Interference-safe CSMA Networks by Local Aggregate Interference Power Measurement

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***Abstract*—Most current wireless IEEE 802.11 networks rely on a power-threshold based carrier-sensing multi-access (CSMA) mechanism to prevent packet collisions, in which a transmitter permits its transmission only if the locally measured *aggregate interference power* from all existing transmissions is below a pre-specified power-sensing threshold. However, such a mechanism can not completely guarantee interference-safe transmissions, leading to the so-called hidden-node problem, which causes degradation in throughput and fairness performance. Traditionally, ensuring interference-safe transmissions was addressed by simple models of conflict graphs, rather than by the realistic signal-to-interference-and-noise ratio (SINR) model. This paper presents the first viable solution for fully interference-safe transmissions that (1) assumes an accurate SINR model, and (2) is compatible with the carrier-sensing mechanism in existing CSMA networks. Specifically, we determine a proper interference-safe power-sensing threshold by considering both the effects of (*i*) arbitrary ordering of local interference power measurements, and (*ii*) ACK frames. We compare our interference-safe solution with other solutions, and provide extensive evaluation on its throughput and fairness performance.**

***Index Terms*—Carrier-Sensing Multi-Access, Interference-Safeness, SINR Model, Interference Power Measurement**

I. Introduction

Carrier-sensing multi-access (CSMA) networks (e.g., Wi-Fi), a class of distributed and randomized medium-access protocols, are widely deployed in enterprises and homes. Despite its simplicity of implementation, CSMA is plagued with performance issues. Substantial improvements have been proposed to improve the efficiency of CSMA networks with higher capacity and lower control overhead. However, many previous studies are either 1) based on an over-simplified model that relies on a simple notion of "conflict graphs" and ignores the physical layer characteristic of signal-to-interference-and-noise ratio (SINR) in wireless communications, or 2) not compatible with the distributed carrier-sensing mechanism in today's CSMA protocol. As a result, the effectiveness of these proposed solutions remains dubious. This paper focuses on an important performance problem in CSMA networks, caused by failure of CSMA protocol to ensure *interference-safe* transmissions among simultaneously transmitting links (also known as hidden node problem). Previous studies have addressed interference-safe transmissions and hidden node problem are based on "conflict graph" models for the interference and carrier-sensing operations, which are crude approximations to the more realistic physical-layer wireless communication model.

This paper presents a viable solution that (1) uses an accurate SINR model, considering the effects of arbitrary ordering of carrier-sensing operations, and the presence of ACK frames; and (2) is compatible with the existing CSMA networks, relying on only basic power-threshold based carrier-sensing operations. Our results close a vital gap in enhancing the performance of practical CSMA in a realistic setting.

A. Overview of Results

The idea of CSMA is that before a transmitter attempts its transmission, it needs to infer the channel condition. If the transmitter infers that its transmission is not interference-safe, namely possibly upsetting (or be upset by) any on-going transmissions, then it defers its transmission. A common approach in the extant CSMA protocol (which we call *Aggregate-interference-Power Carrier-Sensing* (APCS)) is to let the transmitter measure the aggregate interference power – the total power of all concurrent transmissions and the background noise at the pending transmitter. A transmission will proceed only if the locally measured aggregate interference power is below a pre-specified *power-sensing threshold*.

It is vital to set the power-sensing threshold properly. An improperly high threshold fails to safe-guard interference, leading to the hidden node problem [1], whereas an improperly low threshold causes over-conservative protection against interference and inefficiency in throughput, leading to the exposed node problem [1]. The difficulty to determine a proper threshold is due to two salient effects of CSMA:

1) (*Effect of Ordering*): CSMA is a distributed protocol, such that the transmitters decide their transmissions without global coordination, using APCS before each transmission. However, the transmitters may follow an arbitrary order, and this introduces a unique challenge in guaranteeing interference-safe transmission that has not been studied before. Specifically, an earlier transmitter that measured low interference power before the transmission may be disrupted by a later transmitter that causes unforeseen higher interference power to it. A proper power-sensing threshold should tolerate arbitrary ordering of local measurements of transmitters, by which the local interference power may rise in the future without the on-going transmitter's awareness. To our best knowledge, all past works on ensuring interference-safe transmission (or removing hidden-nodes) in CSMA networks have ignored this subtle yet important effect.
2) (*Effect of ACKs*): CSMA in wireless networks is often an ACK-based protocol, in which the receivers are required to reply an ACK frame for each successful transmission. Hence, the power-sensing threshold not only must ensure

the successful receptions of DATA frames in one direction, but also the successful receptions of ACK frames in the opposite direction in the presence of other interfering transmitters. The consideration of bi-directional communications in terms of SINR will complicate the determination of power-sensing threshold.

Furthermore, we provide performance evaluation of CSMA networks with APCS, and show that our result is a viable approach. We also observe that the power-sensing threshold provided by our theoretical study is relatively robust in spite of uncertain parameters from the channel model.

B. Comparison to Related Work

Ensuring interference-safe transmissions has been addressed in the extant literature (e.g., [1]–[4]), in which a CSMA network was modeled by a "conflict graph" that is induced by a geometric graph based on the transmitters and receivers. The conflict graph model relies on the binary constraint among pairs of transmitters, which does not consider the additive property of wireless signals. Hence, it calls for a more realistic model with signal-to-interference-and-noise ratio (SINR). The study of hidden node problem in an SINR model has begun recently. Ref. [5] found that the common CTS/RTS mechanism, which relies on the assumption that the decodable range of a CTS/RTS message is comparable to the interfering range, is not sufficient to ensure interference-safe transmissions in an SINR model, because the sum of individually insignificant interference power can still be considerably large in an SINR model, and hence, a transmitter can affect very far-off nodes, other than those that can decode its packets. Two previous studies akin to this work are our prior work [6], [7]. In [6], we proved the scaling law of capacity of CSMA in an SINR model. But the result relies on a simplified model of power-threshold based carrier-sensing, which ignores the effect of ordering of local measurements. This work extends that study to consider the effect of ordering in CSMA.

Previously, [7] proposed an alternate approach called Incremental-Power Carrier-Sensing (IPCS), which ensures interference-safe transmissions under an SINR model by inferring the distances among transmitters with local measurement of incremental power changes at each node. Specifically, in IPCS, a transmitter will defer transmission if the incremental power change measured recently is above a power threshold. IPCS requires modification to the existing standard CSMA protocol, because the carrier-sensing mechanism of the latter compares the current absolute power measured (not incremental power changes recently) against a threshold. In other words, IPCS is not an off-the-shelf solution to the hidden node problem. In contrast, this work is based on the simpler idea of Aggregate-interference-Power Carrier-Sensing (APCS), which requires no modification to the existing CSMA protocol, because it also compares the absolute power measured against a threshold. We compare the two carrier-sensing approaches by evaluation and analysis. Note that for benchmarking IPCS with standard carrier sensing, [7] also used the same power-sensing threshold optimized for IPCS for standard carrier sensing. However, the power-sensing threshold for IPCS may not be optimal for standard carrier sensing. In this paper, we provide a power-sensing threshold appropriate for standard carrier sensing. Our solution of APCS can be shown to have comparable performance with IPCS.

There are other studies [1], [8] to address a related problem of exposed nodes. We note that there is an inevitable trade-off between hidden node problem and exposed node problem [1], as a solution addressing one of the problems often causes the other problem. Since there is a lack of thorough study for hidden node problem in a realistic setting of SINR model, it is difficult to properly address both problems simultaneously. Our results in this paper solve the former problem and provide a cornerstone for a complete solution for both problems in an SINR model.

II. Motivating Examples

We first illustrate the effects of ACK frames and ordering of local measurements in SINR model, through some motivating examples. Consider Fig. 1 with three transmitter-receiver pairs with the same transmission distance d arranged in parallel.

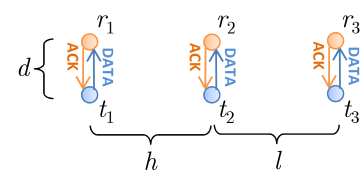

Fig. 1. Three transmitter-receiver pairs arranged in parallel.

The signal-to-interference-and-noise ratio (SINR) from node x to node y in the presence of a set of concurrent transmitters $\mathcal{N}$ is defined as:

$$\mathrm{SINR}_{x \to y}^{\mathcal{N}} \triangleq \frac{\mathsf{P}_{\mathsf{tx}} |x-y|^{-\alpha}}{\mathsf{N}_0 + \sum_{z \in \mathcal{N}} \mathsf{P}_{\mathsf{tx}} |z-y|^{-\alpha}} \tag{1}$$

where P_{tx} is the transmission power, N_0 is a background noise and α is the path-loss exponent.

