MATH 2050A - HW 2 - Comments and Common Mistakes:

General Comments:

1. Q1 carries 6 marks; Q2 carries 3 marks; 1 mark is for presentation and effort.

2. Please be reminded to change the filename of the PDF. That can save me lots of time when uploading the graded assignment. Thank you very much. If you have done so, thank you and please keep doing so in the future.

3. This assignment tests your understanding mainly on the Archimedean Property and the $\epsilon - N$ definition of sequences convergence. These involve a lot of predicate quantifiers (the universal quantifier $\forall$ and the existential quantifier $\exists$) and hence a lot of *variables*. A rule of thumb is that you should never use a variable in a statement if it is not declared as an element of a certain set, in particular, if it is not defined or whose range is not specified. For instance, when proving the convergence of some sequence, some students start immediately by writing *"By Archimedean's Property, there exists natural number $N > 1/\epsilon$"*. Here, ϵ is not yet declared. Sentences like *"Let* $\epsilon > 0$ should be written first.

4. In addition to the introduction of more variables, there are also more *types* of variables, like sets, real numbers and sequences. You should always be aware of the type whenever you use a variable. Normally you cannot assert equality between different types of variables. In addition, some operations are only okay for some types of variables. For example it is not okay to say *Let* $x = (x_n) = 1/n \in \mathbb{R}$, in which x represents a sequence of real numbers, not a real number. Write instead *Let $x = (x_n)$ be a sequence where $x_n = 1/n \in \mathbb{R}$ for all $n \in \mathbb{N}$*. You cannot take limit on sets as well.

Common Misktakes:

Question 1:

1. The marking scheme of Question one is simple: 1.5 points for each limit in which each of the following is 0.5 point worth
   (a) You have used the Archimedean Property to find an $N \in \mathbb{N}$ for every $\epsilon > 0$
   (b) You have considered the *eventual behaviour* of the sequence. For example, you have written something like *when* $n \geq N$*,...*
   (c) You have considered the distance of elements of the sequence with the limit point and showed it to be less than ϵ. For example, you have said $|x_n - L| \leq \cdots <$

   Missing any of these will have a large chance of losing points.

2. As a practice of giving rigorous justification, I expected you to justify all your statements as clearly as possible. Therefore, I do not accept if you use the Archimedean's Property without explicitly mentioning the term (at least in this HW in which this property is one of the subjects.), that is, marks will be deducted if you directly mention the existence of some natural N with $N > 1/\epsilon$ without things like *By Archimedean Property.*

3. When finding the $N \in \mathbb{N}$ after which the elements of the sequence are within $\epsilon-$ unit from the limit, some students write take $N = 1/\epsilon$. This is not ok! A sequence is always indexed by *natural numbers* and you do not quarantee if $1/\epsilon$ is one: this indeed emphasizes the real power of the Archimedean Property, which allows you to find one N greater than $1/\epsilon$ always when given $\epsilon > 0$, so you do not have to worry the exact identity of N.

4. Before using the Archimedean Property, some students had chosen to verify, for example, $1/\epsilon$, is positive. This is un-ncecessary. The Archimedean Property allows you to bound above *any* given real numbers with some natural numbers. Indeed if the real number is negative, any natural number will do.

Question 2a:

1. Your performance in this question is great in general.

2. Some students conclude the equivalence of $\lim_n x_n = 0$ and $\lim_n |x_n| = 0$ by simply writing *because* $|x_n - 0| = ||x_n| - 0|$ *for all* $n \in \mathbb{N}$
   Still the point: I expected you to practise writing the $\epsilon - N$ definition of a limit, so some points will be deducted if you do not write even *"let* $\epsilon > 0$*"*. Indeed, the convergence of a sequence depends only on its final behaviour of a sequence. It is a good practise to emphasize that identities are true for $n \geq N$ for some $N \in \mathbb{N}$ instead of simply every $n \in \mathbb{N}$ though the latter gives valid proofs as well.

Question 2b:

1. Nearly all of you used the example $(x_n) = (-1)^n$. That is good as this is both an easy and standard example.

2. I expected you to give a proof to show *why* your chosen sequence is divergent. The absence of such a proof will lead to mark deduction.

3. Some students prove the divergence of $(x_n) = (-1)^n$ by claiming that (x_{2n}) and (x_{2n-1}) converge to different value. This is in fact possible *only after you have learnt* ***that subsequences of a convergent sequence should converge to the same limit.*** It is not (mainly) because of the uniqueness of limit as (x_{2n}) and (x_{2n-1}) are two *different* sequences. (You may see my Solution on how to prove the divergence using this approach.)

4. Following the above point some students write the limit of the "subsequences" as "$\lim_n x_n = 1$ *for even n*". This is not ok. You should note that *taking limit* is an *operation* on *sequences*. So $\lim x_n$ makes sense even without declaring what n is. There is nothing to declare: the notion (x_n) represents a legitimate object, a sequence and the lim operation acts on (x_n)