The Chinese University of Hong Kong
Department of Mathematics

MATH3280 Introductory Probability

Solutions to Assignment 1

Solutions to problems

p.50 Q3(1 mark)

EF is the event that the sum of the dice is odd and at least one of the dice lands on 1.
$E \cup F$ is the event that the sum of the dice is odd or at least one of the dice lands on 1.
FG and EFG are both the event that one of the dice lands on 1 and the other lands on 4.
EF^c is the event that the sum of the dice is odd and none of the dice lands on 1.

p.50 Q4(2 marks)

(a) An element $00\dots01$ in S represents the outcome of coin flips in the order of $A, B, C, A, \dots$, where 0 represents a tail and 1 represents a head. The element $000\dots$ is the outcome that A, B and C get a tail every time and they flip the coin endlessly.

(b) Define $0^n1 := \underbrace{0\dots\dots\dots0}_{n \text{ times}}1$ and $0^\infty := 000\dots$

(i) $A = \{1, 000\ 1, 000\ 000\ 1, \dots\} = \cup_{k=0}^{\infty} \left\{0^{3k}1\right\}$.

(ii) $B = \{01, 000\ 01, 000\ 000\ 01, \dots\} = \cup_{k=0}^{\infty} \left\{0^{3k+1}1\right\}$.

(iii) $(A \cup B)^c = C$ wins or no one wins $= \left(\cup_{k=0}^{\infty} \left\{0^{3k+2}1\right\}\right) \cup \{0^\infty\}$.

p.51 Q8(1 mark)

(a) The event that either A or B occurs is $A \triangle B := (A \backslash B) \cup (B \backslash A)$. Given $A \cap B = \emptyset$, we have $A \backslash B = A$, $B \backslash A = B$, and $(A \backslash B) \cap (B \backslash A) = \emptyset$. Therefore, the answer is

$$\Pr((A \backslash B) \cup (B \backslash A)) = \Pr(A \backslash B) + \Pr(B \backslash A) = \Pr(A) + \Pr(B) = 0.3 + 0.5 = 0.8.$$

(b) The event that A occurs but B does not is AB^c. Since $\Pr(A) = \Pr(AB) + \Pr(AB^c)$, the answer is

$$\Pr(AB^c) = \Pr(A) - \Pr(AB) = \Pr(A) - \Pr(\emptyset) = 0.3 - 0 = 0.3.$$

(c) The event that both A and B occur is AB which is the empty set. Hence the answer is 0.

p.51 Q11(1 mark)

Let C_1 be the event that a randomly chosen American male smokes cigarettes, C_2 be the event that a randomly chosen American male smokes cigars.

(a) Since

$$\Pr(C_1^c C_2^c) = \Pr((C_1 \cup C_2)^c) = 1 - \Pr(C_1 \cup C_2) = 1 - (\Pr(C_1) + \Pr(C_2) - \Pr(C_1 \cap C_2)),$$

the percentage of males who smoke neither cigars nor cigarettes is $[1 - (0.28 + 0.07 - 0.05)] \times 100\% = 70\%$.

(b) Since $\Pr(C_2 C_1^c) = \Pr(C_2) - \Pr(C_2 C_1)$, the percentage of males who smoke cigars but not cigarettes is $(0.07 - 0.05) \times 100\% = 2\%$.

p.53 Q44(3 marks)

(a) The total number of permutations of the five people is 5!. There are C_1^3 ways to choose a person X from C, D, E who is arranged between A and B as AXB. We have 3! ways to permute the people when we regard AXB as an object. We also have 2 ways to arrange within the object AXB because we can switch A and B. Hence the probability that there is exactly one person between A and B is

$$\frac{C_1^3 \times 3! \times 2!}{5!} = 0.3.$$

(b) There are C_2^3 ways to choose two people X, Y from C, D, E who are arranged between A and B as $AXYB$. We have 2! ways to permute the people when we regard $AXYB$ as an object. We also have $2! \times 2!$ ways to arrange within the object $AXYB$ because we can switch A and B and switch X and Y. Hence the probability that there are exactly two people between A and B is

$$\frac{C_2^3 \times 2! \times 2! \times 2!}{5!} = 0.2.$$

(c) There is only one way to choose three people X, Y, Z from C, D, E who are arranged between A and B as $AXYZB$. We have $2! \times 3!$ ways to arrange within the object $AXYZB$ because we can switch A and B and permute X, Y and Z. Hence the probability that there are exactly three people between A and B is

$$\frac{2! \times 3!}{5!} = 0.1.$$

Solutions to theoretic exercises

p.55 Q7(1 mark)

(a) $(E \cup F)(E \cup F^c) = E \cup (FF^c) = E \cup \emptyset = E$.

(b) $(E \cup F)(E^c \cup F)(E \cup F^c) = ((EE^c) \cup F)(E \cup F^c) = F(E \cup F^c) = (EF) \cup (FF^c) = EF$.

(c) $(E \cup F)(F \cup G) = F \cup (EG)$.

p.55 Q10(1 mark)

By the inclusion-exclusion principle,

$$
\begin{aligned}
&\Pr(E \cup F \cup G) \\
&= \Pr(E) + \Pr(F) + \Pr(G) - \Pr(EF) - \Pr(FG) - \Pr(EG) + \Pr(EFG) \\
&= \Pr(E) + \Pr(F) + \Pr(G) - (\Pr(EFG) + \Pr(EFG^c)) - (\Pr(EFG) + \Pr(E^cFG)) \\
&\qquad - (\Pr(EFG) + \Pr(EF^cG)) + \Pr(EFG) \\
&= \Pr(E) + \Pr(F) + \Pr(G) - \Pr(EFG^c) - \Pr(E^cFG) - \Pr(EF^cG) - 2\Pr(EFG).
\end{aligned}
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