A receiver can successfully receive the data from its transmitter, if the SINR at the receiver is above a certain threshold β. For instance, if r_1 can receive data from t_1 in the presence of a set of concurrent transmitters $\{t_2, t_3\}$, it must satisfy:

$$\mathrm{SINR}_{t_1 \to r_1}^{\{t_2,t_3\}} = \frac{\mathsf{P}_{\mathsf{tx}} |t_1 - r_1|^{-\alpha}}{\mathsf{N}_0 + \mathsf{P}_{\mathsf{tx}} |t_2 - r_1|^{-\alpha} + \mathsf{P}_{\mathsf{tx}} |t_3 - r_1|^{-\alpha}} \geq \beta$$

For simplicity, we let $\mathsf{P}_{\mathsf{tx}} = 1$, $\mathsf{N}_0 = 0$, $\alpha = 2$ and $\beta = 1$.

Example 1: (*Effect of ACKs*) Let $h = l = 1.2d$ in Fig. 1. We obtain the SINRs at the receivers as:

$$\begin{aligned} &\mathrm{SINR}_{t_1 \to r_1}^{\{t_2,t_3\}} = \mathrm{SINR}_{t_3 \to r_3}^{\{t_1,t_2\}} = \tfrac{1}{(2.6)^{-2} + (1.56)^{-2}} = 1.79, \\ &\mathrm{SINR}_{t_2 \to r_2}^{\{t_1,t_3\}} = \tfrac{1}{(1.56)^{-2} + (1.56)^{-2}} = 1.22 \end{aligned} \tag{2}$$

Since $\beta = 1$, it appears that all receivers can receive the data from their transmitters in spite of interfering transmissions.

However, CSMA protocol also requires the transmission of an ACK frame after the transmission of DATA frame to confirm successful reception. Otherwise, the transmitter will take it as a failed transmission, and re-transmit data later. If r_2 begins to transmit ACK to t_2, the SINR at t_2 becomes:

$$\mathrm{SINR}_{r_2 \to t_2}^{\{t_1,t_3\}} = \tfrac{1}{(1.2)^{-2} + (1.2)^{-2}} = 0.72 \tag{3}$$

The reverse transmission $r_2 \to t_2$ is not successful, and hence the transmission $t_2 \to r_2$ is considered unsuccessful by t_2.

In general, we have to consider the interference from all possible transmitters (for DATA frames) and receivers (for ACK frames). For instance, if t_1 and r_3 are transmitting,

$$\mathrm{SINR}_{t_2 \to r_2}^{\{t_1, r_3\}} = \frac{1}{(1.56)^{-2} + (1.2)^{-2}} = 0.9 \tag{4}$$

Hence, r_2 is also unable to receive DATA frame from t_2 in the presence of interfering t_1 and r_3. ■

To prevent collisions, CSMA relies on local interference power measurement. The *locally measured interference-and-noise power* from the set of concurrent transmitters $\mathcal{N}$ and the background noise obtained at transmitter x before the transmission is denoted by:

$$\mathbf{P}_x[\mathcal{N}] \triangleq \mathsf{N}_0 + \sum_{z \in \mathcal{N}} \mathsf{P}_{\mathsf{tx}} |z - x|^{-\alpha} \tag{5}$$

By Aggregate-interference-Power Carrier-Sensing (APCS), transmitter x proceeds with transmission, only if $\mathbf{P}_x[\mathcal{N}] \le \mathsf{t}_{\mathsf{cs}}$ for a pre-specified power-sensing threshold t_{cs}.

In Example 1, transmission $t_2 \leftrightarrow r_2$ is unsuccessful, whenever two other transmissions ($t_1 \leftrightarrow r_1$ and $t_3 \leftrightarrow r_3$) are present. To preclude this from happening, one can set:

$$\begin{aligned} \mathsf{t}_{\mathsf{cs}} = \min \Big\{ & \mathbf{P}_{t_1}[\{r_2, r_3\}], \mathbf{P}_{t_1}[\{t_2, r_3\}], \mathbf{P}_{t_1}[\{t_2, t_3\}], \\ & \mathbf{P}_{t_1}[\{r_2, t_3\}], \mathbf{P}_{t_2}[\{r_1, r_3\}], \mathbf{P}_{t_2}[\{t_1, r_3\}], \\ & \mathbf{P}_{t_2}[\{t_1, t_3\}], \mathbf{P}_{t_2}[\{r_1, t_3\}] \Big\} - \epsilon \\ = \mathbf{P}_{t_1}&[\{r_2, r_3\}] - \epsilon = 0.56 - \epsilon \end{aligned}$$

for some small ϵ. Then, any transmitter among t_1, t_2, t_3 is able to infer if other two transmissions are present or not.

But such a setting requires the knowledge of network topology. This poses a question that how we can determine a proper power-sensing threshold in APCS to ensure inteference-safe transmissions without the full knowledge of network topology. The setting of power-sensing threshold should require minimum a-priori information and be applicable to universal topologies.

Example 2: (*Effect of Ordering*) We consider the scenario that with a given power-sensing threshold t_{cs}, we can rearrange the network topology. In fact, whatever given value of t_{cs} for Fig. 1, there always exist values of h, l, such that the local interference power at a transmitter can exceed t_{cs} unawarely. For a given t_{cs}, we set h, l such that they satisfy:

$$h^{-\alpha} = \mathsf{t}_{\mathsf{cs}}, \quad l^{-\alpha} + (h + l)^{-\alpha} = \mathsf{t}_{\mathsf{cs}} \tag{6}$$

We suppose that the transmitters follows the sequence of local measurements as in Fig. 2.

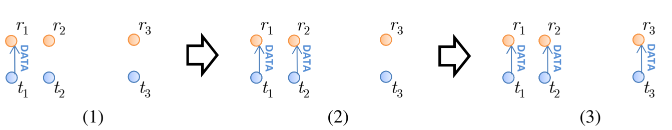

Fig. 2. A sequence of local interference power measurements.

Initially, there is no transmission. First, t_1 measures the interference power as $\mathbf{P}_{t_1}[\varnothing] = 0 \le \mathsf{t}_{\mathsf{cs}}$, and proceeds with transmission. Next, t_2 measures the interference power as $\mathbf{P}_{t_2}[\{t_1\}] = \mathsf{t}_{\mathsf{cs}}$, and proceeds with transmission. Lastly, t_3 measures the interference power as $\mathbf{P}_{t_3}[\{t_1, t_2\}] = \mathsf{t}_{\mathsf{cs}}$, and also proceeds with transmission. However, after t_3's transmission, the interference power at t_2 rises to $\mathbf{P}_{t_2}[\{t_1, t_3\}] > \mathbf{P}_{t_2}[\{t_1\}] = \mathsf{t}_{\mathsf{cs}}$, which prevents inteference-safe transmissions. This highlights the situation where an early transmitter (t_2) may be unaware that a later transmitter (t_3) can increase its interference power in the course of its transmission, even though the later transmitter (t_3) ensures its locally measured aggregate interference power to be below t_{cs}. ■

Based on Example 2, the ordering effect always happens regardless of the value of power-sensing threshold t_{cs}. However, depending on the value of t_{cs}, such effect may or may not prevent inteference-safe transmissions. This raises the question that how we can set a proper power-sensing threshold to ensure inteference-safe transmissions despite of the ordering effect.

III. Model and Notations

Before presenting the results for a proper power-sensing threshold to avoid the problems in Examples 1-2, we provide some formal definitions. Consider a dense network setting with a set of links X. For $i \in X$, we denote t_i as the transmitter, and r_i as the receiver. Note that a receiver may be associated with more than one transmitters, and some receiver may be also a transmitter in another link. We also write t_i and r_i as the respective coordinates. Let

$$\mathsf{dist}(i, j) \triangleq \min(|t_j - r_i|, |r_j - t_i|, |r_j - r_i|, |t_j - t_i|) \tag{7}$$

which is the minimum distance among the transmitters and receivers between a pair of links i, j.

To capture the feasible states that are interference-safe in the presence of possible ACKs, we define the following feasible family of subsets of concurrently transmitting links.

