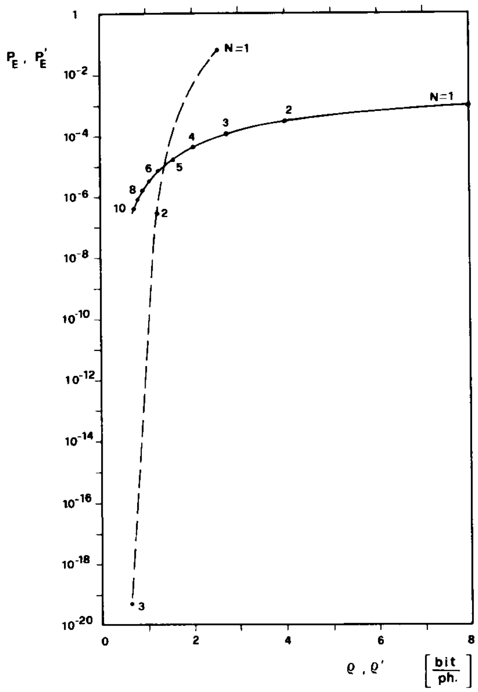

Fig. 7. Bit error probabilities against the transmission efficiency, for binary (continuous line) and PPM transmission (dashed line). The two techniques exhibit approximately the same bandwidth expansion factor ($\beta = \beta' \simeq 12.4$).

are depicted in continuous and in dashed lines, respectively. These curves exhibit an intersection at $\rho = \rho' \simeq 1.55$ bit/photon. For higher values of the efficiency, the binary transmission appears, even in this case, to be preferable. On the other hand, for lower values of the efficiency, the PPM transmission achieves bit error probabilities extremely low. ρ and ρ' increase as N decreases, as expected. When $N = 1$, which is the smallest integer values for N, the PPM transmission gives $\rho' \simeq 2.5$ bit/photon, whereas the binary transmission approaches an efficiency ρ more than three times larger.

V. Conclusions

The objective of this paper was to investigate the maximum transmission efficiency one can reach over an ideal photon counting channel, having fixed the bandwidth expansion factor. First, the ideal situation, represented by Shannon theorem for discrete channels, has been analyzed. A low average number of photons per pulse is demonstrated to be preferable. A binary transmission, with different *a priori* probabilities of the two transmitted symbols, exhibits a higher efficiency than that of an orthogonal PPM transmission, whose M-ary symbols are equiprobable, for an equal bandwidth expansion.

Then practical transmissions have been considered. The PPM technique can be very efficiently coded, and, in some situations, is characterized by a bit error probability lower than that of the uncoded binary technique. However, uncoded binary transmission remains extremely attractive for the achievement of ultra-high transmission efficiencies.

References

[1] J. R. Pierce, "Optical channels: Practical limits with photon counting," *IEEE Trans. Commun.*, vol. COM-26, pp. 1819–1821, 1978.

[2] J. R. Pierce, E. C. Posner, and E. R. Rodemich, "The capacity of the photon counting channel," *IEEE Trans. Inform. Theory*, vol. IT-27, pp. 61–77, 1981.

[3] R. J. McEliece, "Practical codes for photon communication," *IEEE Trans. Inform. Theory*, vol. IT-27, pp. 393–398, 1981.

[4] C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication*. Urbana, IL: The Univ. Illinois Press, 1959.

[5] M. Charbit and C. Benjaballah, "Probability of error and capacity of PPM photon counting channel," *IEEE Trans. Commun.*, vol. COM-34, pp. 600–605, 1986.

Comments on "Fundamental Conditions Governing TDM Switching Assignments in Terrestrial and Satellite Networks"

SOUNG C. LIEW

Abstract—**The problem in the above paper[1] is formulated in terms of a max-flow network problem. The main theorem in the paper can be proved quite simply using the max-flow–min-cut theorem once the proper way of looking at the problem is identified.**

An alternative, and perhaps simpler proof of Theorem I of the above paper[1] is presented. This proof uses the max-flow network formulation. Since max-flow is a well-studied combinatorial optimization problem, it may shed some light on designing good algorithms for the problem concerned.

Unless otherwise defined, the notation here is adopted from the paper. The assumptions are listed here for reference

$$\sum_{i=1}^{M}\sum_{j=1}^{M} t_{ij} = NC \tag{1}$$

$$\sum_{j=1}^{M} t_{ij} \le C, \quad i = 1 \text{ to } M \tag{2}$$

$$\sum_{i=\Omega_1(g)}^{\Omega_2(g)}\sum_{j=1}^{M} t_{ij} = K_g C, \quad g = 1 \text{ to } G \tag{3}$$

$$\sum_{i=1}^{M} t_{ij} \le C, \quad j = 1 \text{ to } M \tag{4}$$

$$\sum_{i=1}^{M}\sum_{j=\Omega_1'(g')}^{\Omega_2'(g')} t_{ij} = K'_{g'} C, \quad g' = 1 \text{ to } G'. \tag{5}$$

In addition, (2) and (4) are satisfied with equality only if P_i and P'_j are equal to 1, respectively.

