**THE CHINESE UNIVERSITY OF HONG KONG**
**Department of Mathematics**
**MATH4210 Financial Mathematics 2020-2021 T1**
**Assignment 2**
**Due date: 16 October 2020 11:59 p.m.**

Please submit this assignment on blackboard. If you have any questions regarding this assignment, please email your TA Wong Wing Hong (whwong@math.cuhk.edu.hk).

1. Suppose the continuous compounding interest rate is $r$ and the price of a stock is $S(t)$ at time $t$. If it pays dividend $d \times S(t_D)$ at time $t_D$, where $0 < t_D < T$ and $0 < d < 1$, show that its forward price $F(0,T)$ satisfies $F(0,T) = \frac{1}{1+d} S(0)e^{rT}$ under no arbitrage opportunity assumption.

2. (Put-Call Parity Relation with Dividend) Assume that the value of the dividends of the stock paid during $[t,T]$ is a deterministic constant $D$ at time $t_D \in (t,T]$. Let $S(t)$ be the stock price, $r$ be the continuous compounding interest rate, $C_E(t,K)$ and $P_E(t,K)$ be the prices of European call and put option at time $t$ with strike $K$ and maturity $T$ respectively. Show that

$$C_E(t,K) - P_E(t,K) = S(t) - Ke^{-r(T-t)} - De^{-r(t_D-t)}$$

for all $t < T$.

3. Assume that the value of the dividends of the stock paid during $[t,T]$ is a deterministic constant $D$ at time $t_D \in (t,T]$. Let $S(t)$ be the stock price, $r$ be the continuous compounding interest rate, $C_A(t,K)$ and $P_A(t,K)$ be the prices of American call and put option at time $t$ with strike $K$ and maturity $T$ respectively. Show that

$$C_A(t,K) - P_A(t,K) < S(t) - Ke^{-r(T-t)}$$

for all $t < T$.

4. Suppose the continuous compounding interest rate is $r$, two European call options has same strike $K$ and different maturity $T_1 < T_2$. Suppose

the underlying asset pays a deterministic dividend D at $t_D \in (T_1, T_2]$. Prove

$$C_E(t, T_1) < C_E(t, T_2) + (De^{-r(t_D - t)} - K(e^{-r(T_1 - t)} - e^{-r(T_2 - t)}))^+$$

for all $t < T_1$.

5. Suppose that we have the following 3 European call with the same maturity $T$ in the financial market:

| Type | Strike Price | Price at time 0 |
|---|---|---|
| Call | 90 | 15 |
| Call | 100 | 12 |
| Call | 110 | 5 |

Suppose that the continuous compounding interest rate is $r = 0$ in the market and the maturity time is $T = 1$. Can you construct an arbitrage portfolio with the above options?