Definition 1: (*Bi-directional interference-safe feasible family* $\mathscr{B}^{\mathsf{ag}}_{\mathsf{N}_0, \mathsf{P}_{\mathsf{tx}}}[X, \beta] \subseteq 2^X$): A subset of links S are interference-safe ($S \in \mathscr{B}^{\mathsf{ag}}_{\mathsf{N}_0, \mathsf{P}_{\mathsf{tx}}}[X, \beta]$), if and only if for all $i \in S$,

$$\frac{\mathsf{P}_{\mathsf{tx}} |t_i - r_i|^{-\alpha}}{\mathsf{N}_0 + \sum_{j \in S \setminus \{i\}} \mathsf{P}_{\mathsf{tx}} \cdot \mathsf{dist}(i, j)^{-\alpha}} \ge \beta \tag{8}$$

$\mathscr{B}^{\mathsf{ag}}_{\mathsf{N}_0, \mathsf{P}_{\mathsf{tx}}}[X, \beta]$ generalizes the commonly used feasibility condition in SINR model for uni-directional communications. CSMA is supposed to enable the links in X to operate within the constraint of $\mathscr{B}^{\mathsf{ag}}_{\mathsf{N}_0, \mathsf{P}_{\mathsf{tx}}}[X, \beta]$, requiring no coordination among the links, for instance by only local interference power measurement. That is, if the subsets of links allowed to transmit simultaneously by a carrier-sensing mechanism are always within the feasible family $\mathscr{B}^{\mathsf{ag}}_{\mathsf{N}_0, \mathsf{P}_{\mathsf{tx}}}[X, \beta]$, then the carrier sensing mechanism is said to be hidden-node free. To model APCS, a simple approach is to consider the following feasible family as in [6].

Definition 2: (*Simple aggregate carrier-sensing feasible family* $\mathscr{C}^{\mathsf{simp}}_{\mathsf{N}_0, \mathsf{P}_{\mathsf{tx}}}[X, \mathsf{t}_{\mathsf{cs}}] \subseteq 2^X$): A subset of links S are permitted by carrier-sensing ($S \in \mathscr{C}^{\mathsf{simp}}_{\mathsf{N}_0, \mathsf{P}_{\mathsf{tx}}}[X, \mathsf{t}_{\mathsf{cs}}]$), if and only if for all $i \in S$,

$$\mathsf{N}_0 + \sum_{j\in S} \mathsf{P}_{\rm tx} |t_j - t_i|^{-\alpha} \le \mathsf{t}_{\rm cs} \tag{9}$$

"Aggregate" carrier-sensing refers to sensing the aggregate interference power in APCS, rather than the pairwise interference power as in IPCS. Hidden node problem can be resolved, if we can find a proper power-sensing threshold $\mathsf{t}_{\rm cs}$ such that

$$\mathscr{C}^{\sf simp}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \mathsf{t}_{\rm cs}\big] \subseteq \mathscr{B}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \beta\big] \tag{10}$$

However, $\mathscr{C}^{\sf simp}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \mathsf{t}_{\rm cs}\big]$ does not take into account the ordering of local measurements, which can cause under-estimation of interference by the transmitters that started to transmit earlier. More accurately, a feasible state in CSMA with local interference power measurement is characterized by a sequence of transmitters, instead of a subset of concurrent links. Thus, we define a more appropriate notion of feasible family for APCS as follows.

Definition 3: (*Aggregate carrier-sensing feasible family* $\mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \mathsf{t}_{\rm cs}\big]$): We write $\mathcal{S}$ as a sequence $(i_1, ..., i_{|\mathcal{S}|})$ and each $i_k \in X$. A sequence of transmitters $\mathcal{S}$ is permitted by carrier-sensing ($\mathcal{S} \in \mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \mathsf{t}_{\rm cs}\big]$), if and only if for all i_k,

$$\mathsf{N}_0 + \sum_{i_j \in \{i_1,...,i_{k-1}\}} \mathsf{P}_{\rm tx} |t_{i_j} - t_{i_k}|^{-\alpha} \le \mathsf{t}_{\rm cs} \tag{11}$$

That is, when each transmitter i_k sees the interference power from other concurrent transmitters that have started transmissions before is below the power-sensing threshold $\mathsf{t}_{\rm cs}$, i_k decides that it is allowed to transmit.

For brevity of presentation, we also denote a sequence $\mathcal{S}$ as the set of its ordered items. For example, if $\mathcal{S} = (i_1, i_2, i_3)$, we also denote $\mathcal{S}$ as a set, such that $\mathcal{S} = \{i_1, i_2, i_3\}$.

Our goal is to study how to set the power-sensing threshold $\mathsf{t}_{\rm cs}$ to eliminate hidden node. Namely, we aim at finding a proper value of $\mathsf{t}_{\rm cs}$, such that

$$\mathcal{S} \in \mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \mathsf{t}_{\rm cs}\big] \quad \Rightarrow \quad \mathcal{S} \in \mathscr{B}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \beta\big] \tag{12}$$

We are interested in the maximal value of $\mathsf{t}_{\rm cs}$ without the complete knowledge of X. In the following, we suppose that we only know the maximum transmission distance $\mathsf{r}_{\rm tx} \triangleq \max_{i\in X} |t_i - r_i|$ in a priori, when setting $\mathsf{t}_{\rm cs}$. Alternatively, $\mathsf{r}_{\rm tx}$ can be interpreted as the maximum tolerable transmission distance for a given $\mathsf{t}_{\rm cs}$. Note that even without the knowledge of $\mathsf{r}_{\rm tx}$, for fixed constants $\alpha, \beta, \mathsf{N}_0, \mathsf{P}_{\rm tx}$, the upper bound for transmission distance is $|t_i - r_i| \le (\frac{\mathsf{P}_{\rm tx}}{\beta \mathsf{N}_0})^{1/\alpha}$.

IV. Power-Sensing Threshold

This section presents a general approach to eliminate hidden nodes by local interference power measurement by setting a proper power-sensing threshold $\mathsf{t}_{\rm cs}$.

To eliminate hidden node, we rely on the notion of *interference level* at transmitter t_i with respect to a subset of links S, which is defined as:

$$\mathbf{I}_{t_i}[S, \alpha] \triangleq \sum_{j\in S} |t_j - t_i|^{-\alpha} \tag{13}$$

We denote the maximal interference level in Euclidean space $\mathbb{R}^{\sf d}$, subject to $\mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}[X, \mathsf{t}_{\rm cs}]$ with background noise N_0, by:

$$\mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}_{\rm cs}, \alpha] \triangleq \max_{X,\mathcal{S}\in\mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}[X,\mathsf{t}_{\rm cs}], i\in\mathcal{S}} \mathbf{I}_{t_i}[S, \alpha] \tag{14}$$

Lemma 1: $\mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}_{\rm cs}, \alpha]$ has the following properties:

$$\mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}_{\rm cs}, \alpha] = (\mathsf{t}_{\rm cs}\text{-}\mathsf{N}_0)\mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{N}_0 + 1, \alpha] \tag{15}$$

$$\mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}_{\rm cs}, \alpha] = \frac{1}{\mathsf{P}_{\rm tx}} \mathbf{I}^{\sf ag,d}_{\max\,\mathsf{N}_0,\mathsf{P}_{\rm tx}=1}[\mathsf{t}_{\rm cs}, \alpha] \tag{16}$$

Proof: See the Appendix. ∎

Let the normalized maximal interference level be

$$\mathbf{I}^{\sf ag,d}_{\max}[\alpha] \triangleq \mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0=0,\mathsf{P}_{\rm tx}=1}[1, \alpha] \tag{17}$$

Note that $\mathbf{I}^{\sf ag,d}_{\max}[\alpha] \triangleq \mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0=0,\mathsf{P}_{\rm tx}=1}[1, \alpha]$ is a fundamental parameter depending on the the dimension of the space, and provides a key theoretical tool to determine a proper power-sensing threshold to ensure interference-safe transmissions.

