MATH 2050A - HW 1 - Comments and Common Mistakes:

General Comments:

1. Each question carries 3 marks. 1 mark is given as long as we see your effort.
2. I would be very grateful if you can try the following in your HW Submission
   (a) Write down your name and student id in your homework
   (b) Include your name and student id in your pdf's filename as well.
   (c) If your work is hand-written, try those scanner apps that turn your mobile phone into a scanner. This can drastically enhance the readability of your work.
   (d) Adopt suitable paragraphing and do not write in a rush.
3. The grade of this assignment depends on your familiarity with the basic definitions concerning $\mathbb{R}$, namely set boundedness and supremum (infimum), you have shown in your work. You are expected to show *detailed arguments* using (equivalence) definitions and theorems you have learnt in Lectures. For example, when you prove identities concerning supremums, you should prove one-side to be an upper bound and *then* the least upper bound. Or, you prove inequalities for *2 sides* and apply the partial ordering of $\mathbb{R}$.
4. As one of the beginning courses emphasizing the rigor of your argument, you may find some of the statement very intuitive or even obvious. Nonetheless, please *be patient and work out your argument step-by-step.* As a rule of thumb, please try to (a) ensure every line follows from the line before. (b) If the reason of the line is here but is far above, please recall it or use some ways to cite it. (c) Proofread at least once so that you can ensure at least some people (you) understand what you have written. (d) Please avoid using the 3-dot symbols $\because$ and $\therefore$ and replace them with words. They are often abused. Meanwhile, writing words can make your argument look more formal (and readable) so that you would pay more attention to the structure and possible loopholes of your argument.

   Sometimes, after a careful deduction, what you found intuitive/obvious/trivial may become highly non-trivial.

Common Misktakes:

Question 1:

1. Some students are still not familiar with handling variables in arguments. For example, S_4 is a set. Please do not write something like $1/2 \leq S_4 \leq 2$. If you want to address elements in S_4, declare a notation, by saying like *let* $x \in S_4$ or *define* $x_n := 1 - (-1)^n/n$ *for* $n \in \mathbb{N}$.

2. Some students stop after showing $1/2 \leq y \leq 2$ for all $y \in S_4$. Please remember supremum and infimnum are stronger definitions than upper bounds and lower bounds.

Question 2:

1. A number of students have problem showing $A \cup B$ is a bounded set. They argue like

   For all $x \in A \cup B$, *either* $x \in A$ *or* $x \in B$, *so either* $\inf A \leq x \leq \sup A$ *or* $\inf B \leq x \leq \sup B$. *Since in any case,* $A \cup B$ *is bounded so* $A \cup B$ *is bounded.*

   This is wrong: having different bounds at different moments does NOT mean there is a common bound for all cases; that common bound is what is needed to prove the boundedness of $A \cup B$. The fallacy becomes clear when we consider infinite union. Just take $A := \bigcup_n [n, n+1]$. The individual intervals are bounded below and above by different bounds but there is no common upper bound for them. In fact, A is not bounded above.

2. Some students forget to show $A \cup B$ is bounded below. Please remember bounded sets are both bounded above and below.

Question 3:

1. There is a tricky part in this question: the conclusion of this question is very similar to that of Q10. Nonetheless, in Q10 the sets A, B are assumed to be bounded while in Q12 the sets are only bounded above. Therefore, you cannot use Q10 directly. If you want to use Q10, you may say that the proof in Q10 follows as well if we only assume bounded above. Some students do not give justifications and so marks are deducted.

2. It seems to me some students are tempted by the simplicity of the statement of the question: now one of the set $\{u\}$ is just a singleton. So a number of students seem to have chosen to skip steps and drop the rigor of their arguments. That is not recommended and will lead to mark deduction. For instance, I expected you to say something to verify $\sup\{u\} = u$ like, at least, *since* u *is the maximum* instead of merely saying it is obvious (even though I did not deduct marks from you for this identity).

3. Some students write that since $u \notin S$, u is either an upper bound or lower bound of S. This is NOT true and is easy to be countered by taking S to be any subset with 2 elements.