Math 3280 20-09-14

Review.

- Axiomatic approach to probability.

  A prob. P on the sample space S satisfies:

  Axiom I: $0 \leq P(E) \leq 1$, $\forall$ any Event E

  Axiom II: $P(S) = 1$.

  Axiom III: If $E_1, E_2, \cdots,$ is a sequence of events which are mutually exclusive,

  then $$P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$$

  (Countable additivity of prob.).

- Properties derived from the above axioms:

  - Inclusive-exclusive identity.

    $$P(E \cup F) = P(E) + P(F) - P(EF).$$

$$P\left(\bigcup_{k=1}^{n} E_k\right) = \sum_{r=1}^{n} (-1)^{r+1} \sum_{i_1 < i_2 < \cdots < i_r} P(E_{i_1} E_{i_2} \cdots E_{i_r}).$$

- (Countable sub-additivity of Prob)

$$P\left(\bigcup_{k=1}^{\infty} E_k\right) \leq \sum_{k=1}^{\infty} P(E_k).$$

- (Continuity of Prob.)
  - $P\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} P(E_n)$ if $E_1 \subset E_2 \subset \cdots$
  - $P\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} P(E_n)$ if $E_1 \supset E_2 \supset \cdots$

Example 1. If $P(E) = 0.8$, $P(F) = 0.9$

Show that $P(E \cap F) \geq 0.7$.

Pf. Recall

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Hence

$$\begin{aligned} P(E \cap F) &= P(E) + P(F) - P(E \cup F) \\ &= 0.8 + 0.9 - P(E \cup F) \\ &\geq 0.8 + 0.9 - 1 = 0.7. \quad \blacksquare \end{aligned}$$

Example 2.

If $P(E) = 0.8$, $P(F) = 0.9$, $P(E \cap F) = 0.75$

find the probability that exactly one of E or F occurs.

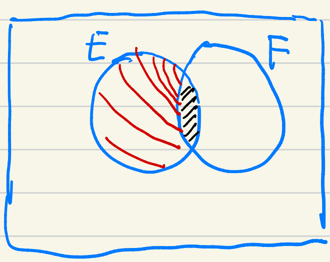
Solution: Let G denote the event that exactly one of E or F occurs.

$$G = (E \setminus F) \cup (F \setminus E)$$

(disjoint union)

Hence $P(G) = P(E \setminus F) + P(F \setminus E)$.

Notice that $E = (E \setminus F) \cap (E \cap F)$



Hence $P(E) = P(E \setminus F) + P(E \cap F) \Rightarrow P(E \setminus F) = P(E) - P(E \cap F)$

$= 0.8 - 0.75$

$= 0.05$.

$$P(F\setminus E) = P(F) - P(E\cap F)$$
$$= 0.9 - 0.75$$
$$= 0.15$$

Hence

$$P(G) = P(E\setminus F) + P(F\setminus E)$$
$$= 0.05 + 0.15$$
$$= 0.20.$$

$\boxed{/\!/}$

§2.6 Sample space having equally likely outcomes.

In many experiments, it is natural to assume that all outcomes have the same chance to occur.

In this case,

$$P(E) = \frac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in } S} = \frac{\# E}{\# S}.$$

Example 3. If two dices are rolled, what is the prob. that the sum of two outcomes is equal to 5 ?

Solution: Let E be the event that the sum of two outcomes is equal to 5. Then

$$E = \{(i,j) : \quad i, j \in \{1, 2, \cdots, 6\}, \ i+j=5\}$$
$$= \{(1,4), (2,3), (3,2), (4,1)\},$$

and

$$S = \{(i,j) : \quad i, j \in \{1, 2, \cdots, 6\}\}$$

Hence $P(E) = \dfrac{\# E}{\# S} = \dfrac{4}{36} = 1/9.$ $\blacksquare$

Example 4.

A committee of 5 is to be selected from a group of 6 men and 9 women. If the
selection is made randomly, what is the probability that the committee consists of 3
men and 2 women?

Solution: Let E denote the event that the selected committee consists of 3 men and 2 women.

Let S be whole sample space.

Then

$$\#S = \binom{15}{5},$$

$$\#E = \binom{6}{3} \cdot \binom{9}{2}.$$

Hence $P(E) = \frac{\#E}{\#S} = \frac{\binom{6}{3}\binom{9}{2}}{\binom{15}{5}}.$

$$\binom{n}{m} = \frac{n!}{m!\,(n-m)!}.$$

$\big(n! = n \times (n-1) \times \cdots 1$
$0! = 1 \big)$