. Describe the tropical variety of the Grassmannian $Gr(2, 4)$. How many cones of each dimension are there?