The Chinese University of Hong Kong

Department of Mathematics

MMAT5380 Graph Theory and Networks

Assignment 2

Please hand in your assignment to the assignment box or the tutor before 6:30p.m. on Oct. 14, 2019 (Monday).

The assignment box is located at the 2nd floor of LSB and opposites to the Room 223.

2-1: Let $\overrightarrow{G}$ be a digraph with vertex set $V = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ and adjacent matrix

$$A = \begin{pmatrix} 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

What are indegree ($\deg^-$) and outdegree ($\deg^+$) of each vertex? Draw this digraph.

2-2: Let G be a graph with vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and adjacent matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(a) Find the eccentricity of each vertex.

(b) Find all center(s) of G.

(c) What are the radius and diameter of G.

(d) Find a longest path (not unique).

2-3: Let $G = (V, E)$ be a graph with $V = \{a, b, c, d, e, f\}$ and $E = \{ab, ad, ae, bc, be, bf, ce, de, df\}$.

(a) Provide five (a, f)-walks of length 4 in G.

(b) Find all (a, f)-trails of length 6 in G.

(c) Find all spanning (a, f)-paths in G.

2-4: A man want to bring a sheep, a dog, and a bag of cabbage across a river from side A to side B on a rowboat. The boat is so small that he can carry only one of these items on the boat at a time. But, he cannot leave the dog alone with the sheep nor the sheep alone with the cabbage.

In order to determine how he should process, we shall describe the possible steps he can take by a graph. Let the vertices (x_1, x_2, x_3, x_4) represent all the allowable situations, where x_1, x_2, x_3, x_4 represent the man, the sheep, the dog, and the cabbage on which side of the river, respectively. For example, the man, the sheep, the dog and the cabbage are on side A of the river is the initial situation. We use (A,A,A,A) (or (AAAA), for short) to represent this situation. The man, the sheep and the dog are on side A while the cabbage is on side B is one allowable intermediate situation. We use (A,A,A,B) (or (AAAB), for short) to represent this situation. Two vertices are adjacent if the

man can make a trip across the river so that the situation corresponding to one of the vertices can be transformed into that corresponding to the other vertex, and conversely.

Construct the graph and determine all possible shortest ways for the man to transport the items across the river.

2-5: Prove that in any graph G the distance $d(u, v)$, $u, v \in V(G)$, satisfies the following statements:

(a) $d(u, v) \geq 0$;

(b) $d(u, v) = 0$ if and only if $u = v$;

(c) $d(u, v) = d(v, u)$;

(d) $d(u, v) + d(v, w) \geq d(u, w)$ (triangle inequality).

2-6: Let G be a graph with $\delta(G) \geq 2$. Show that G contains a cycle.

2-7: Let G be a (p, q)-graph. Show that $\kappa(G) \leq \lfloor \frac{2q}{p} \rfloor$.
($\lfloor x \rfloor$ denotes the largest integer that less than or equal to x.)

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