As suggested, it suffices to show how to find a 0-1 matrix $T' = (t'_{ij}) \le T = (t_{ij})$ such that

$$\sum_{i=1}^{M}\sum_{j=1}^{M} t'_{ij} = N \tag{6}$$

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[1] K. Y. Eng and A. S. Acampora, *IEEE Trans. Commun.*, vol. COM-35, pp. 755–761, July 1987.

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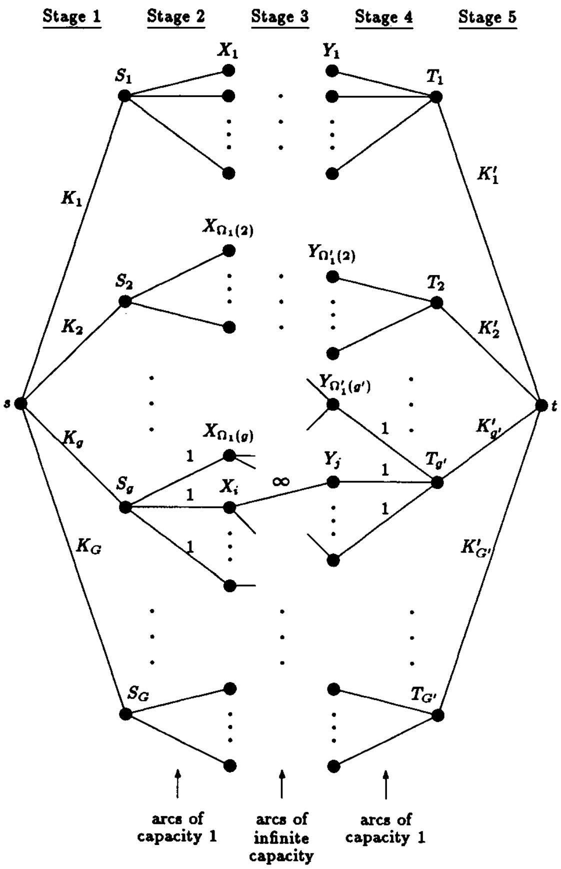


Note: implicit directions of arcs are from left to right.

Fig. 1. Max-flow network model.

$$\sum_{i=\Omega_1(g)}^{\Omega_2(g)} \sum_{j=1}^{M} t'_{ij} = K_g, \quad g = 1 \text{ to } G \tag{7}$$

$$\sum_{i=1}^{M} \sum_{j=\Omega'_1(g')}^{\Omega'_2(g')} t_{ij} = K'_{g'}, \; g' = 1 \text{ to } G'. \tag{8}$$

The associated max-flow network that will be used in the proof here is depicted in Fig. 1. Stage 3 models the traffic t_{ij}. Arc (X_i, Y_j) of infinite capacity[2] exists iff $t_{ij} > 0$. Stage 2 models the traffic originating from the individual sources. Arc (S_g, X_i) of capacity 1 exists iff X_i belongs to the gth source group and $\Sigma_{j=1}^{M} t_{ij} > 0$, i.e., there is traffic out of X_i. Similarly, in Stage 4, arc $(Y_j, T_{g'})$ of capacity 1 exists iff Y_j belongs to the g'th destination group and $\Sigma_{i=j}^{M} t_{ij} > 0$. Finally, in Stage 1, for each g such that $1 \leq g \leq G$, introduce an arc (s, S_g) of capacity K_g, and in Stage 5, for each g' such that $1 \leq g' \leq G'$, introduce an arc $(T_{g'}, t)$ of capacity $K'_{g'}$.

The max-flow problem is to find the maximum flow from node s to node t subject to the arc-capacity constraints. Let $F_{X_iY_j}$ denote the flow from node X_i to node Y_j in Stage 3. $F_{X_iY_j}$ is guaranteed to be either 0 or 1 for most max-flow algorithms thanks to the total unimodularity of the associated node-arc incidence matrix [1]. We claim that the matrix $(F_{X_iY_j})$ is a valid candidate for T'.

The reader can easily verify that the arc-capacity constraints guarantee the left sides of (6), (7), and (8) to be less than or equal to the corresponding right-sides. Therefore, the resulting solution will not violate the conditions dictated by the original physical situation.

