Many-to-One Throughput Capacity of IEEE 802.11 Multihop Wireless Networks

Chi Pan Chan, *Student Member*, *IEEE*, Soung Chang Liew, *Senior Member*, *IEEE*, and An Chan, *Student Member*, *IEEE*

Abstract—This paper investigates the many-to-one throughput capacity (and by symmetry, one-to-many throughput capacity) of IEEE 802.11 multihop networks, in which many sources send data to a sink. An example of a practical scenario is that of a multihop mesh network connecting source and relay nodes to an Internet gateway. In the trivial case where all source nodes are just one hop from the sink, the system throughput can approach L_s, where L_s is the throughput capacity of an isolated link consisting of just one transmitter and one receiver. In the nontrivial case where some source nodes are more than one hop away, one can still achieve a system throughput of L_s by sacrificing and starving the non-one-hop source nodes—however, this degenerates to an unacceptable trivial solution. We could approach the problem by the following partitioning: preallocate some link capacity aL_s $(0 \leq a \leq 1)$ at the sink to the one-hop source nodes and then determine the throughput for the source nodes that are two or more hops away based on the remaining capacity $L = (1 - a)L_s$. The throughput of the one-hop nodes will be around aL_s. This paper investigates the extent to which the remaining capacity L can be used efficiently by the source traffic that is two or more hops away. We find that for such source traffic, a throughput of L is not achievable under 802.11. We introduce the notion of "canonical networks," a general class of regularly structured networks that allow us to investigate the system throughput by varying the distances between nodes and other operating parameters. When all links have equal length, we show that $2L/3$ is the upper bound for general networks, including random topologies and canonical networks. When the links are allowed to have different lengths, we show that the throughput capacity of canonical networks has an analytical upper bound of $3L/4$. The tightness of the bound is confirmed by simulations of 802.11 canonical networks, in which we obtain simulated throughputs of $0.74L$ when the source nodes are two hops away and $0.69L$ when the source nodes are many hops away. We conjecture that $3L/4$ is also the upper bound for general networks. Our simulations show that 802.11 networks with random topologies operated with AODV routing typically achieve throughputs far below $3L/4$. Fortunately, by properly selecting routes near the gateway (or by properly positioning the relay nodes leading to the gateway) to fashion after the structure of canonical networks, the throughput can be improved by more than 150 percent: indeed, in a dense network, deactivating some of the relay nodes near the sink can lead to a higher throughput.

Index Terms—Wireless mesh networks, many-to-one, one-to-many, data-gathering networks, 802.11, Wi-Fi, multihop networks.

1 Introduction

Many-to-one communication is a common communication mode. In multihop wireless mesh networks (e.g., IEEE 802.11s network [1]), there is likely to be little traffic between client stations. Most clients would only want to connect to the core wired Internet via an Internet gateway, using relay nodes if necessary. The client stations and the Internet gateway form a many-to-one relationship. The placement of the relay nodes and the routing of traffic from the clients to the Internet gateway can affect the throughput significantly.

This paper investigates the many-to-one throughput capacity of IEEE 802.11 multihop networks. By symmetry, the throughput capacity thus found is also the same as that in a one-to-many scenario in which a source node generates multiple *distinct* data streams to be forwarded to their respective sinks (note that this is not to be confused with the multicast scenario in which the same data is to be forwarded to multiple sinks).

Gupta and Kumar [2] analyzed the capacity in the many-to-many situation. It provides the basic model that can be adapted for use in the analysis of the many-to-one communication. As a loose bound, it is obvious that the many-to-one throughput capacity is upper bounded by L_s [2], [3], [4], where L_s is the single-link throughput capacity, since this is the rate at which the sink can receive data. There is a high probability, however, that the throughput capacity is lower than L_s for a many-to-one network [4]. This paper follows the approach used in [2], [3], and [4] in characterizing which nodes can transmit together without packet collisions. The main difference is that here, we are interested in the throughput capacity obtained under the IEEE 802.11 distributed MAC protocol [5]. Specifically, we integrate into our analysis the effects of carrier sensing, the existence of an ACK frame for each DATA frame transmission, and the distributed nature of the CSMA protocol, while [2], [3], and [4] do not, and their bounds are obtained with the implicit assumption of perfectly scheduled transmissions.

One can in principle achieve the throughput capacity of L_s in a many-to-one network by sacrificing and starving the

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source nodes that are more than one hop away from the sink. However, this degenerates to an unacceptable trivial solution with only the one-hop source nodes getting any throughput. We could approach the problem by the following partitioning: preallocate some link capacity aL_s $(0 \leq a \leq 1)$ at the sink to the one-hop source nodes and then determine the throughput capacity for the source nodes that are two or more hops away based on the remaining capacity $L = (1 - a)L_s$. For example, the airtime at the sink could be partitioned so that one-hop traffic to the sink and multihop traffic to the sink do not use overlapped airtime. Since the one-hop source nodes do not need any relay, each unit of traffic consumes one unit of the link capacity of the sink. The throughput of the one-hop nodes will be around aL_s. Because of the need for relay, each unit of source traffic that is two or more hops away in general consumes more than one unit of the link capacity of the sink. This paper investigates the extent to which the allocated capacity L can be used efficiently by the source traffic that is two or more hops away.

Indeed, based on the solution to the case where all source nodes are two or more hops away, one could turn around to determine the capacity to preallocate to the one-hop nodes to achieve fairness among nodes. For example, suppose that there are six sources nodes that are two or more hops away and that the throughput capacity for them is found to be $2L/3$. Furthermore, suppose that there are three one-hop source nodes to which we would also like to allocate a throughput capacity of $L/3$ so that the ratio of allocations is 2 : 1 (in general, the allocation to the one-hop source nodes could be ρL for any $\rho \geq 0$ to achieve a ratio of $2/3 : \rho$). Solving $aL_s = L/3$, $L = (1 - a)L_s$, gives $L = 3L_s/4$ and $a = 1/4$. That is, one quarter of the link capacity at the sink should be preallocated to the one-hop source traffic.

