MATH2230B
Complex Variables with Applications

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Definition

Let f be a function defined on an open set $\Omega \subset \mathbb{C}$. f is called analytic at a point $z_0 \in \Omega$ if f is differentiable on a neighborhood $B_\varepsilon(z_0) \subset \Omega$ for some $\varepsilon > 0$. If f is differentiable at every point in Ω, then we also called f is analytic on Ω. Moreover, if f is analytic on $\mathbb{C}$, we call f an entire function.

Example

$f(z) = 1/z$ is differentiable on $\mathbb{C}\backslash\{0\}$with $f'(z) = -1/z^2$ for $z \neq 0$. So f is analytic on $\mathbb{C}\backslash\{0\}$. $g(z) = |z|^2$ is differentiable only at $z = 0$. Thus, g is not analytic anywhere. Finally, we have that every polynomial is an entire function.

Theorem

If $f'(z) = 0$ on an open connected set Ω, then f is a constant on Ω.

Lemma

If an open set Ω is connected, then it is polygonally connected. That is, for any $z_1, z_2 \in \Omega$, z_1 and z_2 can be connected by a polygonal line consisting of finitely many line segments in Ω.