Theorem I is proved by showing that the maximum flow is N. A cut partitions the nodes in Fig. 1 into two sets, Z and Z^c: $s \in Z$ and $t \in Z^c$. The cut value is the sum of the capacities of the arcs originating in Z and terminating in Z^c. The max-flow-min-cut theorem says that the maximum flow is equal to the minimum cut. We know that the maximum flow cannot exeed N. Hence, to prove Theorem I, it suffices to show the value of any cut is at least N.

Consider Fig. 2 in which we show a typical cut. Four sets of node groups are defined as follows: $S = \{S_g : 1 \leq g \leq G\}$, $T = \{T_{g'} : 1 \leq g' \leq G'\}$, $X = \{X_i : \Sigma_{j=1}^{M} t_{ij} > 0]$, and $Y = \{Y_j : \Sigma_{i=1}^{M} t_{ij} > 0]$. A cut partitions each of these sets into two sets, the set of nodes belonging to Z and those belonging to Z^c. Since an arc from X to Y has infinite capacity, we will consider only cuts such that there are no arcs from $X \cap Z$ to $Y \cap Z^c$. The lower part of Fig. 2 depicts the situation. Each arrow, called a *composite arc*, represents a group of arcs from one node set to another. The letter adjacent to a composite arc, not incident on s or t, is the number of the arcs it contains. The letter adjacent to a composite arc incident on s or t is the sum total of the capacities of the arcs it represents. The letters will be referred to as the capacities of the composite arcs. Note that the capacity of a composite arc in Stage 3 is finite even though the capacity of the arc it contains are infinite.

In order to simplify the proof, we will use the following mental picture of messages traversing the network shown in Fig. 1. For each pair (X_i, Y_j), we have a set of t_{ij} messages. All messages corresponding to a pair (X_i, T_j) will traverse the unique path $(s, S_g, X_i, Y_j, T_{g'}, t)$ in the network in Fig. 1 where g and g' are such that $\Omega_1(g) \leq i \leq \Omega_2(g)$ and $\Omega'_1(g') \leq j \leq \Omega'_2(g')$. Equation (3) implies that there are exactly K_gC messages that traverse arc (s, S_g) in Stage 1. Similarly, there are exactly K'_gC messages that traverse arc $(T_{g'}, t)$. By (2) and (4), there are no more than C messages that traverse any arc in Stages 2, 3, or 4. The above facts lead to the following observation.

Observation 1: Let x be a composite arc and r be the total number of messages that traverse the group of arcs represented by x. We refer to r as the number of messages that traverse composite arc x. If u is the capacity of x then $r \leq uC$, with equality if x is in Stage 1 or Stage 5.

Note that by conservation of messages, the number of messages entering equals to the number of messages leaving a set of nodes that does not contain s or t.

The value of an arbitrary cut, as shown in the lower part of Fig. 2, is

$$b + e + l + n. \tag{9}$$

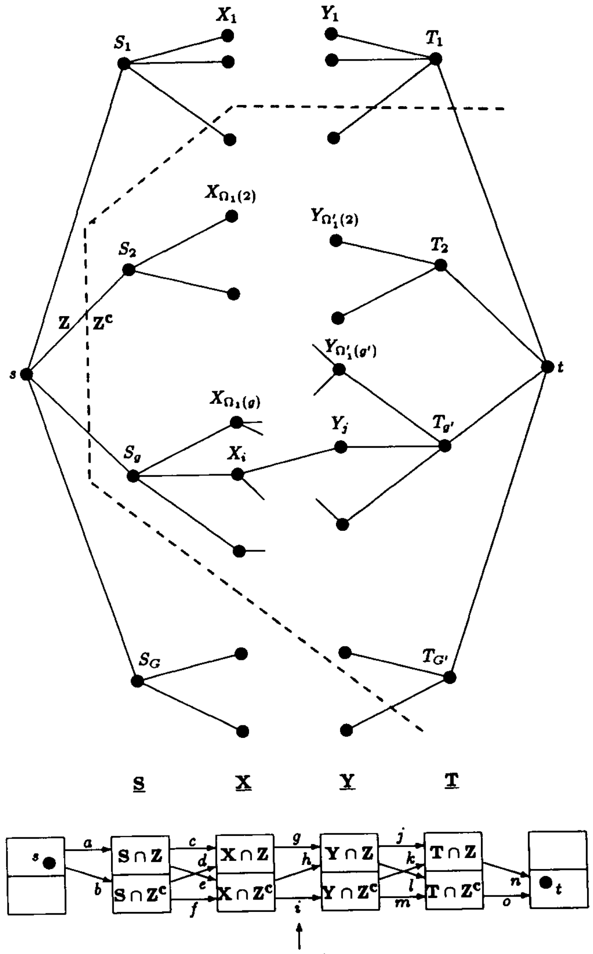
Also, by definition, we have

$$n + o = \sum_{g'=1}^{G'} K'_{g'} = N. \tag{10}$$

Proof of Theorem I: We now show $b + e + l \geq o$ and therefore the cut value $b + e + l + n \geq n + o = N$. Referring to Fig. 2, Observation 1 implies the number of messages that leave $T \cap Z^c$ is exactly oC. By the conservation of messages, the number of messages entering $T \cap Z^c$ must also be oC. These messages come from Y, and of these, the number of messages from $Y \cap Z$ is at most lC by Observation 1. Furthermore, all messages from $Y \cap Z^c$ must come from $X \cap Z^c$. Hence,