This paper introduces the notion of "canonical networks," a general class of regularly structured networks that allows us to investigate the throughput capacity systematically by varying the distances between nodes and other operating parameters. There are three main contributions:

1. When all links have equal length, we establish that the throughput capacity for 802.11 many-to-one networks is upper bounded by $2L/3$ regardless of the network structure. The capacity achieved by canonical networks in simulation is $0.619L$, which is close to $2L/3$. When link lengths are allowed to vary, the throughput capacity of canonical networks can be upper bounded by $3L/4$. We conjecture that this is also the upper bound for general networks.
2. Our studies of the canonical networks yield much insight on how a many-to-one network should be designed. Our simulations show that 802.11 networks with random topologies operated with AODV routing achieves throughputs far below the $3L/4$ upper bound of canonical networks. As such, the popular AODV routing protocol is far from optimal for many-to-one communication. For the same random topology, an NP-hard optimization problem formulation yields a solution in which the selected routes near the sink form a structure resembling the canonical network. As a heuristic alternative to the NP-hard problem, routing or network design could be fashioned after the canonical network. Our investigation shows that a "*manifold*" *canonical network* structure near the center may yield a throughput improvement of more than 150 percent relative to that obtained by using AODV routing in general many-to-one networks with random topologies. Indeed, in a dense network, it is worthwhile to deactivate some of the relay nodes near the sink judiciously.
3. We find that ensuring that the many-to-one network is hidden-node free (HNF) leads to a higher throughput as compared to not doing so. This is in contrast to the many-to-many case, in which the large carrier-sensing range required to ensure the HNF property may lower the throughput due to the increased exposed-node (EN) problem [6]. This observation is used as a design principle in much of the study in 1 and 2 above.

The rest of this paper is organized as follows: Section 2 provides the definitions and assumptions used in our analysis. Section 3 derives the throughput capacities of canonical networks and presents simulation results to support our findings. In addition, we demonstrate the desirability of the HNF property in many-to-one networks. Section 4 investigates general networks beyond the canonical network structure. We show that the routes selected by solving an NP-hard optimization problem form a structure near the center that resembles the optimal canonical network. We then apply this insight to demonstrate the desirability of designing the network according to a "manifold" canonical-network structure near the sink. Section 5 concludes this paper.

2 Definitions and Assumptions

2.1 Definitions

We first provide some definitions used in our analysis.

Definition 1. *The* source nodes *are nodes that generate data traffic.*

Definition 2. *The* sink node *is the center to which the data collected at the source nodes are to be forwarded.*

Definition 3. *The* relay nodes *relay data traffic from the source nodes to the sink node.*

Note that a node can be classified as one of the following:

1. a source node,
2. a sink node,
3. a relay node, or
4. both a source node and a relay node.

Fig. 1 shows a general many-to-one network with a random topology. The sink node (black in color) is at the center, surrounded by source nodes and relay nodes.

Definition 4. *The throughput capacity with respect to a multiaccess protocol p (e.g.,* IEEE 802.11*), C_p, is the total rate at which the data can be forwarded to the sink node using that protocol. The transmission schedule by the links is dictated by the protocol.*

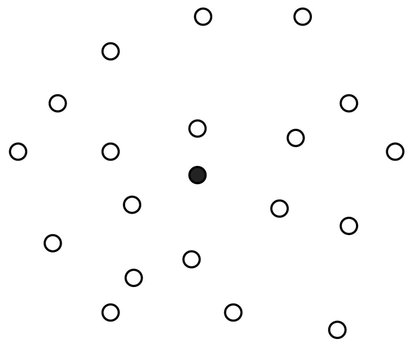

Fig. 1. A general many-to-one network.

This paper focuses on the throughput capacity under the 802.11 CSMA protocol, $C_{802.11}$, assuming that the source nodes are two or more hops away from the sink. Henceforth, by throughput capacity, we mean $C_{802.11}$. For illustration, let us consider the two-chain linear topology shown in Fig. 2. Suppose that only nodes 2 and $2'$ are the source nodes. Under "perfect scheduling," nodes 1 and $2'$ will transmit together, and nodes $1'$ and 2 will transmit together. This yields a throughput capacity of L. Under 802.11, however, the transmissions are usually not perfectly aligned in time. In addition, a DATA frame is followed by an ACK frame in the reverse direction. Consider the case of simultaneous DATA frame transmissions by nodes 1 and $2'$. Suppose that the transmission of node 1 completes first while the transmission node $2'$ is ongoing. When node 0 returns an ACK to node 1, this ACK also reaches node $1'$, causing a collision there. Thus, under 802.11, simultaneous transmissions by nodes 1 and $2'$ will result in a collision unless the completion times of their DATA transmissions are perfectly aligned, which is rare. In this case, $C_{802.11}$ is at best $2L/3$, since at best node 2 and $2'$ can transmit together, and nodes 1 and $1'$ will need to transmit at separate times because they have a common target receiver, the sink.

For many-to-one networks, the capacity bottleneck is likely to be near the sink node because all traffic travels toward the sink node. Specifically, relay nodes near the sink node are responsible for forwarding more traffic, and these nodes contend for access of the wireless medium because they are close to each other. To obtain an idea on the upper limit of the throughput capacity under 802.11, we consider a class of networks referred to as the canonical networks. An example of a canonical network is shown in Fig. 3. We show that $3L/4$ is the upper bound of the throughput capacity of canonical networks, and conjecture that this is also the upper bound for networks with general structures. We will motivate the study of the canonical networks shortly. In the special case in which all links have equal length, the throughput capacity is upper bounded by $2L/3$ regardless

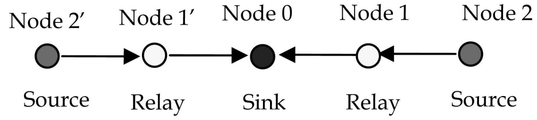

Fig. 2. A two-chain many-to-one network with equal link length.

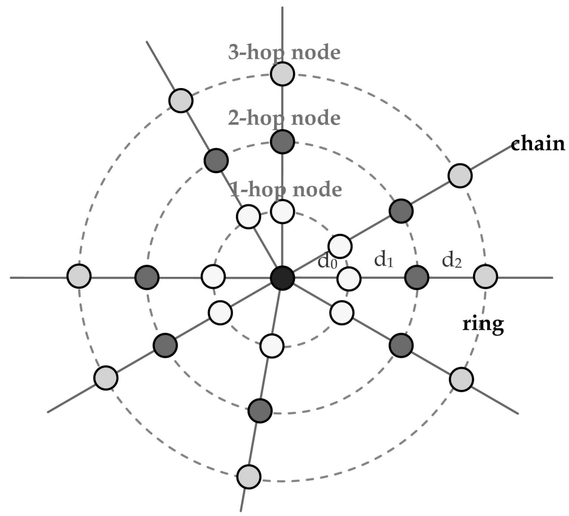

Fig. 3. A canonical network.

of the network structure. We now define the canonical networks.

Definition 5. *A* chain *is formed by a sequence of* at least three *nodes leading to the sink node. Traffic is forwarded from one node to the next node in the sequence on its way to the sink node. A* linear *chain is a chain that is a straight line.*

In Fig. 3, for example, there are eight linear chains.

