MATH 2050A - HW 6
Due Date: 10 Nov 2020, 23:59
*You are reminded that your HW is graded
based on **both** your idea and your presentation*

Problems: P.123: 9, 13; P.129: 4a, 10 (4 Questions in total)

Textbook: Bartle RG, Sherbert DR(2011). Introduction to Real Analysis, fourth edition, John Wiley Sons,Inc.

We type here all the required problems *for your convenience only.* The presentation of the problems here may be different from the original one but the respective solution should be unaffected.

1 (P.123 Q9). Let $a \in \mathbb{R}$. Let $f : (a, \infty) \to \mathbb{R}$ be a function such that $L := \lim_{x \to \infty} xf(x) \in \mathbb{R}$. Show that $\lim_{x \to \infty} f(x) = 0$

2 (P.123 Q13). Let $a \in \mathbb{R}$. Let f, g be defined on (a, ∞). Suppose $\lim_{x \to \infty} f = L$ and $\lim_{x \to \infty} g = \infty$ where $L \in \mathbb{R}$. Show that $\lim_{x \to \infty} f \circ g = L$.

3 (P.129 Q4a). Let $x \in \mathbb{R}$. Define $\lfloor x \rfloor$ to be the greatest integer $n \in \mathbb{Z}$ such that $n \leq x$, for example, $\lfloor \pi \rfloor = 3, \lfloor -\pi \rfloor = -4$. We call $x \mapsto \lfloor x \rfloor$ the floor function, which is defined on $\mathbb{R}$.

  i Determine the points of continuity of $f(x) = \lfloor x \rfloor$. *(Do not forget to prove your assertion).*

  ii **Optional (+1 Bonus):** Show that the floor function is well-defined.

4 (P.129 Q10). Show that the absolute value function $f(x) = |x|$ is continuous everywhere on $\mathbb{R}$.