# MMAT5120 HW1 Due Date: Oct 12, 2020

1. Let $S: S^2 \setminus \{N=(0,0,1)\} \to \mathbb{C}$ be the Stereographic projection given by

$$x+iy = S(a,b,c) = \frac{a+ib}{1-c}.$$

Find $S^{-1}: \mathbb{C} \to S^2 \setminus \{N\}$.

2. Let $D = \{l : l \text{ is a straight line in } \mathbb{R}^2 = \mathbb{C}\}$ and $f: D \to \mathbb{R} \cup \{\infty\}$ be a function on $D$ defined by

$$f(l) = \begin{cases} \infty, & \text{if } l \text{ is parallel to the y-axis.} \\ \text{slope of } l, & \text{otherwise.} \end{cases}$$

(a) Show that $D$ is invariant in the translational geometry and in the Euclidean geometry.

(b) Is $f$ invariant in translational geometry? Jusify your answer.

(c) Is $f$ invariant in Euclidean geometry? Jusify your answer.

(3) (a) Show that $(z_0, \infty, z_2, z_3) = \frac{z_0 - z_2}{z_0 - z_3}$.

(b) Find a Möbius transformation sending $0, i, 2$ to $-2i, 1, 0$ respectively.

(c) Find all Möbius transformations with fixed points $1$ and $i$.

(4) Using Fundamental Theorem of Möbius geometry, show that all clines are congruent in Möbius geometry.

(5) (a) Let $C$ be a cline which is in fact a straight line. Show that the symmetric point $z^*$ is the usual Euclidean reflection of $z$ across $C$.

(b) In addition, let $C'$ be another clines perpendicular to $C$ and passing through the point $z$. Show that $C'$ also passes throught the symmetric point $z^*$.

(c) Conversely, show that if a clines $C'$ passes through $z$ and $z^*$, then $C'$ is perpendicular to $C$.

(End)