Definition 6. *An i-hop* node *is a node that is i hops away from the sink node in a chain (see Fig. 3).*

Definition 7. *A* canonical network *is formed by a number of linear chains leading to a* common *center sink node; the nodes in different chains are distinct, except for the sink node. In addition, the distance between an i-hop node and an $(i+1)$-hop node, d_i, is the same for all the linear chains (see Fig. 3).*

Definition 8. *A* ring *is a circle centered on the sink node. An i-hop ring consists of all the i-hop nodes of the different linear chains in a canonical network (see Fig. 3).*

2.2 Motivation for the Study of Canonical Networks

Canonical networks have regular structures and can be analyzed more easily than general networks. We conjecture that the upper bound of throughput capacity obtained for canonical networks is also the upper bound for general networks. Consider the following intuitive argument: 1) In a densely populated network (say, infinitely dense), we may choose to form linear chains from the source nodes to the center sink node for routing purposes. Since the direction of traffic flow is pointed exactly to the center, there is no "wastage" with respect to the case in which the routing direction is at an angle to the center. 2) We have defined the class of canonical networks to be quite general in that we do not restrict the number of linear chains in it. Neither do we limit the distance d_i. In deriving the capacity of the canonical network later, we allow for the possibility of an infinite number of linear chains and arbitrarily small d_i. This provides us with a high degree of freedom in identifying the best structured canonical networks. The above intuitive reasoning will be validated by simulation results later. In addition, we will show later that in a general many-to-one network with many nodes (so that there is a high degree of freedom in forming routes), establishing a canonical-network-like structure near the center for routing purposes will generally lead to superior throughput performance.

2.3 Assumptions

In this paper, unless otherwise stated, we further assume the following:

1. The nodes and links are homogenous. They are configured similarly, i.e., they have the same transmission power, carrier-sensing range (CSRange), transmission rate, etc.
2. An ACK frame is sent by the receiver when a DATA frame is received successfully, as per the 802.11 DCF operation.
3. The following constraints apply to simultaneous transmissions [2], [7]. Consider two links $(T1, R1)$ and $(T2, R2)$, where $T1$ and $T2$ are transmitters, while $R1$ and $R2$ are receivers. For simultaneous transmissions without collisions, they must satisfy *all* the eight inequalities below:

$$\begin{aligned}
&|X_{T2} - X_{R1}| > (1+\Delta)|X_{T1} - X_{R1}|,\\
&|X_{R2} - X_{R1}| > (1+\Delta)|X_{T1} - X_{R1}|,\\
&|X_{T2} - X_{T1}| > (1+\Delta)|X_{T1} - X_{R1}|,\\
&|X_{R2} - X_{T1}| > (1+\Delta)|X_{T1} - X_{R1}|,\\
&|X_{T1} - X_{R2}| > (1+\Delta)|X_{T2} - X_{R2}|,\\
&|X_{R1} - X_{R2}| > (1+\Delta)|X_{T2} - X_{R2}|,\\
&|X_{T1} - X_{T2}| > (1+\Delta)|X_{T2} - X_{R2}|,\\
&|X_{R1} - X_{T2}| > (1+\Delta)|X_{T2} - X_{R2}|,
\end{aligned} \tag{1}$$

where X_i is the location of node i, $|X_i - X_j|$ is the distance between X_i and X_j, and $\Delta > 0$ is the distance margin (see the next paragraph). These are the physical constraints that prevent DATA-DATA, DATA-ACK, and ACK-ACK collisions.

The received power function $P(d)$ can be expressed in the form of

$$P(d) \propto P_t/d^{\alpha}, \tag{2}$$

where P_t is the transmit power, d is the distance, and α is the path-loss exponent, which typically ranges from two to six according to different environments [9]. By the assumptions that all the nodes have the same transmission power and a Signal-to-Interference Ratio (SIR) requirement of 10 dB, at $R1$, we require

$$\frac{P(|X_{T1} - X_{R1}|)}{P(|X_{T2} - X_{R1}|)} > SIR, \tag{3}$$

giving

$$\frac{|X_{T2} - X_{R1}|}{|X_{T1} - X_{R1}|} > 10^{1/\alpha} = 1 + \Delta.$$

4. In 802.11 networks, there are two types of packet collisions: collisions due to hidden nodes (HNs) (see explanation of *Assumption 5* below or [7]) and collisions due to simultaneous countdown to zero in the backoff period of the MAC of different transmitters. In much of our throughput-capacity analysis, we will neglect the latter collisions and assume that they have only small effects toward throughput capacity, a fact that has been borne out by simulations and that can be understood through intuitive reasoning, particularly for a network in which a node is surrounded by only a few other *active nodes* who may collide with it. As will be shown later in this paper, this is generally a characteristic of a network with good throughput performance (see the results in Figs. 13 and 16, for example). Also, an upper bound on throughput capacity obtained by neglecting the countdown collisions is still a valid upper bound. It is a good upper bound so long as it is tight. We will see later that the upper bounds we obtain are reasonably tight when verified against simulations results in which countdown collisions are taken into account. In the remainder of this paper, unless otherwise stated, the term "collisions" refers to collisions due to HN (i.e., caused by the failure of carrier sensing) rather than the simultaneous countdown to zero.
5. In this paper, unless otherwise specified, we assume the so-called HNF Design (HFD) [7] in the network. That is, we design the network such that simultaneous transmissions that will cause collisions can be carrier sensed by transmitters and be avoided. A reason for this assumption is that for many-to-one communication, eliminating HNs is worthwhile (see the simulation results in Section 3.3). According to [7], HFD requires

   a. the use of Receiver Restart (RS) Mode and
   b. a sufficiently large CSRange.

   This paper assumes the 802.11 basic mode and that RTS/CTS are not used. We briefly describe the HFD requirements here. More details can be found in [7].

   Fig. 4 is an example showing that no matter how large CSRange is, the HN phenomenon can still occur in the absence of RS Mode. In the figure, $T1$ and $T2$ are more than CSRange apart, and so, simultaneous transmissions can occur. Furthermore, the SIR is sufficient at $R1$ and $R2$ so that no "physical collisions" occur. But HNs can still happen, as described below.

   Assume that $T1$ starts first to transmit a DATA frame to $R1$. After the physical-layer preamble of the packet is received by $R2$, the typical behavior is such that $R2$ will "capture" the packet and will not attempt to receive another new packet while $T1$'s DATA is ongoing. If at this time, $T2$ starts to transmit a DATA to $R2$, $R2$ will not receive it and

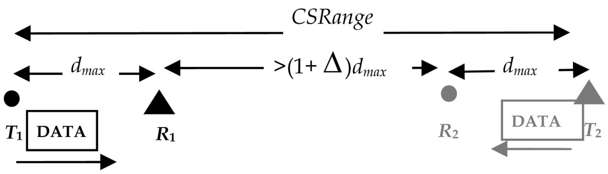

Fig. 4. Lack of RS Mode leads to HNs no matter how large the CSRange and SIR are.