A. Sufficient Condition

We next provide a sufficient condition. Define *bi-directional* interference level at link i w.r.t. a subset of links S as:

$$\mathbf{B}_i[S, \alpha] \triangleq \sum_{j\in S} \mathsf{dist}(i, j)^{-\alpha} \tag{18}$$

We denote the maximal bi-directional interference level in Euclidean space $\mathbb{R}^{\sf d}$, subject to $\mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}[X, \mathsf{t}_{\rm cs}]$ with background noise N_0, by:

$$\mathbf{B}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}_{\rm cs}, \alpha] \triangleq \max_{X,\mathcal{S}\in\mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}[X,\mathsf{t}_{\rm cs}], i\in\mathcal{S}} \mathbf{B}_i[S, \alpha] \tag{19}$$

Lemma 2: Let $\mathsf{r}_{\rm tx} \triangleq \sum_{i\in X} |t_i - r_i|$. If

$$\mathsf{t}_{\rm cs} \le \mathsf{P}_{\rm tx}\Big((\frac{\mathsf{t}'_{\rm cs} - \mathsf{N}_0}{\mathsf{P}_{\rm tx}})^{\frac{-1}{\alpha}} + 2\mathsf{r}_{\rm tx} \Big)^{-\alpha} + \mathsf{N}_0, \tag{20}$$

then

$$\mathbf{B}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}_{\rm cs}, \alpha] \le \mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}'_{\rm cs}, \alpha] \tag{21}$$

Proof: See the Appendix. ∎

Theorem 1: Given a set of links X, which lies in Euclidean space $\mathbb{R}^{\sf d}$, let $\mathsf{r}_{\rm tx} \triangleq \max_{i\in X} |t_i - r_i|$. If we let

$$\mathsf{t}_{\rm cs} \le \mathsf{P}_{\rm tx}\Big(2\mathsf{r}_{\rm tx} + (\frac{1}{\mathbf{I}^{\sf ag,d}_{\max}[\alpha]}(\frac{\mathsf{r}_{\rm tx}^{-\alpha}}{\beta} - \frac{\mathsf{N}_0}{\mathsf{P}_{\rm tx}}))^{\frac{-1}{\alpha}} \Big)^{-\alpha} + \mathsf{N}_0 \tag{22}$$

then it can eliminate hidden nodes in APCS,

$$\mathcal{S} \in \mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \mathsf{t}_{\rm cs}\big] \quad \Rightarrow \quad \mathcal{S} \in \mathscr{B}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \beta\big] \tag{23}$$

Proof: Suppose that $\mathcal{S} \in \mathscr{C}^{\sf ag}_{\mathsf{N}_0,\mathsf{P}_{\rm tx}}\big[X, \mathsf{t}_{\rm cs}\big]$, and $\mathsf{t}'_{\rm cs}$ satisfies Eqn. (20). Recall that $\mathbf{I}^{\sf ag,d}_{\max}[\alpha] = \mathbf{I}^{\rm ag,max}_{\mathsf{d},\mathsf{N}_0=0}[1, \alpha]$. Then, by Lemmas 2 and 1, we obtain:

$$\begin{aligned}
&\frac{\mathsf{P}_{\rm tx}|t_i - r_i|^{-\alpha}}{\mathsf{N}_0 + \sum\limits_{j\in\mathcal{S}\backslash\{i\}} \mathsf{P}_{\rm tx} \cdot \mathsf{dist}(i, j)^{-\alpha}} \\
\ge\ & \frac{\mathsf{P}_{\rm tx}\mathsf{r}_{\rm tx}^{-\alpha}}{\mathsf{N}_0 + \mathsf{P}_{\rm tx}\mathbf{B}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}_{\rm cs}, \alpha]} \ge \frac{\mathsf{P}_{\rm tx}\mathsf{r}_{\rm tx}^{-\alpha}}{\mathsf{N}_0 + \mathsf{P}_{\rm tx}\mathbf{I}^{\sf ag,d}_{\max,\mathsf{N}_0,\mathsf{P}_{\rm tx}}[\mathsf{t}'_{\rm cs}, \alpha]} \\
=\ & \frac{\mathsf{P}_{\rm tx}\mathsf{r}_{\rm tx}^{-\alpha}}{\mathsf{N}_0 + (\mathsf{t}'_{\rm cs}\text{-}\mathsf{N}_0)\mathbf{I}^{\sf ag,d}_{\max}[\alpha]}
\end{aligned} \tag{24}$$

Hence,

$$\frac{1}{\mathbf{I}^{\mathsf{ag,d}}_{\max}[\alpha]}\Big(\mathsf{P_{tx}}(\beta \mathsf{r_{tx}})^{-\alpha} - \mathsf{N_0}\Big) + \mathsf{N_0} \geq \mathsf{t}'_{\mathsf{cs}}$$
$$\Rightarrow \frac{\mathsf{P_{tx}}|t_i - r_i|^{-\alpha}}{\mathsf{N_0} + \sum_{j\in\mathcal{S}\setminus\{i\}} \mathsf{P_{tx}} \cdot \mathsf{dist}(i,j)^{-\alpha}} \geq \beta \tag{25}$$

Next,

$$\begin{aligned}&\tfrac{1}{\mathbf{I}^{\mathsf{ag,d}}_{\max}[\alpha]}\Big(\mathsf{P_{tx}}(\beta \mathsf{r_{tx}})^{-\alpha} - \mathsf{N_0}\Big) + \mathsf{N_0}\\ &\geq \mathsf{P_{tx}}\Big(\big(\tfrac{\mathsf{t_{cs}}-\mathsf{N_0}}{\mathsf{P_{tx}}}\big)^{\frac{-1}{\alpha}} - 2\mathsf{r_{tx}}\Big)^{-\alpha} + \mathsf{N_0}\\ &\Rightarrow \mathsf{t_{cs}} \leq \mathsf{P_{tx}}\Big(2\mathsf{r_{tx}} + \big(\tfrac{1}{\mathbf{I}^{\mathsf{ag,d}}_{\max}[\alpha]}(\tfrac{\mathsf{r_{tx}^{-\alpha}}}{\beta} - \tfrac{\mathsf{N_0}}{\mathsf{P_{tx}}})\big)^{\frac{-1}{\alpha}}\Big)^{-\alpha} + \mathsf{N_0}\end{aligned} \tag{26}$$

■

Theorem 1 provide the proper values of $\mathsf{t_{cs}}$ without the knowledge of the network topology, and hence are useful to generic CSMA networks. Although the values appear to dependent on the parameters of channel model α, our results are relatively conservative, and in the evaluation section. We observe that the thesholds are relatively robust against uncertain parameters from the channel model.

In particular, when $\mathsf{N_0} = 0$, then

$$\mathsf{t_{cs}} \leq \mathsf{P_{tx}}\Big(\big(2 + (\beta \mathbf{I}^{\mathsf{ag,d}}_{\max}[\alpha])^{\frac{1}{\alpha}}\big)\mathsf{r_{tx}}\Big)^{-\alpha} \tag{27}$$

In the following, we derive $\mathbf{I}^{\mathsf{ag,d}}_{\max}[\alpha]$ for the 1-D and 2-D cases in the next sections.

V. Power-sensing Threshold: 1-D Case

Although the 1-D case $\mathbb{R}$ (the real line) seems too simple, the obtained result is however critical to the more realistic 2-D case in the next section. In order to use Theorem 1 to determine a proper power-sensing threshold, we consider the worst case topology that generates $\mathbf{I}^{\mathsf{ag,1}}_{\max}[\alpha]$. We consider a large set of points with coordinates labeled as $(t_0, t_1, t_2, ..t_T)$. It can be shown that $\mathbf{I}^{\mathsf{ag,1}}_{\max}[\alpha]$ is obtained via solving the following optimization problem:

$$(\mathbf{MaxI}) : \max \sum_{i=1}^{T} |t_i - t_0|^{-\alpha} \text{ subject to}$$
$$\textstyle\sum_{j=1}^{k-1} |t_k - t_j|^{-\alpha} \leq 1 \text{ for all } k = 1, ..., T$$

Because of the restriction of placement of nodes in $\mathbb{R}$, we can also make use of a *greedy placement algorithm* to solve **MaxI**. The basic idea is to sequentially place the i-th node t_i in $\mathbb{R}$ such that the distance $|t_i - t_0|$ is minimized subject to:

$$\mathbf{I}_{t_i}[\{t_0, ..., t_{i-1}\}, \alpha] = \sum_{t_j = t_0, ..., t_{i-1}} \big(|t_i - t_j|\big)^{-\alpha} = 1 \tag{28}$$

The interference level at t_0 w.r.t. an infinite sequence $\{t_1, t_2, ...\}$ is the maximal interference level $\mathbf{I}^{\mathsf{ag,1}}_{\max}[\alpha]$. In fact, greedy placement algorithm can be implemented by balanced placement around t_0, in the positive and negative real lines alternatively, as in Algo. 1.