$$lC + \sum_{i: X_i \in X \cap Z^c} \sum_{j=1}^{M} t_{ij} \geq oC; \tag{11}$$

[2] Alternatively, we may assume unit capacity, but the proof later needs to be modified.



No arc from $\mathbf{X} \cap \mathbf{Z}$ to $\mathbf{Y} \cap \mathbf{Z}^c$ for a noninfinite flow.

Fig. 2. An arbitrary cut with value below ∞.

the second term on the left side sums to the number of messages that go into $X \cap Z^c$. But the messages that go into $X \cap Z^c$ come from S. There are at most eC messages from $S \cap Z$, and those from $S \cap Z^c$ must not exceed bC. Hence,

$$bC + eC \geq \sum_{i: X_i \in X \cap Z^c} \sum_{j=1}^{M} t_{ij} \tag{12}$$

Applying the above to (11), we have $b + e + l \geq o$ and the proof is completed.

Concluding Remarks

Using the algorithm in [2], the complexity of the max-flow problem is $O(M^3)$. Since there are C time slots, each requiring solving a max-flow problem, the complexity of the overall problem is $O(CM^3)$. Solving the C individual max-flow problems separately is wasteful and further work on how to integrate them together will be worthwhile.

References

[1] C. H. Papadimitriou and K. Steiglitz, *Combinatorial Optimization*. Englewood Cliffs, NJ: Prentice-Hall, 1982.

[2] V. M. Malhotra, M. P. Kumar, and S. N. Maheshwari, "An $O(|v|^3)$ algorithm for finding maximum flows in networks," *Inform. Proc. Letter*, vol. 7, no. 6, pp. 135–150, Oct. 1978.

Encoding of Images Based on a Lapped Orthogonal Transform

PHILIPPE M. CASSEREAU, DAVID H. STAELIN, AND GERHARD DE JAGER

***Abstract*—Unitary transform image coding has been successfully applied to image data compression. However, traditional block transform image coding systems generate artifacts near block boundaries which degrade low bit rate coded images. To reduce these artifacts a new class of unitary transformations, defined here as lapped orthogonal transforms (LOT), has been investigated.**

The basis functions upon which the signal is projected are overlapped for adjacent blocks. An example of a LOT optimized in terms of energy compaction was numerically derived using an augmented Lagrangian optimization algorithm.

Using this LOT, intraframe coding experiments for 256 × 240 pixel images were performed at bit rates between 0.1 and 0.35 bits/pixel. The LOT improved the coded image subjective quality over other transforms such as the discrete cosine transform (DCT) and the short-space Fourier transform (SSFT). The LOT was also used in interframe full-motion video coding experiments for head and shoulder sequences at 28 and 56 kbits/s. Experiments designed to measure the subjective quality assessment showed that significant improvement resulted at low data rates and if no motion compensation were used. However, the improvement was no longer significant at 56 kbits/s with full motion compensation.

I. Introduction

Transform coding is recognized as one of the most successful methods for digital image data compression. In transform coding systems the digital video signal is typically divided into blocks, perhaps containing 8 × 8 pixels, which are then subjected to an energy-preserving unitary transformation. The aim of the transformation is to convert statistically dependent picture elements (pixels) into a set of essentially independent transform coefficients, preferably packing most of the signal energy (or information) into a minimum number of coefficients. The resulting transform coefficients are quantized, coded, and transmitted. At the receiver the video signal is recovered by computing the inverse transformation after decoding the transmitted data [1]–[3].

The input signal F represents the digitized image which can be viewed as a matrix of size $R \times R$ where R is the resolution of the image. The representation of the video signal in the transform domain is the matrix F_1 comprising $R \times R$ real transform coefficients. With a separable two-dimensional transformation, the matrix F_1 is derived as follows:

$$F_1 = TFT^t \tag{1}$$

where T^t indicates the transposed matrix. The $R \times R$ matrix T is unitary and represents the one-dimensional transform kernel. The rows of the transform matrix T are defined as the transform basis functions.

In most transform coding systems, prior to transformation

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