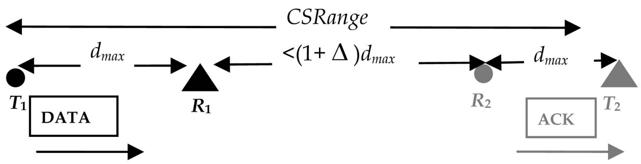

Fig. 5. With the RS Mode, a CSRange that is not sufficiently large still leads to HNs due to an insufficient SIR.

will not reply with an ACK to $T2$, causing a transmission failure on link $(T2, R2)$. This is the default receiver mode assumed in the NS-2 simulator [10] and most 802.11 commercial products. Note that the example in Fig. 4 is independent of the size of CSRange.

This HN problem can be solved with the RS Mode, which can be enabled in some 802.11 products (e.g., Atheros Wi-Fi chips; however, the default is that this mode is not enabled). With RS Mode, a receiver will switch to receive the stronger packet if its power is, say, 10 dB higher than the power of the current packet. The example in Fig. 4 will not give rise to HNs with RS if CSRange is sufficiently large.

The RS Mode alone, however, cannot prevent HNs without a sufficiently large CSRange. To see this, consider the example in Fig. 5. Assume that $T1$ transmits a DATA to $R1$ first. During the DATA's period, $T2$ starts to send a shorter DATA packet to $R2$. With the RS Mode, $R2$ switches to receive $T2$'s DATA and sends an ACK after the reception. If $T1$'s DATA is still in progress, $R2$'s ACK will corrupt the DATA at $R1$, since the distance between $R1$ and $R2$ is within interference range $((1+\Delta)d_{\max})$. To prevent $T2$ from transmission (hence, the collision), the following must be satisfied:

$$|X_{T1} - X_{T2}| \leq CSRange. \tag{4}$$

Reference [7] proved that in general, if $CSRange > (3+\Delta)\, d_{max}$, where d_{max} is the maximum link length, then HN can be prevented in any network. However, for a specific network topology, e.g., the canonical network, the required CSRange can be smaller.

6. Throughout this work, we primarily focus on the pairwise-interference model [2], [7]. The concept of CSRange and the constraints in (1) rely on this assumption. An analysis that at the outset takes into account the simultaneous interferences from more than one source will complicate things significantly. Therefore, given a network topology, our approach is to first identify the capacity based on pairwise interference analysis only and then verify that the capacity is still largely valid under multiple interferences (this verification is done in Section 3.4).

3 Canonical Networks

In this section, we derive the throughput capacities of canonical networks. Section 3.1 analyzes two kinds of networks: equal-link-length and variable-link-length networks. Simulation results are presented and discussed in Section 3.2. Section 3.3 compares the performance of HFD and non-HFD networks, and Section 3.4 verifies the results under multiple interferences.

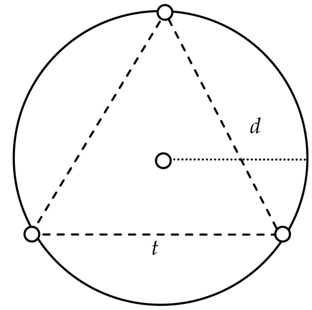

Fig. 6. Equilateral triangle inscribed in a circle.

3.1 Theoretical Analysis

3.1.1 *Equal-Link-Length Networks*

We first consider the case where all links have the same length d, i.e., $d_0 = d_1 = \cdots = d$. For concreteness, we first assume that $\alpha = 4$. We will generalize the result to other values of α later. When $\alpha = 4$, Theorem 1, which follows from Lemma 1 and Corollary 1 below, states that the throughput capacity in an equal-link-length network is upper bounded by $2L/3$.

Lemma 1. *Given three nodes on the periphery of a circle of radius d, we can identify two nodes with a distance smaller than $(1+\Delta)d$ between them.*

Proof. The three nodes form the vertices of a triangle. Consider the *equilateral* triangle inscribed on the circle of radius d and let t be the length of one side (see Fig. 6). Then,

$$t = 2d\cos\frac{\pi}{6} = 1.731\ d < (1+\Delta)d.$$

For $\alpha = 4$ and an SIR requirement of 10 dB, $\Delta = 0.78$. That is, it is not possible to inscribe a triangle with all sides no less than $(1+\Delta)d$ on the circle. □

Corollary 1. *At any time, at most two two-hop nodes can transmit at the same time.*

Proof. With reference to Fig. 7, suppose that three two-hop nodes can transmit together. In order for the ACK of any

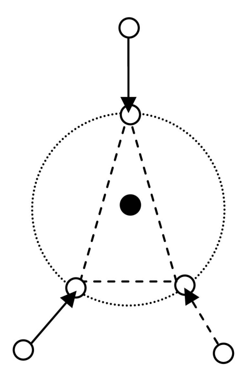

Fig. 7. At most two simultaneous transmissions from two-hop nodes.

one-hop node to not interfere with the reception of DATA packet of another transmission, the distances between the three one-hop nodes must all be larger than $(1+\Delta)d$. By Lemma 1, this is not possible. □

Theorem 1. *For equal-link-length canonical networks,* $C_{802.11} \leq 2L/3$.

Proof. Define the "airtime" usage of a node to include the transmission time of DATA packets, as well as the ACK from the receiver [8]. Let S_{ij} be the airtime occupied by the transmission of the i-hop node on the jth chain over a long time interval $[0, Time]$.

Let S_1 = the union of airtimes occupied by all one-hop nodes S_{1j}. Similarly, let S_2 = the union of airtimes occupied by all two-hop nodes S_{2j}. That is, $S_1 = S_{11} \cup S_{12} \cup \cdots \cup S_{1N}$, and $S_2 = S_{21} \cup S_{22} \cup \cdots \cup S_{2N}$. We further define $x_{ij} = |S_{ij}|/Time$.