We show the correctness of Algo. 1 by Lemma 3.

Lemma 3: $\mathbf{I}_T$ is the output of Algo. 1, $\lim_{T\to\infty} \mathbf{I}_T = \mathbf{I}^{\mathsf{ag,1}}_{\max}[\alpha]$. We defer the proof to the technical report [9].

Algo. 1 Greedy1d: Input (α, max step T), Output ($\mathbf{I}_T$)

Put t_o at the origin of $\mathbb{R}$
$\mathcal{S}_0 \leftarrow (t_0)$
for $i = 1, ..., T$ **do**
  if i is odd **then**
    Place t_i in $\mathbb{R}$ such that $t_i > \max \mathcal{S}_{i-1}$ and $\sum_{t_j\in\mathcal{S}_{i-1}} \big(|t_i - t_j|\big)^{-\alpha} = 1$
  else
    Place t_i in $\mathbb{R}$ such that $t_i < \min \mathcal{S}_{i-1}$ and $\sum_{t_j\in\mathcal{S}_{i-1}} \big(|t_i - t_j|\big)^{-\alpha} = 1$
  end if
  $\mathcal{S}_i \leftarrow (t_0, ..., t_{i-1}, t_i)$
end for
return $\mathbf{I}_T = \sum_{s_j \in \mathcal{S}_T \setminus \{t_0\}} \big(|t_0 - t_j|\big)^{-\alpha}$

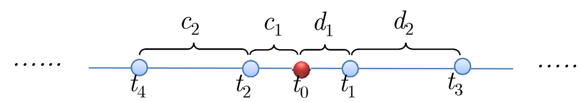

Fig. 3. Nodes are placed in $\mathbb{R}$ according to Algo. 1.

We next evaluate $\mathbf{I}_T$ from Algo. 1. We let $d_i \triangleq |t_{2i-1} - t_{2i-3}|$ and $c_i \triangleq |t_{2i} - t_{2i-2}|$ (see Fig. 3). We obtain:

$$\begin{aligned}
1 &= (d_1)^{-\alpha} \;\Rightarrow\; d_1 = 1\\
1 &= (c_1)^{-\alpha} + (c_1 + d_1)^{-\alpha}\\
1 &= (d_2)^{-\alpha} + (d_1 + d_2)^{-\alpha} + (c_1 + d_1 + d_2)^{-\alpha}\\
1 &= (c_2)^{-\alpha} + (c_2 + c_1)^{-\alpha} + (c_2 + c_1 + d_1)^{-\alpha}\\
&\quad + (c_2 + c_1 + d_1 + d_2)^{-\alpha}\\
&\;\;\vdots\\
1 &= (d_k)^{-\alpha} + (d_{k-1} + d_k)^{-\alpha} + ... + (d_1 + ... + d_k)^{-\alpha}\\
&\quad + (c_1 + d_1 + ... + d_k)^{-\alpha} + ...\\
&\quad + (c_{k-1} + ... + c_1 + d_1 + ... + d_k)^{-\alpha}\\
1 &= (c_k)^{-\alpha} + (c_k + c_{k-1})^{-\alpha} + ... + (c_k + ... + c_1)^{-\alpha}\\
&\quad + (c_k + ... + c_1 + d_1)^{-\alpha} + ...\\
&\quad + (c_k + ... + c_1 + d_1 + ... + d_k)^{-\alpha}
\end{aligned} \tag{29}$$

One can solve $\{c_i, d_i\}_{i=1,2,...}$ from the above equations. Then the maximal interference level is obtained by

$$\mathbf{I}^{\mathsf{ag,1}}_{\max}[\alpha] = \sum_{n=1}^{\infty}(c_1 + ... + c_n)^{-\alpha} + \sum_{n=1}^{\infty}(d_1 + ... + d_n)^{-\alpha} \tag{30}$$

Numerically, we evaluate $\{c_i, d_i\}_{i=1,...,20}$ for the case $\alpha = 2$, and estimate that $\mathbf{I}^{\mathsf{ag,1}}_{\max}[2] \approx 2.59$

However, it is difficult to obtain $\{c_i, d_i\}_{i=1,2,...}$ for general α. Next, we obtain a upper bound for $\mathbf{I}^{\mathsf{ag,1}}_{\max}[\alpha]$.

First, we note that

$$d_1 < c_1 < d_2 < c_2 < ... < d_{k-1} < c_{k-1} < d_k < c_k < ... \tag{31}$$

It is because that the locally measured interference level w.r.t. $\mathcal{S}_{i-1}$ at each t_i is 1 in Algo. 1, and hence t_i should have an increasing distance to the closest node in $\mathcal{S}_{i-1}$.

Hence, we obtain:

$$\begin{aligned}
c_1 + d_1 &< c_1 + c_1\\
(c_1 + d_1)^{-\alpha} &> (c_1 + c_1)^{-\alpha}\\
1 = (c_1)^{-\alpha} + (c_1 + d_1)^{-\alpha} &> (c_1)^{-\alpha}(1 + 2^{-\alpha})\\
\Rightarrow\; c_1 &> \tfrac{1}{(1+2^{-\alpha})^{-1/\alpha}}
\end{aligned} \tag{32}$$

Similarly, we obtain:

$$\begin{aligned}1 = &(d_k)^{-\alpha} + (d_{k-1}+d_k)^{-\alpha} + ... + (d_1+...+d_k)^{-\alpha}\\ &+(c_1+d_1+...+d_k)^{-\alpha}+...\\ &+(c_{k-1}+...+c_1+d_1+...+d_k)^{-\alpha}\\ > &(d_k)^{-\alpha}(1+2^{-\alpha}+...+(2k-1)^{-\alpha})\end{aligned} \tag{33}$$

$$\Rightarrow \quad d_k > \Big(\sum_{i=1}^{2k-1} i^{-\alpha}\Big)^{1/\alpha} \tag{34}$$

$$\begin{aligned}1 = &(c_k)^{-\alpha} + (c_k+c_{k-1})^{-\alpha} + ... + (c_k+...+c_1)^{-\alpha}\\ &+(c_k+...+c_1+d_1)^{-\alpha}+...\\ &+(c_k+...+c_1+d_1+...+d_k)^{-\alpha}\\ > &(c_k)^{-\alpha}(1+2^{-\alpha}+...+(2k)^{-\alpha})\end{aligned} \tag{35}$$

$$\Rightarrow \quad c_k > \Big(\sum_{i=1}^{2k} i^{-\alpha}\Big)^{1/\alpha} \tag{36}$$

Thus, we obtain an upper bound $\bar{\mathbf{I}}_1[\alpha] > \mathbf{I}^{\mathsf{ag},1}_{\max}$ as follows:

$$\begin{aligned}\mathbf{I}^{\mathsf{ag},1}_{\max}[\alpha] = &\sum_{n=1}^{\infty}(c_1+...+c_n)^{-\alpha} + \sum_{n=1}^{\infty}(d_1+...+d_n)^{-\alpha}\\ < \bar{\mathbf{I}}_1[\alpha] \triangleq &\sum_{n=1}^{\infty}\Big(\sum_{k=1}^{n}\Big(\sum_{i=1}^{2k} i^{-\alpha}\Big)^{1/\alpha}\Big)^{-\alpha}\\ &+\sum_{n=1}^{\infty}\Big(\sum_{k=1}^{n}\Big(\sum_{i=1}^{2k-1} i^{-\alpha}\Big)^{1/\alpha}\Big)^{-\alpha}\end{aligned} \tag{37}$$

Numerically, we evaluate $\bar{\mathbf{I}}_1[\alpha]$ in Fig. 4 by summing only the first n terms in the outmost summation of $\bar{\mathbf{I}}_1[\alpha]$. We observe that $\bar{\mathbf{I}}_1[\alpha]$ converges quickly (see Fig. 4).

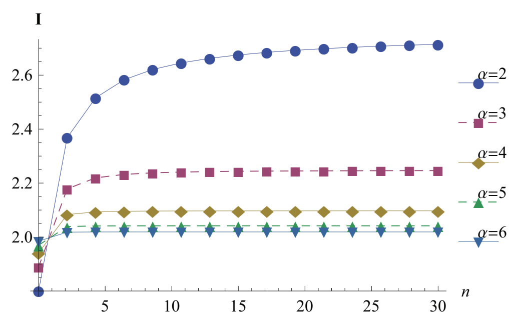

Fig. 4. Numerical values of $\bar{\mathbf{I}}_1[\alpha]$ of the first n terms in the summation.

