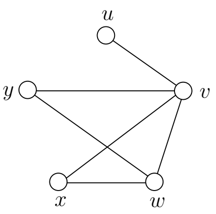
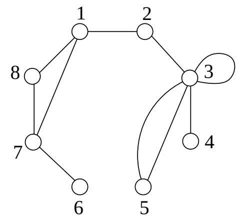
MMAT5380 Graph Theory and Networks

Suggested Solution for Assignment 1

1-1: (a)



(b)



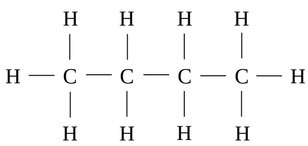
1-2: (a) $(6, 3, 3, 2, 2, 2, 1, 1)$.

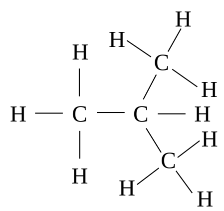
(b) There are 10 edges. The degree sum is $6 + 3 + 3 + 2 + 2 + 2 + 1 + 1 = 20$. So it satisfies the handshaking lemma.

(c) $A(G) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$. $M(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$.

1-3: (a) Each carbon atom vertex has degree 4 and each hydrogen atom vertex has degree 1.

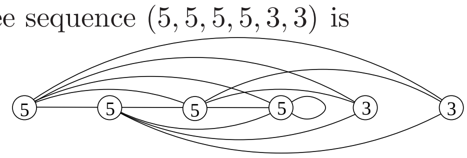
(b) The graphs are as follows:



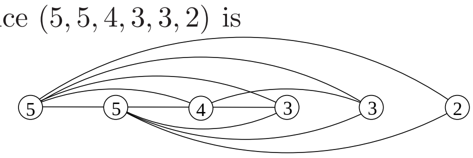


1-4: (a) $(5, 5, 5, 5, 3, 3) \longrightarrow (4, 4, 4, 2, 2) \longrightarrow (3, 3, 1, 1) \longrightarrow (2, 0, 0)$. The last sequence is not graphical. So there is no simple graph with degree sequence $(5, 5, 5, 5, 3, 3)$.

A graph of order 6 with degree sequence $(5, 5, 5, 5, 3, 3)$ is



(b) $(5, 5, 4, 3, 3, 2) \longrightarrow (4, 3, 2, 2, 1) \longrightarrow (2, 1, 1, 0)$. The last sequence is graphical which corresponds to P_3. So there is a simple graph with degree sequence $(5, 5, 4, 3, 3, 2)$. A graph of order 6 with degree sequence $(5, 5, 4, 3, 3, 2)$ is

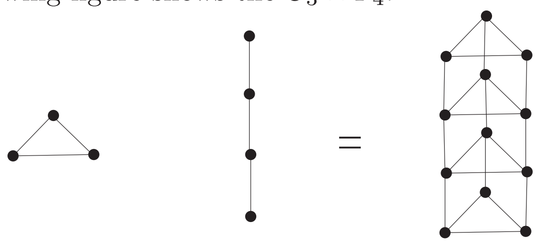


1-5: (a) Graphs H and K are subgraphs of G. The graph J is not, because J contains edge bd which is not an edge of G.

(b) From (a), only H and K are needed to consider. For those two graphs, H is an induced subgraph. An induced subgraph is the maximal subgraph containing the given vertex set. For the graph H, all necessary edges appear. Thus, H is induced. For the graph K, edge ce would also need to appear since that edge is in G. Thus, K is not an induced subgraph of G.

1-6: Define $F(1) = a, F(2) = c, F(3) = e, F(4) = b, F(5) = d$ and $F(6) = f$. The mapping F induces a correspondence from the edges of G to the edges of H as follows: $14 \to ab$; $15 \to ad$; $16 \to af$; $24 \to cb$; $25 \to cd$; $26 \to cf$; $34 \to eb$; $35 \to ed$ and $36 \to ef$. Thus the mapping F preserves the adjacency. So we have shown that G is isomorphic to H.

1-7: (a) The right graph of the following figure shows the $C_3 \times P_4$.



(b) Consider a vertex (u_i, v_j) in $G \times H$.

For this fixed vertex v_j in H, vertices (u_i, v_j) and (u_l, v_j) are adjacent if and only if vertices u_i and u_l are adjacent in G. So (u_i, v_j) has $\deg_G(u_i)$ neighbors of the form (u_l, v_j).

For this fixed vertex u_i in G, vertices (u_i, v_j) and (u_i, v_k) are adjacent if and only if vertices v_j and v_k are adjacent in H. So (u_i, v_j) has $\deg_H(v_j)$ neighbors of the form (u_i, v_k). Therefore, totally, $\deg_{G\times H}(u_i, v_j) = \deg_G(u_i) + \deg_H(v_j)$.

1-8: (a) For $v \in N(A \cup B)$, there is a vertex $u \in A \cup B$ such that v is adjacent with u, i.e., $v \in N(u)$. Since $u \in A$ or $u \in B$, $v \in N(u) \subseteq N(A)$ or $v \in N(u) \subseteq N(B)$. Hence $u \in N(A) \cup N(B)$. Thus, $N(A \cup B) \subseteq N(A) \cup N(B)$.

Conversely, for $v \in N(A) \cup N(B)$, we have $v \in N(A)$ or $v \in N(B)$. By definition $N(A) \subseteq N(A \cup B)$ and $N(B) \subseteq N(A \cup B)$. So, $N(A) \cup N(B) \subseteq N(A \cup B)$.

We have $N(A \cup B) = N(A) \cup N(B)$.

(b) For $v \in N(A \cap B)$, there is a $u \in A \cap B$ such that v is adjacent with u. Since $u \in A$ and $u \in B$, $v \in N(A)$ and $u \in N(B)$. Thus $v \in N(A) \cap N(B)$. We obtain $N(A \cap B) \subseteq N(A) \cap N(B)$.