By definition

$$|S_1 \cup S_2| \leq Time. \tag{5}$$

According to *Assumption 3*, when any one-hop node transmits, none of the other one-hop nodes or two-hop nodes can transmit at the same time if collisions are not to happen. Thus, if carrier sensing works perfectly and collisions due to simultaneous countdown to zero in the 802.11 backoff algorithm are negligible (see *Assumptions 4* and *5* in Section 2), then

$$S_1 \cap S_2 = \emptyset \tag{6}$$

and

$$S_{1i} \cap S_{1j} = \emptyset \qquad \text{for } i \neq j. \tag{7}$$

This implies that

$$|S_1| + |S_2| = |S_1 \cup S_2| \leq Time \tag{8}$$

and

$$|S_1| = |S_{11}| + |S_{12}| + \cdots + |S_{1N}|. \tag{9}$$

By Corollary 1,

$$|S_2| \geq \frac{|S_{21}| + |S_{22}| + \cdots + |S_{2N}|}{2}. \tag{10}$$

Since we focus on source traffic that is two or more hops away here, all traffic transmitted by one-hop nodes under consideration here must therefore come from two-hop nodes. By the "no collision" assumption, the sum of the airtimes of one-hop nodes must not be greater than the sum of airtimes of two-hop nodes. We have

$$|S_{11}| + |S_{12}| + \cdots + |S_{1N}| \leq |S_{21}| + |S_{22}| + \cdots + |S_{2N}|. \tag{11}$$

From (8)-(10), we have

$$|S_{11}| + |S_{12}| + \cdots + |S_{1N}| + (|S_{21}| + |S_{22}| + \cdots + |S_{2N}|)/2 \leq Time.$$

Applying (11), we get

$$(x_{11} + x_{12} + \cdots + x_{1N}) + \frac{(x_{11} + x_{12} + \cdots + x_{1N})}{2} \leq 1,$$

giving

$$x_{11} + x_{12} + \cdots + x_{1N} \leq \frac{2}{3},$$

where $(x_{11} + x_{12} + \cdots + x_{1N})L$ is the throughput. □

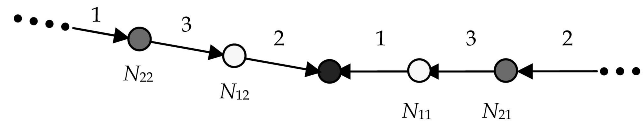

Fig. 8. Example of an equal-link-length topology, $CSRange = 2.9d$.

We now show a specific schedule on a two-chain network that achieves the capacity of $2L/3$. Consider the topology shown in Fig. 8. Following the notations in canonical networks, we use N_{ij} to denote the i-hop node in the jth chain. There are two chains, having link distance d and $CSRange = 2.9d$, which removes HNs. Recall that the general HFD has two requirements: 1) RS mode and 2) $CSRange > (3+\Delta)\, d_{max}$ [7]. For the topology in Fig. 8, it turns out that $CSRange = 2.9d$ is enough for avoiding simultaneous transmissions that lead to collision. In particular, the distance between N_{11} and N_{22} is less than $2.9d$ to prevent simultaneous transmissions, and the distance between N_{21} and N_{22} is greater than $2.9d$ to allow simultaneous transmissions that will not lead to collision.

The numbers shown on the links in Fig. 8 represent a possible transmission schedule. Links with the same number transmits at the same time. Following this pattern, the throughput capacity of $2L/3$ is "potentially" achievable. Our simulation results in Section 3.2 below show that the 802.11 protocol throughput capacity is below but close to this upper bound.

Before going to the next section, we note that Theorem 1 actually applies to not just canonical networks (the proof does not require it) but also general networks in which all links are of the same length. In other words, the chains leading to the data center need not be straight-line linear chains. Thus, Theorem 1 can be stated more generally as Theorem 1′ below.

Theorem 1′. *For equal-link-length general networks,* $C_{802.11} \leq 2L/3$.

Proof. The proof is the same as Theorem 1 since Lemma 1 and Corollary 1 apply to general networks with equal link length also. □

It turns out the results of Theorem 1 and Theorem 1′ are valid for any $\alpha < 4.192$, assuming the same SIR requirement of 10 dB. To see this, we need only to consider the inequality $t = 2d\cos(\pi/6) \leq 10^{\frac{1}{\alpha}}d$ with respect to Figs. 6 and 7, which gives $\alpha < 4.192$.

When $\alpha \geq 4.192$, a better bound than $C_{802.11} \leq 2L/3$ can be obtained since the inequality $t = 2d\cos(\pi/6) \leq 10^{\frac{1}{\alpha}}d$ that bars three simultaneous transmissions is no longer satisfied. We should then consider inscribing a square in the circle to get the condition under which four simultaneous transmissions are not possible. Doing this yields $\alpha < 6.644$. Thus, in the range $4.192 \leq \alpha < 6.644$, $C_{802.11} \leq 3L/4$.

TABLE 1
Upper Bound of $C_{802.11}$ for Equal-Link-Length Networks as Path-Loss Exponent α Varies

α	$C_{802.11}$ upperbound
$2.000 \le \alpha < 2.096$	$2L/5$
$2.096 \le \alpha < 3.322$	$L/2$
$3.322 \le \alpha < 4.192$	$2L/3$
$4.192 \le \alpha < 6.664$	$3L/4$

By similar reasoning, we can show that $C_{802.11}$ is more severely limited when $\alpha < 3.322$. In this case, at most one two-hop node can transmit at a time. When $\alpha < 2.096$, the one-hop, two-hop, three-hop, four-hop, and five-hop nodes on the same chain cannot transmit simultaneously. Table 1 summarizes the bounds for $C_{802.11}$ for different ranges of α.

3.1.2 Variable-Link-Length Networks

In this section, we consider canonical networks in which the distance between adjacent rings can be varied (i.e., d_i, $d_j, \ldots$ may be distinct, for $i \ne j$). With this assumption, the capacity is upper bounded by $3L/4$ for $2 \le \alpha \le 6$. This is proved in Theorem 2 after Lemma 2 in the following.

Lemma 2. *At any time, at most three two-hop nodes can transmit at the same time.*

Proof. Assume the contrary that we can have four two-hop nodes belonging to four different chains transmitting at the same time. With respect to Fig. 9, consider the four straight lines formed by the four nodes to the center (note that the network could have more chains, it is just that we are focusing on the four chains of the four two-hop nodes here). Four angles are formed between adjacent lines. Let $\theta \le \pi/2$ be the minimum of the four angles. Four angles are also formed between nonadjacent lines. Let $\beta \le \pi$ be the angle encompassing θ (see Fig. 9).

For simultaneous transmissions of two-hop nodes, the transmitters should not be able to carrier sense each other. This implies an upper bound for CSRange as follows:

$$CSRange < 2(d_0 + d_1)\sin\frac{\theta}{2}. \qquad (12)$$

In addition, by *Assumption 5*, to prevent collisions of the transmissions of one-hop nodes and two-hop nodes, they should be able to carrier sense each other. This implies a lower bound for CSRange. By (4), we get

$$CSRange \ge \sqrt{(d_0 + d_1)^2 + d_0^2 - 2d_0(d_0 + d_1)\cos\beta}. \qquad (13)$$

By *Assumption 3*, the receivers of simultaneous transmissions should not violate the physical constraints. By (1), we get

$$(1 + \Delta)d_1 < 2d_0 \sin\frac{\theta}{2}. \qquad (14)$$

Since there are four chains, $\theta \le \pi/2$ and $\beta \le \pi$. From the definitions of θ and β, we have

$$2\theta \le \beta \le \pi. \qquad (15)$$

In the range $0 \le \phi \le \pi$, $\cos\phi$ is a decreasing function (i.e., $-\cos\beta \ge -\cos(2\theta)$). We could replace (13) with the more relaxed condition below since we intend to prove a negative result:

$$CSRange \ge \sqrt{(d_0 + d_1)^2 + d_0^2 - 2d_0(d_0 + d_1)\cos(2\theta)}. \qquad (16)$$

