TA's remarks on 5011 homework 1

1. The mark distribution for Hw1 is:
   Q1 (1 mark); Q4 (3 marks); Q5 (3 marks); Q7 (3 marks).

2. For Q5, observe that $g - f$ may not be well-defined as we may encounter $\infty - \infty$. Consider in particular $f(x) \equiv \infty$, $g(x) \equiv \infty$ for all $x \in X$. Alternatively, we may consider

$$\{x \in X : f(x) < g(x)\} = \bigcup_{r \in \mathbb{Q}} \left(f^{-1}[-\infty, r) \cap g^{-1}(r, \infty] \right)$$

   and

$$\{x \in X : f(x) = g(x)\} = \{x \in X : f(x) < g(x)\}^c \cap \{x \in X : f(x) > g(x)\}^c .$$

3. Q7 concerns Borel-Cantelli lemma. It is the first part of the following result in probability theory:

   > **Theorem** (Borel-Cantelli)**.** *If* $\{A_n, n \geq 1\}$ *is a sequence of events with* $\sum_1^\infty \Pr(A_n) < \infty$, *then* $\Pr(A_n, \text{infinitely often}) = 0$. *Conversely, if the events* $\{A_n, n \geq 1\}$ *are independent and* $\sum_1^\infty \Pr(A_n) = \infty$, *then* $\Pr(A_n, \text{infinitely often}) = 1$.

   According to Chow & Teicher's *Probability Theory*,

   > The Borel-Cantelli theorem is a *sine qua non* of probability theory and is instrumental in proving strong laws of large numbers, the law of the iterated logarithm, etc.

   One may consult their book for a proof of this result.