P31-3

$(-8-8\sqrt{3}i)^{\frac{1}{4}} = (16e^{(\frac{4}{3}\pi + 2n\pi)i})^{\frac{1}{4}} = 2e^{(\frac{\pi}{3} + \frac{n\pi}{2})i}$

$\Rightarrow n=0: 1+\sqrt{3}i,\ n=1: -\sqrt{3}+i,\ n=2: -1-\sqrt{3}i,\ n=3: \sqrt{3}-i$

P31-4

(a) $(-1)^{\frac{1}{3}} = e^{\frac{1}{3}\log(-1)} = e^{\frac{1}{3}(\ln 1 + (\pi + 2n\pi)i)} = e^{(\frac{\pi}{3} + \frac{2n\pi}{3})i}$

$n=0: \frac{1}{2} + \frac{\sqrt{3}}{2}i,\ n=1: -1,\ n=2: \frac{1}{2} - \frac{\sqrt{3}}{2}i$

(b) $8^{\frac{1}{6}} = e^{\frac{1}{6}\log 8} = e^{\frac{1}{6}(\ln 8 + 2n\pi i)} = \sqrt{2}e^{\frac{n\pi}{3}i}$

$n=0: \sqrt{2},\ n=1: \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i,\ n=2: -\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$

$n=3: -\sqrt{2},\ n=4: -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i,\ n=5: \frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$

P35-4 skipped.

P35-5 skipped.

P95-1

(a) $\mathrm{Log}(-ei) = \cancel{\ln e} \ \mathrm{Log}(e\, e^{-\frac{\pi}{2}i}) = \ln e - \frac{\pi}{2}i = 1 - \frac{\pi}{2}i$

(b) $\mathrm{Log}(1-i) = \ln\sqrt{2} + \mathrm{Log}(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}i$

P95-2

(c) $\log(-1+\sqrt{3}i) = \log(2e^{(\frac{2}{3}\pi + 2n\pi)i}) = \ln 2 + (\frac{2}{3}\pi + 2n\pi)i,\ n \in \mathbb{Z}.$

(a), (b) skipped.

P96-5

(a) $z^2 = i \Rightarrow z^2 = e^{(\frac{\pi}{2} + 2n\pi)i} \Rightarrow z = e^{(\frac{\pi}{4} + n\pi)i}$

$\Rightarrow e^{\frac{\pi}{4}i}$ & $e^{\frac{5}{4}\pi i}$

$\log(e^{\frac{\pi}{4}i}) = \ln 1 + (\frac{\pi}{4} + 2n\pi)i = (\frac{\pi}{4} + 2n\pi)i = (\frac{\pi}{4} + m\pi)i,\ m$ even.

$\log(e^{\frac{5}{4}\pi i}) = \ln 1 + (\frac{5}{4}\pi + 2n\pi)i = ((2n+1) + \frac{1}{4})\pi i$, $n \in \mathbb{Z}$. $(m + \frac{1}{4})\pi i$, m odd

$\Rightarrow \log(i^{\frac{1}{2}}) = (n + \frac{1}{4})\pi i$, $n \in \mathbb{Z}$

(b) $\frac{1}{2}\log i = \frac{1}{2}(\frac{\pi}{2} + 2n\pi)i = (\frac{1}{4} + n)\pi i$

P103-1

(a) $(1+i)^i = e^{i \log(1+i)} = e^{i(\ln\sqrt{2} + (\frac{\pi}{4} + 2n\pi)i)} = e^{-\frac{\pi}{4} + 2n\pi} e^{\frac{\ln 2}{2} i}$, $n \in \mathbb{Z}$.

(b) $\frac{1}{i^{2i}} = \frac{1}{e^{2i \log i}} = \frac{1}{e^{2i(\frac{\pi}{2} + 2n\pi)i}} = 1/e^{-(\pi + 4n\pi)} = e^{(4n+1)\pi}$, $n \in \mathbb{Z}$.

P103-2

(a) $(-i)^i = e^{i \log(-i)} = e^{i(-\frac{\pi}{2} + 2n\pi)i} = e^{\frac{\pi}{2} - 2n\pi} \Rightarrow n = 0 : e^{\frac{\pi}{2}}$

(b) $[\frac{e}{2}(-1-\sqrt{3}i)]^{3\pi i} = \exp(3\pi i \log(\frac{e}{2}(-1-\sqrt{3}i)))$

$= \exp(3\pi i(\ln e + (-\frac{2}{3}\pi + 2n\pi)i))$

$= \exp(3\pi i + 2\pi^2 - 6n\pi^2)$

$\Rightarrow n = 0 : \exp(3\pi i + 2\pi^2) = -e^{2\pi^2}$

(c) $(1-i)^{4i} = \exp(4i \log(1-i)) = \exp(4i(\ln\sqrt{2} + (-\frac{\pi}{4} + 2n\pi)i))$

$= \exp(2\ln 2\, i + \pi - 8n\pi)$

$\Rightarrow n = 0 : \exp(2\ln 2\, i + \pi) = e^{\pi}(\cos(2\ln 2) + \sin(2\ln 2)i)$

P103-6

$|z^a| = \cancel{\exp(a\ln|z|)}\ |e^{a \log z}| = |e^{a(\ln|z| + (\Theta + 2\pi n)i)}|$

$= |z|^a$

P112-16 (Not following the step in the question.)

(a) $\sinh z = \frac{e^z - e^{-z}}{2} = i \Rightarrow e^{2z} - 1 = 2ie^z \Leftrightarrow (e^z - i)^2 = 0$

$\Rightarrow e^z = i \Rightarrow z = (\frac{1}{2} + 2n)\pi i$

(b) $\cosh z = \frac{1}{2} \Rightarrow e^z + e^{-z} = 1 \Rightarrow e^{2z} + 1 = e^z \Rightarrow (e^z - \frac{1}{2})^2 = -\frac{3}{4}$

$\Rightarrow e^z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i \Rightarrow \begin{cases} e^z = e^{(\frac{\pi}{3} + 2n\pi)i} \Rightarrow z = (\frac{\pi}{3} + 2n\pi)i \\ e^z = e^{(-\frac{\pi}{3} + 2n\pi)i} \Rightarrow z = (-\frac{\pi}{3} + 2n\pi)i \end{cases}$

P112 - 17

$\cosh z = \frac{e^z + e^{-z}}{2} = \cancel{0}\ -2 \Rightarrow e^{2z} + 1 = -4e^z \Rightarrow (e^z + 2)^2 = 3 \Rightarrow e^z = -2 \pm \sqrt{3}$

$\Rightarrow \begin{cases} \cancel{e^{x+yi} = -2+\sqrt{3}} \Rightarrow & z = \log(-2+\sqrt{3}) = \ln(2-\sqrt{3}) + (2n+1)\pi i \\ \cancel{e^{x+yi} = -2-\sqrt{3}} & z = \log(-2-\sqrt{3}) = \ln(2+\sqrt{3}) + (2n+1)\pi i \end{cases}$

Notice that $(2-\sqrt{3})(2+\sqrt{3}) = 1$, $\ln(2-\sqrt{3}) = -\ln(2+\sqrt{3})$

$\Rightarrow z = \pm \ln(2+\sqrt{3}) + (2n+1)\pi i$