Let $d_1 = c\, d_0$. We can form two inequalities from (12), (14), and (16):

$$c < \frac{\sqrt{2(1-\cos\theta)}}{(1+\Delta)}, \qquad (17)$$

$$\begin{cases} c > \frac{1-2\cos^2\theta}{1-2\cos\theta} + \frac{\sqrt{(2\cos^2\theta-1)^2+1-2\cos\theta}}{1-2\cos\theta} - 1, \\ or \\ c < \frac{1-2\cos^2\theta}{1-2\cos\theta} - \frac{\sqrt{(2\cos^2\theta-1)^2+1-2\cos\theta}}{1-2\cos\theta} - 1. \end{cases} \qquad (18)$$

The solution to c (or the lack of it) could be seen by plotting (17) and (18). Fig. 10 plots the case $\alpha = 6$ (the least stringent α for concurrent transmissions); the shadowed region is the area of solution. In general, for $2 \le \alpha \le 6$, $\theta > \pi/2$ in order that there is a solution to c. This contradicts (15). Hence, the assumption that four two-hop nodes can transmit together successfully is not valid. □

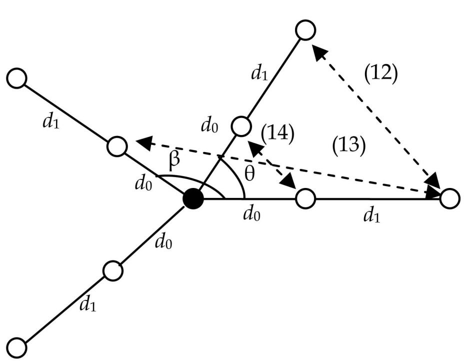

Fig. 9. Example of a four-chain canonical network.

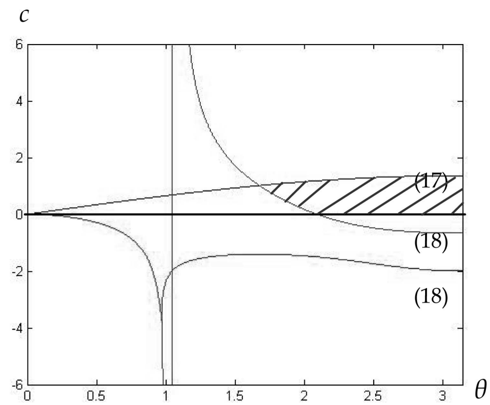

Fig. 10. Plot of (17) and (18).

Theorem 2. *For variable-link-length canonical networks,* $C_{802.11} \le 3L/4$.

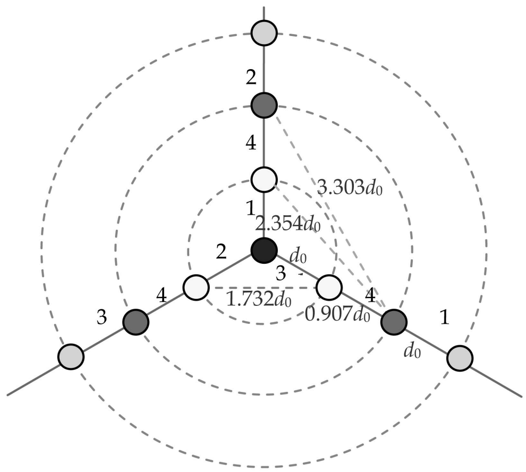

Fig. 11. Example of a three-chain canonical network, $CSRange = 2.7d$.

Proof. Similar to the proof of Theorem 1, from Lemma 2

$$|S_2| \geq \frac{|S_{21}| + |S_{22}| + \cdots + |S_{2N}|}{3}. \tag{19}$$

Hence,

$$(x_{11} + x_{12} + \cdots + x_{1N}) + \frac{(x_{11} + x_{12} + \cdots + x_{1N})}{3} \leq 1$$

or

$$x_{11} + x_{12} + \ldots + x_{1N} \leq \frac{3}{4},$$

where $(x_{11} + x_{12} + \cdots + x_{1N})L$ is the throughput. □

Fig. 11 shows an example of a three-chain canonical network and a schedule of four time slots to deliver 3 units of data to the sink from the nodes that are two or more hops away. The CSRange has to be set larger than $2.354d_0$ but smaller than $3.303d_0$. This example works for $3.564 < \alpha \leq 6$. In Fig. 11, the three two-hop nodes can transmit together and the transmissions of three-hop nodes can reuse the time slots of one-hop nodes without collisions. The example indicates that the bound of $3L/4$ may be tight[1] (i.e., the bound will be tight if 802.11 scheduling also achieves a capacity close to the perfect scheduling in Fig. 11, a fact borne out by our simulation study in Section 3.2). It can also be shown (omitted here due to the space limit) that if all the source nodes of the canonical network are exactly two hops away, perfect scheduling similar to that in Fig. 11 is possible for $2 \leq \alpha \leq 6$. However, when the source nodes are three or more hops away and $\alpha \leq 3.564$, similar perfect scheduling cannot be found, and the upper bound of $3L/4$ may not be as tight.

3.2 Simulation

We use the network simulator NS2 [10] to simulate the canonical network shown in Fig. 11. In the simulation, the RS Mode is enabled. Table 2 shows the details of the simulated configuration. Recall that this paper focuses on source traffic that is two or more hops away. To consider the "worst case" where traffic requires many hops to reach the sink, only the n-hop nodes at the boundary generate traffic in our simulations. Hence, in our simulations, $L = L_s$. Offered-load control [8] is applied to prevent the sources from injecting too much traffic into the network. For the interested reader, it has been shown in [8] that offered-load control can yield a higher throughput in multihop networks. We use constant-bit-rate applications in NS2 to generate the source traffic. For each session i, the packet generation times are t_i, $t_i + T$, $t_i + 2T, \ldots$, where t_i is the session start time, and T is the packet-generation period. The start times of different sessions are randomized. It is worth noting that the packet-generation period is different from the packet transmission schedule. The latter is governed by the 802.11 multiple-access protocol (i.e., perfect scheduling is not assumed).

