The Chinese University of Hong Kong
MATH3280 Introductory Probability 2020-2021
Mid-term Examination

Oct 29 10:30am–12:25pm (noon)

Instructions

- This is an open-book examination. You may **ONLY** refer to printed/written materials during the examination. Assessing information on the internet is not allowed.
- You are allowed to use a calculator in the approved list during the examination.
- You shall take the examination in isolation and shall not communicate with any person during the examination other than the course teacher(s) concerned. Please kindly be reminded of the following regulations enforced by the university:

  Honesty in Academic Work: *The Chinese University of Hong Kong places very high importance on honesty in academic work submitted by students, and adopts a policy of zero tolerance on cheating and plagiarism. Any related offence will lead to disciplinary action including termination of studies at the University.*

- There are a total of 100 points.
- Answer **ALL** questions.
- Show all steps clearly in your working. **NO** point will be given if suffcient justification is not provided.
- Only handwritten answers on papers or electronic devices will be accepted. Typed answer will **NOT** be accepted.
- Please follow the instruction of submission below:
  1. Write your answers on papers or electronic devices. Only handwritten answers will be accepted and typed answer will **NOT** be accepted.
  2. Scan or take photos of your work (if you write on papers).
  3. Combine your work into a single pdf file.
  4. Name the pdf file by your student id (e.g. 1155123456.pdf).
  5. You must upload the pdf file to Blackboard before 12:25pm (noon), 29 Oct, 2020. Mark deduction will be made for late submission.
  6. Please check your file carefully to make sure no missing pages.

1 (10 pts). Suppose that E, F, G are independent events in a probability space. Let E^c and F^c denote the complements of E and F, respectively.

(i) Show that E^c and F are independent;

(ii) Is $E \cup F$ independent of G? Justify your answer.

2 (10 pts) Suppose events A, B, and C are independent with probabilities 1/2, 1/6, and 1/3, respectively. Calculate the following probabilities:

(a) $P(A \cap B \cap C)$;

(b) $P(A \cup B \cup C)$;

3 (10 pts) Suppose that the cumulative distribution function of a random variable X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{b}{4} & 0 \le b < 1 \\ \frac{1}{2} + \frac{b-1}{4} & 1 \le b < 2 \\ \frac{11}{12} & 2 \le b < 3 \\ 1 & 3 \le b \end{cases}$$

(a) Find $P\{X = i\}$, $i = 1, 2, 3$.

(b) Find $P\{1 \le X < 3\}$.

——————(Please Turn Over)——————

4 (10 pts) A box contains 40 black balls and 50 red balls. Five balls are drawn at random from the box, one after the other, without replacement. Find the chance that the first black ball appears on the last draw.

5 (10 pts) Let X be a standard normal random variable.

(a) Find the probability density function of $Y = 2X + 1$;

(b) Find the probability density function of $Z = X^2$.

6 (20 pts) There are two boxes, the odd box containing 3 black marble and 5 white marbles, and the even box containing 2 black marbles and 4 white marbles. A box is selected at random, and a marble is drawn at random from the selected box.

(a) What is the probability that the marble is black?

(b) Given the marble is white, what is the probability that it came from the odd box?

7 (20 pts) Suppose 3 numbers are picked at random without replacement from 1, 2, 3, . . . , 9, 10.

(a) Find the chance that the number 1 is among these 3 numbers.

(b) Let X be the smallest number among these 3 numbers. Find $E[X]$.

8 (10 pts) Prove that for any events $E_1, \ldots, E_n$,

$$P(E_1 \cap E_2 \cap \cdots \cap E_n) \geq P(E_1) + \cdots + P(E_n) - (n - 1).$$

——— End ———