We tabulate the values of $\bar{\mathbf{I}}_1[\alpha]$ via numerical study in Table V.

	$\alpha=2$	$\alpha=3$	$\alpha=4$	$\alpha=5$	$\alpha=6$
$\bar{\mathbf{I}}_1[\alpha]$	2.74438	2.24708	2.09705	2.04166	2.01887

Note that $\mathbf{I}^{\mathsf{ag},1}_{\max}[2] \approx 2.59$ by numerical study. Hence, the upper bound appears to be tight.

VI. Power-Sensing Threshold: 2-D Case

Obtaining the maximal interference level $\mathbf{I}^{\mathsf{ag},2}_{\max}[\alpha]$ for Theorem 1 in the 2-D case is more complicated, because there are many more possible locations for nodes in Euclidean space $\mathbb{R}^2$. By numerical solving $(\mathbf{MaxI})$ for the case of $\alpha = 3$ with a large T, we obtain $\mathbf{I}^{\mathsf{ag},2}_{\max}[3] \approx 4.2$.

We next give a upper bound for $\mathbf{I}^{\mathsf{ag},2}_{\max}[\alpha]$. Recall the sequence of separation distances $(d_1, d_2, ...)$ from the definition in Fig. 3 for the 1-D case. We set the spacing distance in the hexagonal grid to be $d_1 = 1$.

Nodes are placed as hexagonal rings around t_0 (see Fig. 6). We denote the set of nodes in hexagonal grid for the i-th ring by $\mathcal{H}_i = \{t_i^1, ..., t_i^{|\mathcal{H}_i|}\}$. Particularly, t_i^1 are placed on the positive horizontal real line in $\mathbb{R}^2$. We set the separation distance between the rings according to sequence $(d_1, d_2, ...)$. Namely, the location of t_i^1 in $\mathbb{R}^2$ is $\big(\lfloor\sum_{j=1}^{i} d_i\rfloor, 0\big)$.

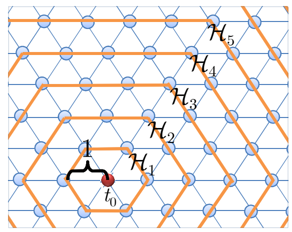

Fig. 5. Nodes are placed evenly around t_0 in hexagonal grid.

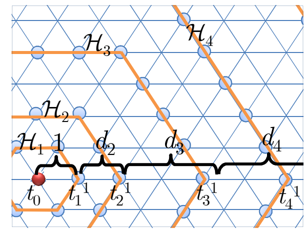

Fig. 6. Nodes are placed as hexagonal rings, separated by d_i.

We can upper bound $\mathbf{I}_{t_0}[\bigcup_{j=1}^{\infty}\mathcal{H}_j]$ from Fig. 6. For each i-th ring, $|\mathcal{H}_i| = 6\big(\lfloor\sum_{j=1}^{i} d_i\rfloor\big)$. By Eqn. (34), we obtain:

$$\begin{aligned}\mathbf{I}^{\mathsf{ag},2}_{\max}[\alpha] &< \mathbf{I}_{t_0}[\textstyle\bigcup_{j=1}^{\infty}\mathcal{H}_j]\\ &< \sum_{n=1}^{\infty} 6(d_1+...+d_n)^{-\alpha+1}\\ &< \bar{\mathbf{I}}_2[\alpha] \triangleq 6\sum_{n=1}^{\infty}\Big(\sum_{k=1}^{n}\Big(\sum_{i=1}^{2k-1} i^{-\alpha}\Big)^{1/\alpha}\Big)^{-\alpha+1}\end{aligned} \tag{38}$$

Numerically, we evaluate $\bar{\mathbf{I}}_2[\alpha]$ in Fig. 7 by summing only the first n terms in the outmost summation of $\bar{\mathbf{I}}_2[\alpha]$. We observe that $\bar{\mathbf{I}}_1[\alpha]$ converges quickly as n increases. We tabulate the values of $\bar{\mathbf{I}}_2[\alpha]$ via numerical study in Table VI.

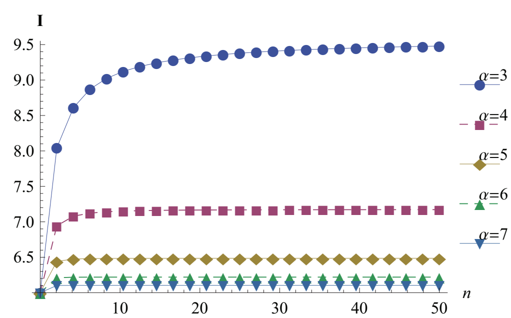

Fig. 7. Numerical values of $\bar{\mathbf{I}}_2[\alpha]$ of the first n terms in the summation.

	$\alpha=3$	$\alpha=4$	$\alpha=5$	$\alpha=6$	$\alpha=7$
$\bar{\mathbf{I}}_2[\alpha]$	9.56077	7.17297	6.48636	6.21992	6.10368

VII. Comparison to IPCS

Besides of APCS, an alternate approach is to use the *pairwise* interference power as in IPCS [7]. The basic idea of Incremental-Power Carrier-Sensing (IPCS) is that t_i can estimate the distance to each individual concurrent transmitter t_k by measuring the change of interference power. Suppose that initially t_i measures the aggregate interference power as: $\mathbf{P}_{t_i}[\mathcal{N}\backslash\{t_k\}]$. Then when t_k transmits, the measured change of interference power at t_i becomes:

$$\Delta \mathsf{P}_i = \mathbf{P}_{t_i}[\mathcal{N}] - \mathbf{P}_{t_i}[\mathcal{N}\backslash\{t_k\}] = \mathsf{P}_{\mathsf{tx}}|t_k - t_i|^{-\alpha}, \tag{39}$$

which reveals the distance $|t_k - t_i|$. Suppose that each transmitter t_i maintains a counter cnt_i (initially set as 0). When t_i detects any change $\Delta\mathsf{P}_i$,

- if $\Delta\mathsf{P}_i \geq \mathsf{P}_{\mathsf{tx}}\mathsf{r}_{\mathrm{cs}}^{-\alpha}$, then $\mathsf{cnt}_i \leftarrow \mathsf{cnt}_i + 1$.
- if $\Delta\mathsf{P}_i \leq -\mathsf{P}_{\mathsf{tx}}\mathsf{r}_{\mathrm{cs}}^{-\alpha}$, then $\mathsf{cnt}_i \leftarrow \mathsf{cnt}_i - 1$.

Transmitter t_i is allowed to transmit only if $\mathsf{cnt}_i = 0$.

In idealized CSMA protocol, congestion avoidance countdown is based on a continuous random variable. Hence, it is likely that no transmitters will simultaneously start to transmit at the same time. In such a setting, IPCS realizes a pairwise carrier-sensing feasible family as follows:

Definition 4: (*Pairwise carrier-sensing feasible family* $\mathscr{C}^{\mathsf{pw}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X, \mathsf{r}_{\mathrm{cs}}] \subseteq 2^X$): A subset of links are permitted by pairwise carrier-sensing $S \in \mathscr{C}^{\mathsf{pw}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X, \mathsf{r}_{\mathrm{cs}}]$, if and only if for all $i, j \in S$,

$$|t_j - t_i| \geq \mathsf{r}_{\mathrm{cs}} \tag{40}$$

We denote the maximal interference level in Euclidean space $\mathbb{R}^{\mathsf{d}}$, subject to $\mathscr{C}^{\mathsf{pw}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X, \mathsf{r}_{\mathrm{cs}}]$ with background noise $\mathsf{N_0}$, by:

$$\mathbf{I}^{\mathsf{pw,d}}_{\max}[\mathsf{r}_{\mathrm{cs}}, \alpha] \triangleq \max_{X, S \in \mathscr{C}^{\mathsf{pw}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X,\mathsf{r}_{\mathrm{cs}}], i \in \mathcal{S}} \mathbf{I}_{t_i}[S, \alpha] \tag{41}$$

A natural question is which mechanism among APCS and IPCS is better. One may compare the size of feasible families between aggregate carrier-sensing and pairwise carrier-sensing. Nonetheless, we note that the feasible family of aggregate carrier-sensing $\mathscr{C}^{\mathsf{ag}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X, \mathsf{t}_{\mathrm{cs}}]$ is a collection of sequences of transmitters, whereas that of pairwise carrier-sensing $\mathscr{C}^{\mathsf{pw}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X, \mathsf{r}_{\mathrm{cs}}]$ is a collection of subsets of links. To have a fair comparison, we set t_{cs} and r_{cs}, such that $\mathbf{I}^{\mathsf{pw,d}}_{\max}[\mathsf{r}_{\mathrm{cs}}, \alpha] = \mathbf{I}^{\mathsf{ag,d}}_{\max}[\mathsf{t}_{\mathrm{cs}}, \alpha]$. Namely, the maximum interference level at a transmitter by APCS or IPCS is the same.

