MATH3060 HW5 Due date: Nov 6, 2020 (at 12:00 noon)

1. (Generalization of Contraction Mapping Principle)
   Let X be a complete metric space. And let $T: X \to X$ be a continuous map such that the k-time composition T^k is contraction. Show that T has a unique fixed point.

2. Show that the equation $\cos x + 2x^4 + x = 1.001$ has a solution near $x = 0$.

3. Let A be an $n \times n$ symmetric matrix and $V \in \mathbb{R}^n$. Show that there exist $r > 0$ and $R > 0$ such that $\forall\, y \in \overline{B_R(0)} \subset \mathbb{R}^n$, there exists a unique $x \in \overline{B_r(0)} \subset \mathbb{R}^n$ such that
   $$x = y + (x^T A x) V.$$

4. Let $K(x,t) \in C([0,1] \times [0,1])$. Show that there exists $\lambda > 0$ such that for all $g \in C[0,1]$, there exists a unique solution $y \in C[0,1]$ of the integral equation
   $$y(x) = g(x) + \lambda \int_0^1 K(x,t)\, y(t)\, dt.$$

(End)