TABLE 2
Simulation Configuration for Variable-Link-Length Canonical Networks

Number of chains	3
d_0	250m
d_1	242m
d_i for i>1	250m
Transmission Range	250m
Carrier Sensing Range	675m
Routing Protocol	AODV
Propagation Model	Two Ray Ground
Packet Data Size	1460 bytes

Fig. 12 shows the simulation result assuming the setup in Table 2 and $\alpha = 4$. The x-axis is the number of nodes per chain, including the sink. Given a number of nodes per chain, we vary the offered load in the simulation to identify an offered load that achieves the highest average throughput. By simulating one single link, L is determined to be around 6.24 Mbps. When the number of nodes per chain is three, i.e., the two-hop nodes are the source nodes, the throughput is 4.62 Mbps $(0.74L)$, which is very close to the theoretical capacity $3L/4$. But when the number of nodes per chain increases, the throughput drops to 4.30 Mbps $(0.69L)$.

An explanation for this phenomenon is that the scheduling scheme of IEEE 802.11 does not result in the optimal transmission schedule presented in Fig. 11 needed to achieve the $3L/4$ upper bound. That is, the incorporation of random backoff countdown time in 802.11 causes imperfect scheduling. Consider Fig. 11; it is possible for two-hop and three-hop nodes of different chains to

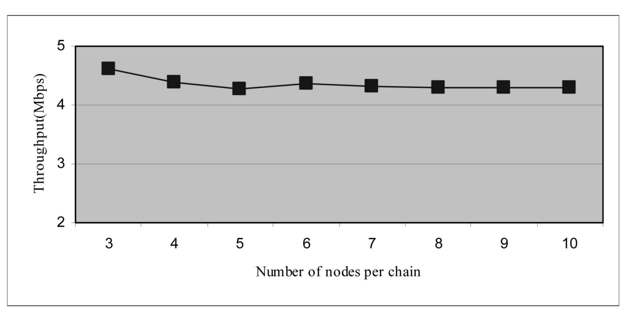

Fig. 12. Simulated throughput of a three-chain canonical network with offered-load control with $\alpha = 4$.

1. For the one-to-many network (i.e., the sink becomes the source, and the sources become the sinks with respect to the many-to-one case here), some parameters should be changed to attain the capacity of $3L/4$. Specifically, $CSRange = 1.7d_0$, and $d_i = 0.7d_0$, for $i = 1, 2, \ldots$.

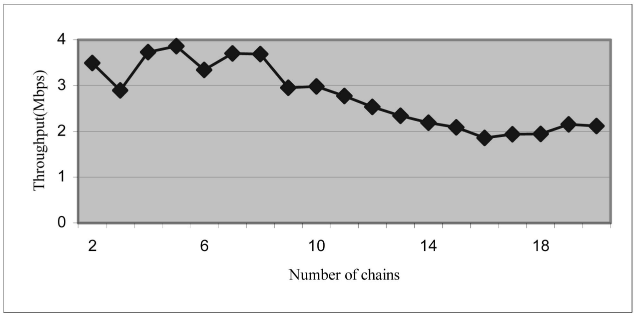

Fig. 13. Simulated throughput of equal-link-length canonical networks with offered-load control.

transmit at the same time in 802.11, since they are out of the carrier-sensing range of each other. To achieve capacity $3L/4$, however, all the two-hop nodes must transmit together. However, a three-hop transmission may prevent this, resulting in only some of the two-hop nodes transmitting together. In other words, there are times when not all two-hop nodes transmit together, meaning that $|S_2|$ cannot reach the lower bound in (19). Meeting the lower bound, however, is essential to achieving the optimal throughput $3L/4$.

Fig. 13 shows the simulation results of canonical networks with different numbers of chains but with equal link length. The simulated configuration is shown in Tables 3 and 4, and $\alpha = 4$ is assumed. The CSRange for each topology is determined by minimizing its value while preventing HNs. The throughput is obtained by varying the offered load and choosing the highest one. From the graph, the highest throughput is 3.86 Mbps $(0.62L)$, which is slightly smaller than the theoretical capacity of $2L/3$ stated in Theorem 1 and Theorem 1′. This is due to the imperfect scheduling by 802.11, which has been discussed in the previous paragraph.

In Fig. 13, the throughput converges to around 2.0 Mbps $(0.32L)$ when the number of chains increases. The convergence can be explained as follows: From the analysis in Section 3.1, we see that the bottleneck is around the center. When the number of chains is large, the area near the center will become dense. The possible transmission patterns are similar in this area, and thus, the throughput converges. In addition, note that the converged value, $0.32L$, is considerably smaller than the value achieved when the number of chains is three, $0.62L$. This is again due to the imperfect scheduling of the 802.11 MAC protocol. An interesting insight is that when the number of chains is small, the

TABLE 3
Simulation Configuration for
Equal-Link-Length Canonical Networks

Number of nodes per chain	8
d_i for all i	250m
Transmission Range	250m
Carrier Sensing Range	Refer to Table IV
Routing Protocol	AODV
Propagation Model	Two Ray Ground
Packet Data Size	1460 bytes

TABLE 4
Carrier-Sensing Range for
Equal-Link-Length Canonical Networks

Number of chains	Carrier Sensing Range
2	725m
3	875m
4	750m
5	725m
6	875m
7	800m
8	750m
9	875m
10	825m
>10	900m

possible transmission patterns that arise from "random" 802.11 MAC scheduling are more limited. And by limiting this degree of freedom, a higher throughput can actually be achieved because random transmission patterns that degrade throughputs are eliminated.

The above observation has two implications: 1) *For network design, we may want to structure the network in such a way that the number of routes leading to the center is limited*. 2) *Even for a general noncanonical network densely populated with nodes and with many routes leading to the center, it is better to selectively turn on only a subset of the nodes to limit the routes to the center*. This principle will be further discussed in Section 4.

We have also conducted experiments to investigate the tightness of the analytical upper bounds in Table 1 for a range of path-loss exponents. For $\alpha = 2$, 3, 4, 6, the measured maximum throughputs are respectively $0.36L$, $0.48L$, $0.62L$, and $0.69L$. These results show that random multiple-access scheduling by 802.11 can potentially achieve throughputs close to that of perfect scheduling.

3.3 HFD versus Non-HFD Performance

In the preceding sections, we have assumed HFD networks to simplify the analysis by eliminating the effect of collision. We now investigate the performance of HFD versus that of non-HFD networks. As a reminder, HFD requires

i. the use of RS Mode and
ii. a sufficiently large CSRange.

From [11], we know that increasing CSRange increases the number of ENs and decreases the number of HNs and vice versa. When the HN phenomenon is removed, say, with HFD, the EN phenomenon will be more severe, which lowers the throughput. However, that is the case for many-to-many data delivery only. For this paper, we are interested in many-to-one data delivery. Table 5 shows the simulation results with the same configuration as in Table 3 with varying CSRange, assuming that $\alpha = 4$. The shaded entries correspond to HFD. From the table, when the number of chains is between 2 and 10, the highest throughput is achieved if we choose the smallest CSRange within HFD. This shows that the best HFD configuration generally works better than non-HFD.

