

MATH 2050A - HW 7

Due Date: 17 Nov 2020, 23:59

*You are reminded that your HW is graded
based on **both** your idea and your presentation*

Problems: P.134: 7, 15

(2 Questions in total)

Textbook: Bartle RG, Sherbert DR(2011). Introduction to Real Analysis, fourth edition, John Wiley Sons,Inc.

We type here all the required problems *for your convenience only*. The presentation of the problems here may be different from the original one but the respective solution should be unaffected.

1 (P.134 Q7). Give an example of a function $f : [0, 1] \rightarrow \mathbb{R}$ such that f is discontinuous at every point of $[0, 1]$, but $|f|$ is continuous on $[0, 1]$.

2 (P.134 Q15). Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be continuous at a point $c \in \mathbb{R}$. For all $x \in \mathbb{R}$, define $h(x) := \sup\{f(x), g(x)\}$. Show that the function $h : \mathbb{R} \rightarrow \mathbb{R}$ satisfies that

i. $h(x) = \frac{1}{2}(f(x) + g(x)) + \frac{1}{2}|f(x) - g(x)|$ for all $x \in \mathbb{R}$

ii. Hence, h is continuous at c .