In such a setting, we show in [9] that there exists a set of links X, such that there is a feasible sequence in $\mathscr{C}^{\mathsf{ag}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X, \mathsf{t}_{\mathrm{cs}}]$, but there is no corresponding feasible set of its ordered members in $\mathscr{C}^{\mathsf{pw}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X, \mathsf{r}_{\mathrm{cs}}]$. Also, we show in [9] that there exists a set of links X, such that there is a feasible set in $\mathscr{C}^{\mathsf{pw}}_{\mathsf{N_0},\mathsf{P_{tx}}}[X, \mathsf{r}_{\mathrm{cs}}]$, but not all sequences of its members are in $\mathscr{C}^{\mathsf{ag}}[X, \mathsf{t}_{\mathrm{cs}}]$. Hence, neither approach is superior to one another in terms of feasibility.

VIII. Performance Evaluation

We perform simulations to evaluate the relative performance of APCS as comparing to IPCS. We also consider a "benchmark" approach, as proposed by [7]. This is essentially APCS, but the power-sensing threshold t_{cs} is set as:

$$\mathsf{t}_{\mathsf{cs}} \leq \mathsf{P}_{\mathsf{tx}}(\mathsf{r}_{\mathsf{cs}})^{-\alpha} + \mathsf{N}_0 \tag{42}$$

where r_{cs} is the carrier-sensing range for IPCS obtained from [7]. Namely, we set t_{cs} to implement IPCS by APCS.

In our simulations, the nodes are placed in a square area of 300m $\times$ 300m. The locations of the transmitters are generated according to a Poisson point process and clustered Poisson point process. The length of a link is uniformly distributed between 10 and 20 meters. The receiver associated with a transmitter is randomly located between the two concentric circles of radii 10m and 20m centered on the transmitter. We study the performance under different values of countdown window (CWin) in the CSMA protocol.

We consider $\alpha = 3$, and use $\mathbf{I}^{\mathsf{ag},2}_{\max}[3] \approx 4.2$. We compare the average throughput among the links and fairness measured by Jain index, $\mathrm{JI} = \frac{(\sum_{i=1}^{n} \lambda_i)^2}{n \sum_{i=1}^{n} \lambda_i}$, where λ_i is throughput of link i in the total n links. The large value of JI means more fair.

1. (*Known channel parameter*): For $\alpha = 3$, we plot the results in Fig. 8. We observe that APCS has similar average throughput as IPCS, and significantly outperforms IPCS under clustered point model. The fairness of APCS is more biased under Poisson point model, but is same as IPCS under clustered point model. In all cases, APCS outperforms the benchmark approach. It is because APCS depends on only the aggregate interference, rather the pairwise interference in the neighborhood, APCS is more suitable for clustered environment than IPCS, where links are clustered together.

2. (*Uncertain channel parameter*): When α is a independently random variable in the range $[2, 4]$ for each pair of links, we plot the results in Fig. 9. We observe that APCS has a better average throughput than IPCS, and the fairness is similar under Poisson point model and clustered point model. This is due to the fact that APCS measures the sum of interference from all links, and is less vulnerable to the random fluctuation of pathloss exponent α from an individual link as in IPCS. This suggests that APCS is a more robust solution in the presence of uncertain pathloss exponent from the channel model.

IX. Conclusion

Interactions between links in realistic CSMA networks are affected by the special properties attributed to SINR, effects of arbitrary ordering of local measurements, and ACK frames. Without proper consideration of these properties, inteference among simultaneous links can cause considerable performance degradation to CSMA networks. This paper presents the first viable standard-compatible solution to ensure inteference-safe transmissions by determining a proper interference-safe power-sensing threshold in Aggregate-interference-Power Carrier-Sensing (APCS). We remark that our solution does not require modification to the existing CSMA protocol.

References

[1] L. B. Jiang and S. C. Liew, "Improving throughput and fairness by reducing exposed and hidden nodes in 802.11 networks," *IEEE Trans. Mobile Computing*, vol. 7, no. 1, pp. 34–49, January 2008.

[2] Y. Gao, D. M. Chiu, and J. C. S. Lui, "Determining the end-to-end throughput capacity in multi-hop networks: methodology and applications," in *Proc. ACM SIGMETRICS*, 2006, pp. 39–50.

[3] M. Durvy, O. Dousse, and P. Thiran, "On the fairness of large CSMA networks," in *IEEE J. Selected Areas in Communications*, vol. 27, September 2009, pp. 1093–1104.

[4] V. V. Kapadia, S. N. Patel, and R. H. Jhaveri, "Comparative study of hidden node problem and solution using different techniques and protocols," *J. of Computing*, vol. 2, no. 3, March 2010, available on Arxiv: http://arxiv.org/abs/1003.4070.

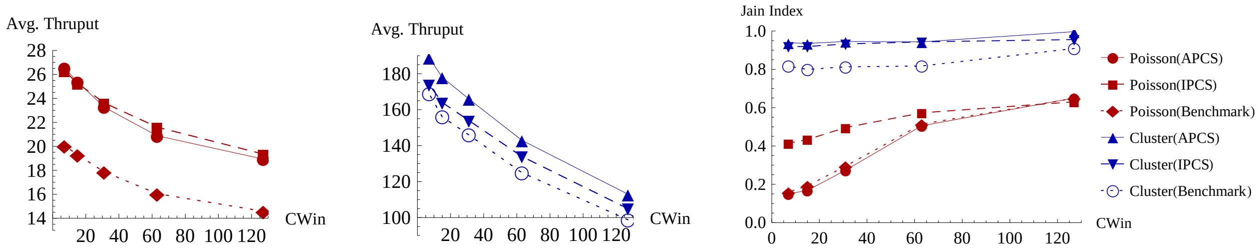

Fig. 8. For $\alpha = 3$, the average throughput and Jain index for APCS, IPCS and benchmark under Poisson point model and clustered point model with different contention windows (CWin).

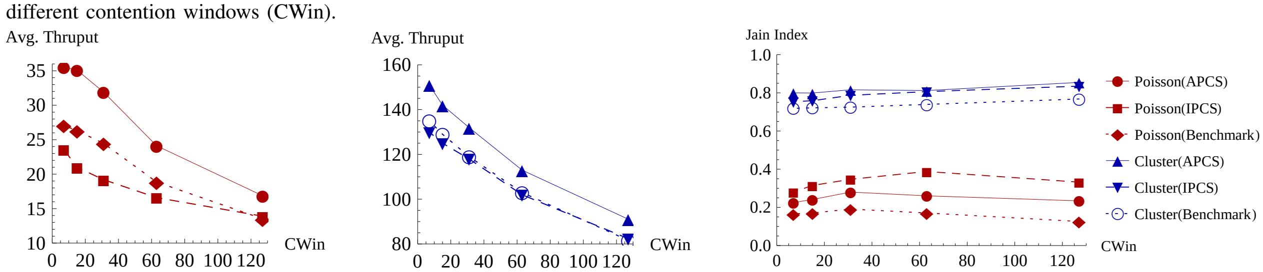

Fig. 9. For independently uniformly random $\alpha \in [2, 4]$, the average throughput and Jain index for APCS, IPCS and benchmark under Poisson point model and clustered point model with different contention windows (CWin).

[5] K. Xu, M. Gerla, and S. Bae, "How effective is the IEEE 802.11 RTS/CTS handshake in ad hoc networks," in *Proc. IEEE GLOBECOM*, 2002.

[6] C.-K. Chau, M. Chen, and S. C. Liew, "Capacity of large-scale CSMA wireless networks," in *Proc. ACM MobiCom*, 2009.

[7] L. Fu, S. C. Liew, and J. Huang, "Effective carrier sensing in CSMA networks under cumulative interference," in *Proc. IEEE INFOCOM*, 2010.

