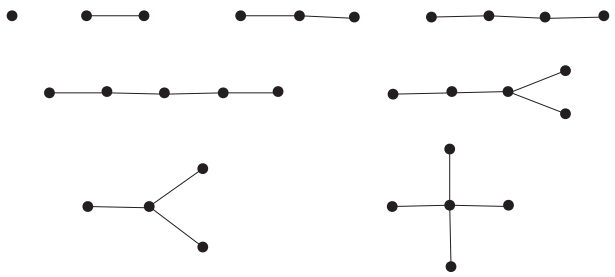
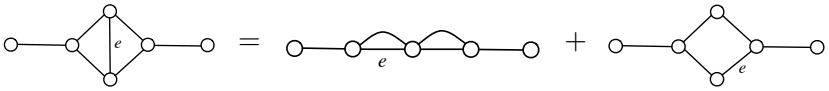
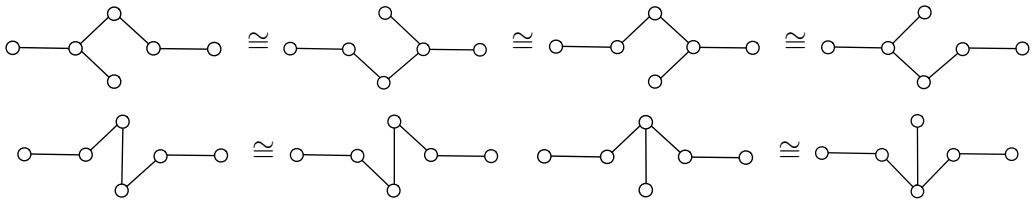
MMAT5380 Graph Theory and Networks
Suggested Solution for Assignment 3

3-1:

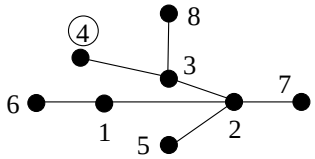
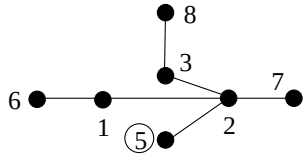
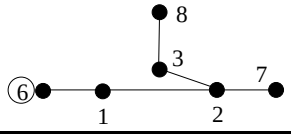


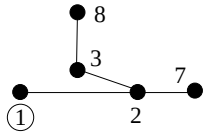
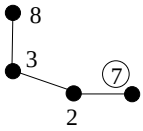
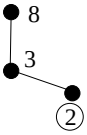
3-2: There are eight spanning tree of the graph G.

$\tau(G) =$  $= 2 + 2 + (1 + 2) + 1 = 8.$

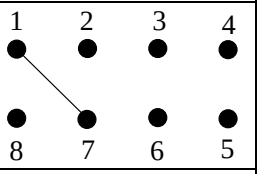
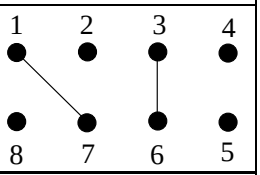
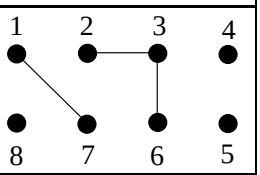
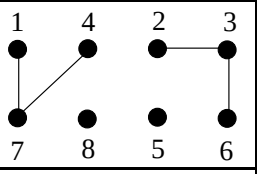
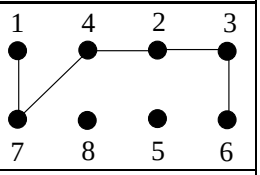
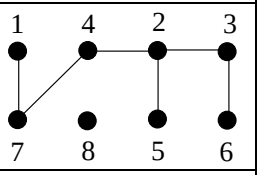
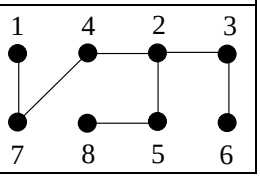


3-3:

labeled tree	Prüfer sequence
	$(3, \cdot, \cdot, \cdot, \cdot, \cdot)$
	$(3, 2, \cdot, \cdot, \cdot, \cdot)$
	$(3, 2, 1, \cdot, \cdot, \cdot)$

	$(3, 2, 1, 2, \cdot, \cdot)$
	$(3, 2, 1, 2, 2, \cdot)$
	$(3, 2, 1, 2, 2, 3)$
	

3-4:

i	augmented Seq.	b_i	B	labeled tree
1	$(7, 3, 2, 4, 2, 5, 8)$	1	2, 3, 4, 5, 6, 7	
2	$(3, 2, 4, 2, 5, 8)$	6	2, 3, 4, 5, 7	
3	$(2, 4, 2, 5, 8)$	3	2, 4, 5, 7	
4	$(4, 2, 5, 8)$	7	2, 4, 5	
5	$(2, 5, 8)$	4	2, 4	
6	$(5, 8)$	2	5	
7	(8)	5		

So the sister sequence is $(1, 6, 3, 7, 4, 2, 5)$ and the labeled tree is described above.

3-5: For any tree with order p, from Theorem 3.1.18 we have

$$k = 2 + \sum_{i=3}^{p-1}(i-2)n_i \geq 2 + \sum_{i=3}^{p-1} n_i = 2 + (p-k) \qquad (*)$$

where n_i is the number of vertices of degree i. Hence $p \leq 2(k-1)$. Therefore Γ_k is a finite set.

[Note that, for a fixed p, there are at most $2^{p(p-1)/2}$ simple graphs of order p.]