MATH 2050A - HW 3
Due Date: 6 Oct 2020, 23:59
(Please submit assignments to Blackboard and follow the instructions there.)

Problems: P.69 Q9, 20, 22 (2 Questions in total)

Textbook: Bartle RG, Sherbert DR(2011). Introduction to Real Analysis, fourth edition, John Wiley Sons,Inc.

We type here all the required problems *for your convenience only.* The presentation of the problems here may be different from the original one but the respective solution should be unaffected.

1 (P.69 Q9)**.** Let $y_n := \sqrt{n+1} - \sqrt{n}$ for all $n \in \mathbb{N}$. Show that $(\sqrt{n}y_n)$ converges and find the limit.

2 (P.69 Q20)**.** Let (x_n) be a sequence of positive real numbers such that $L := \lim(x_n^{1/n}) < 1$.

i. Show that there exists a real number $r \in (0, 1)$ such that $x_n \in (0, r^n)$ for all sufficiently large $n \in \mathbb{N}$.

ii. Hence, show that $\lim x_n = 0$.

3 (P.69 Q22)**.** Let (x_n) be a covergent sequence of real numbers and (y_n) be such that for all $\epsilon > 0$ there exists $M \in \mathbb{N}$ such that $|x_n - y_n| < \epsilon$ for all $n \geq M$. Does it follow that (y_n) is convergent?