[8] D. Shukla, L. Chandran-Wadia, and S. Iyer, "Mitigating the exposed node problem in IEEE 802.11 ad hoc networks," in *Proc. ICCCN*, 2003.

[9] C.-K. Chau, J. Zhang, M. Chen, and S. C. Liew, "Interference-safe CSMA networks by local aggregate interference power measurement," Tech. Rep., 2012, available at: http://www.mit.edu/~ckchau/papers.

X. Appendix: Proofs

A. Proof of Lemma 1

Proof: To prove Eqn. (15), suppose $\tilde{X}$ and $\tilde{\mathcal{S}}$ gives the maximal interference level for $\mathbf{I}^{\mathsf{ag,d}}_{\mathsf{max,N_0,P_{tx}}}[\mathsf{N_0}+1,\alpha]$. Since $\tilde{X}$ lies in $\mathbb{R}^{\mathsf{d}}$, we can multiply the distances between all transmitters in $\tilde{X}$ by a factor of $(\mathsf{t_{cs}}-\mathsf{N_0})^{-1/\alpha}$. We denote such a set of rescaled transmitters as $\tilde{X}'$. Namely,

$$|t'_j - t'_i| = (\mathsf{t_{cs}}-\mathsf{N_0})^{-1/\alpha}|t_j - t_i| \tag{43}$$

where $t'_j, t'_i \in X'$, $t_j, t_i \in X$. It is easy to see that

$$\begin{aligned} &\mathsf{N_0} + \sum_{i_j\in\{i_1,...,i_{k-1}\}} \mathsf{P_{tx}}|t_{i_j} - t_{i_k}|^{-\alpha} \le \mathsf{N_0}+1 \\ \Leftrightarrow\ &\mathsf{N_0} + \sum_{i'_j\in\{i'_1,...,i'_{k-1}\}} \mathsf{P_{tx}}|t_{i'_j} - t_{i'_k}|^{-\alpha} \le \mathsf{t_{cs}} \end{aligned} \tag{44}$$

where $(i_1, ..., i_k) = \tilde{\mathcal{S}}$ and $(i'_1, ..., i'_k) = \tilde{\mathcal{S}}'$, and $\tilde{\mathcal{S}}'$ is the induced sequence of $\tilde{\mathcal{S}}$ in the rescaled $\tilde{X}'$.

Hence, the induced $\tilde{\mathcal{S}}'$ must also give the maximal interference level for $\mathbf{I}^{\mathsf{ag,d}}_{\mathsf{max,N_0,P_{tx}}}[\mathsf{t_{cs}},\alpha]$. Therefore,

$$\begin{aligned} &\mathbf{I}^{\mathsf{ag,d}}_{\mathsf{max,N_0,P_{tx}}}[\mathsf{t_{cs}},\alpha] = \sum_{j\in S'}|t'_j - t'_i|^{-\alpha} = (\mathsf{t_{cs}}\text{-}\mathsf{N_0})\sum_{j\in\mathcal{S}}|t_j - t_i|^{-\alpha} \\ &= (\mathsf{t_{cs}}\text{-}\mathsf{N_0})\mathbf{I}^{\mathsf{ag,d}}_{\mathsf{max,N_0,P_{tx}}}[\mathsf{N_0}+1,\alpha] \end{aligned} \tag{45}$$

To prove Eqn. (16), it follows a similar approach. However, we multiply the distances between all transmitters in $\tilde{X}'$ by a factor of $\mathsf{P_{tx}}^{1/\alpha}$. ■

B. Proof of Lemma 2

Proof: Suppose $\mathcal{S} = (i_1, ..., i_{|\mathcal{S}|}) \in \mathscr{C}^{\mathsf{ag}}_{\mathsf{N_0,P_{tx}}}[X,\mathsf{t_{cs}}]$. By Eqn. (11), for any pair $i_j, i_k \in \mathcal{S}$ and $j < k$, we obtain:

$$\mathsf{N_0} + \sum_{i_j\in\{i_1,...,i_{k-1}\}} \mathsf{P_{tx}}|t_{i_j} - t_{i_k}|^{-\alpha} \le \mathsf{t_{cs}} \tag{46}$$

$$\Rightarrow\ |t_{i_j} - t_{i_k}| \ge \Big(\frac{\mathsf{t_{cs}}-\mathsf{N_0}}{\mathsf{P_{tx}}}\Big)^{\frac{-1}{\alpha}} \ge \Big(\frac{\mathsf{t}'_{\mathsf{cs}}-\mathsf{N_0}}{\mathsf{P_{tx}}}\Big)^{\frac{-1}{\alpha}} + 2\mathsf{r_{tx}} \tag{47}$$

Since $|t_{i_j} - r_{i_j}| \le \mathsf{r_{tx}}$ and $|t_{i_k} - r_{i_k}| \le \mathsf{r_{tx}}$, by triangular inequality,

$$|t_{i_j} - r_{i_k}| \ge |t_{i_j} - t_{i_k}| - |t_{i_k} - r_{i_k}| \ge \Big(\frac{\mathsf{t}'_{\mathsf{cs}}-\mathsf{N_0}}{\mathsf{P_{tx}}}\Big)^{\frac{-1}{\alpha}} + \mathsf{r_{tx}} \tag{48}$$

$$|r_{i_j} - t_{i_k}| \ge |t_{i_j} - t_{i_k}| - |r_{i_j} - t_{i_j}| \ge \Big(\frac{\mathsf{t}'_{\mathsf{cs}}-\mathsf{N_0}}{\mathsf{P_{tx}}}\Big)^{\frac{-1}{\alpha}} + \mathsf{r_{tx}} \tag{49}$$

$$|r_{i_j} - r_{i_k}| \ge |r_{i_j} - t_{i_k}| - |t_{i_k} - r_{i_k}| \ge \Big(\frac{\mathsf{t}'_{\mathsf{cs}}-\mathsf{N_0}}{\mathsf{P_{tx}}}\Big)^{\frac{-1}{\alpha}} \tag{50}$$

Hence,

$$\mathsf{dist}(i,j) \ge \Big(\frac{\mathsf{t}'_{\mathsf{cs}}-\mathsf{N_0}}{\mathsf{P_{tx}}}\Big)^{\frac{-1}{\alpha}} \tag{51}$$

$$\Rightarrow\ \mathsf{N_0} + \sum_{i_j\in\{i_1,...,i_{k-1}\}} \mathsf{P_{tx}}\cdot\mathsf{dist}(i_j,i_k)^{-\alpha} \le \mathsf{t}'_{\mathsf{cs}} \tag{52}$$

We define a new feasible family $\tilde{\mathscr{C}}^{\mathsf{ag}}_{\mathsf{N_0,P_{tx}}}[X,\mathsf{t}'_{\mathsf{cs}}]$, such that $\mathcal{S} \in \tilde{\mathscr{C}}^{\mathsf{ag}}_{\mathsf{N_0,P_{tx}}}[X,\mathsf{t}'_{\mathsf{cs}}]$, if and only if Eqn. (52) holds for all $i_k \in \mathcal{S}$. It follows that

$$\mathcal{S} \in \mathscr{C}^{\mathsf{ag}}_{\mathsf{N_0,P_{tx}}}[X,\mathsf{t_{cs}}] \ \Rightarrow\ \mathcal{S} \in \tilde{\mathscr{C}}^{\mathsf{ag}}_{\mathsf{N_0,P_{tx}}}[X,\mathsf{t}'_{\mathsf{cs}}] \tag{53}$$

Hence,

$$\mathbf{B}^{\mathsf{ag,d}}_{\mathsf{max,N_0,P_{tx}}}[\mathsf{t_{cs}},\alpha] \le \max_{X,\mathcal{S}\in\tilde{\mathscr{C}}^{\mathsf{ag}}_{\mathsf{N_0,P_{tx}}}[X,\mathsf{t}'_{\mathsf{cs}}],i\in\mathcal{S}} \mathbf{B}_i[S,\alpha] \tag{54}$$

We complete the proof by:

$$\max_{X,\mathcal{S}\in\tilde{\mathscr{C}}^{\mathsf{ag}}_{\mathsf{N_0,P_{tx}}}[X,\mathsf{t}'_{\mathsf{cs}}],i\in\mathcal{S}} \mathbf{B}_i[S,\alpha] \le \mathbf{I}^{\mathsf{ag,d}}_{\mathsf{max,N_0,P_{tx}}}[\mathsf{t}'_{\mathsf{cs}},\alpha] \tag{55}$$

■