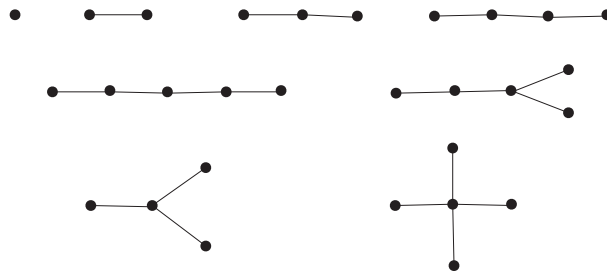


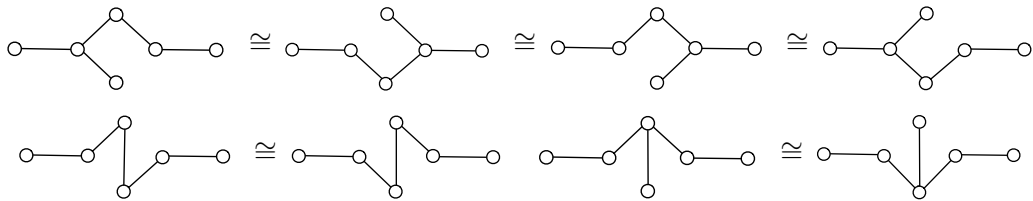
**MMAT5380 Graph Theory and Networks**  
**Suggested Solution for Assignment 3**

3-1:



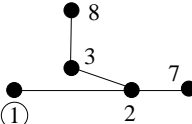
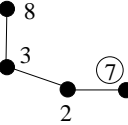
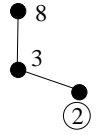

3-2: There are eight spanning tree of the graph  $G$ .

$$\begin{aligned}
 \tau(G) &= \text{graph with diamond and tail} = \text{graph with bridge and tail} + \text{graph with diamond and tail} \\
 &= \left( \text{graph with loop and tail} + \text{graph with bridge and tail} \right) \\
 &+ \left( \text{graph with triangle and tail} + \text{graph with diamond and tail} \right) \\
 &= 2 + 2 + \left( \text{graph with leaf and tail} + \text{graph with bridge and tail} \right) + 1 \\
 &= 2 + 2 + (1 + 2) + 1 = 8.
 \end{aligned}$$

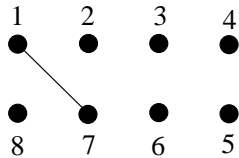
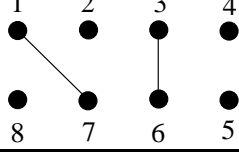
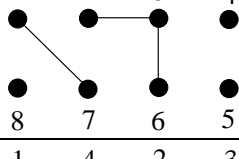
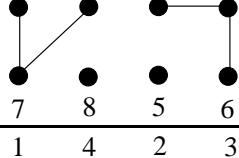
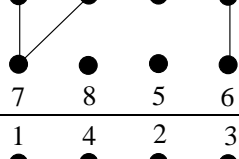
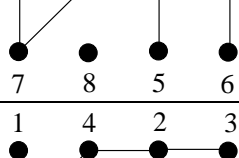
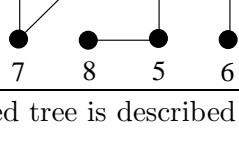


3-3:

labeled tree	Prüfer sequence
	$(3, \cdot, \cdot, \cdot, \cdot)$
	$(3, 2, \cdot, \cdot, \cdot)$
	$(3, 2, 1, \cdot, \cdot)$

	$(3, 2, 1, 2, \cdot, \cdot)$
	$(3, 2, 1, 2, 2, \cdot)$
	$(3, 2, 1, 2, 2, 3)$
	

3-4:

$i$	augmented Seq.	$b_i$	$B$	labeled tree
1	$(7, 3, 2, 4, 2, 5, 8)$	1	2, 3, 4, 5, 6, 7	
2	$(3, 2, 4, 2, 5, 8)$	6	2, 3, 4, 5, 7	
3	$(2, 4, 2, 5, 8)$	3	2, 4, 5, 7	
4	$(4, 2, 5, 8)$	7	2, 4, 5	
5	$(2, 5, 8)$	4	2, 4	
6	$(5, 8)$	2	5	
7	$(8)$	5		

So the sister sequence is  $(1, 6, 3, 7, 4, 2, 5)$  and the labeled tree is described above.

3-5: For any tree with order  $p$ , from Theorem 3.1.18 we have

$$k = 2 + \sum_{i=3}^{p-1} (i-2)n_i \geq 2 + \sum_{i=3}^{p-1} n_i = 2 + (p-k) \quad (*)$$

where  $n_i$  is the number of vertices of degree  $i$ . Hence  $p \leq 2(k-1)$ . Therefore  $\Gamma_k$  is a finite set.

[Note that, for a fixed  $p$ , there are at most  $2^{p(p-1)/2}$  simple graphs of order  $p$ .]