The better performance of HFD could be explained as follows: When CSRange is decreased, the number of HNs

TABLE 5
Simulation Result for Equal-Link-Length Canonical Networks

Throughput (Mbps)		No. of Chains								
		2	3	4	5	6	7	8	9	10
CSRange (m)	975	2.388	2.981	3.355	2.833	2.863	3.022	2.891	3.054	3.114
	925	2.793	2.993	3.329	3.518	2.837	2.805	2.943	3.270	3.108
	875	2.797	**2.999**	3.508	3.535	**3.393**	3.272	3.163	**3.384**	2.883
	825	2.795	2.490	3.513	3.483	2.615	**3.681**	3.575	3.053	**3.366**
	775	2.808	2.473	**3.724**	3.540	2.760	2.754	**3.709**	3.367	3.269
	725	**3.589**	2.226	3.210	**3.854**	2.095	2.264	3.147	3.199	2.686
	675	3.170	2.288	2.398	2.799	2.142	2.261	2.176	2.367	2.633
	625	3.166	1.806	2.219	2.657	1.735	2.020	2.670	1.906	2.156
	575	3.183	1.788	2.168	2.202	1.657	1.609	2.280	1.929	2.041

bold: highest throughput; shaded: HFD

increases and the number of ENs decreases. More links could be active when there are fewer ENs; thus, the throughput in the multiple-source, multiple-destination network could be higher in the non-HNF situation. In a many-to-one network, however, all the traffic is directed toward the same destination. With a non-HFD, although the total throughput on a link basis (point-to-point throughput) may be increased, the many-to-one throughput (or the end-to-end throughput) could not benefit from the increase, because all the traffic in the end will flow toward the bottleneck and be dropped there due to HNs. We will see later that this observation suggests a design in which the area near the center should be made HNF, while areas far away from the data center need not be HNF.

3.4 Multiple Interference

Thus far, we have considered pairwise interferences only. The analysis of pairwise interferences is appealing from the simplicity viewpoint. A more accurate model will take into account the fact that the interferences from several other simultaneously transmitting sources may add up to yield an unacceptable SIR even though each of the interferences may not be detrimental. The multiple-interference throughput capacity is in general less than or equal to that of the pairwise throughput capacity.

Nevertheless, we find that the pairwise-interference capacity is a tight bound for multiple-interference capacity. The analytical argument can be found in our technical report [12]. Here, we present simulation results for general canonical networks with an arbitrary number of chains with multiple-interference effects taken into account. We have modified the NS2 simulator to take into account the effects of multiple interferences (the modified NS2 code can be downloaded from the website in [12]). The throughput results are shown in Fig. 14. The multiple-interference throughput is only lower than the pairwise-interference throughput by a small margin, and therefore, the pairwise-interference throughput serves as a good bound for multiple-interference throughput.

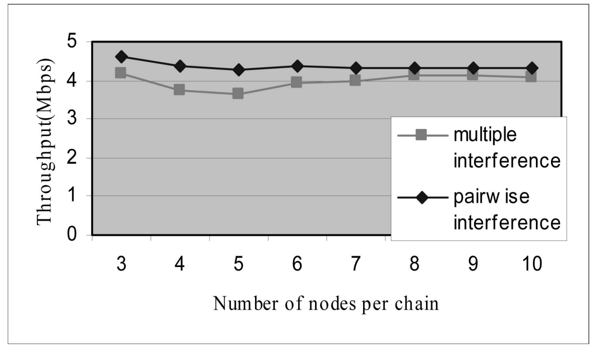

Fig. 14. Simulated throughput of a three-chain canonical network with offered-load control.

4 General Networks

In this section, we consider the throughput of general networks. We propose a method to find the capacity by selecting HNF Paths (HFPs).

4.1 Discussion of HFP

In Section 3.3, we found that the network with HNF outperforms that with HNs in terms of throughput capacity. We could have three schemes that satisfy the HNF condition for general network analysis. As one of the requirements of HFD, we assume that the RS Mode is used in all the analyses and experiments in the remainder of the paper. We assume that all nodes use a common fixed CSRange in each of the following schemes (*Assumption 1* in Section 2); however, the schemes set the fixed CSRange differently.

Scheme 1. CSRange is set to $3.78 \cdot$ TxRange, where TxRange is the transmission range. This is a sufficient condition of HNF for any networks [7].

Scheme 2. CSRange is minimized according to the network topology so that no HN exists with respect to any two links in the network. This scheme, for example, was used in the analysis of canonical networks.

Scheme 3. HFP—we select a subset of links to form paths to the center that are HNF and achieve the highest possible throughput. Since some links are not used, the CSRange can be smaller than in schemes 1 and 2 (i.e., only the links in the path are considered when fixing CSRange).

Based on Table 5, the highest throughput is achieved when we choose the smallest CSRange within HFD. Therefore, we have the following predictions for the throughputs of the different schemes above. The throughput of scheme 1 cannot be higher than that of scheme 2 (because the CSRange of some links are forced to adopt a higher value than necessary in scheme 1). Also, the throughput of scheme 2 cannot be higher than that of scheme 3 (because scheme 3 requires the HN property to be maintained only for links along the paths, and the paths that will be used are optimally chosen with regard to the throughput, whereas scheme 2 requires all links to be HNF, even for links that are not used). For an example where HFP can achieve a higher throughput than scheme 2, we add two nodes to the three-chain canonical network in Fig. 11 to yield the network in Fig. 15. In the network, link BB' interfere with link AA'. If we set CSRange to be less than

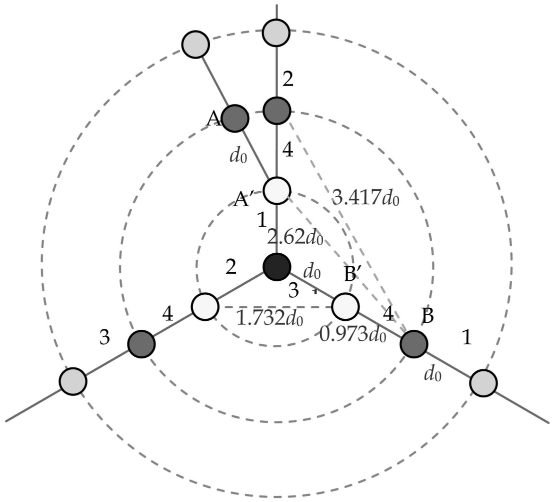

Fig. 15. Example of HFP.

$3.417d_0$, node B will become an HN of link AA'. If we set CSRange larger than $3.417d_0$, the capacity upper bound $3L/4$ cannot be achieved. On the other hand, if we use HFP, we could select the links in the canonical network only. Therefore, node A could be "switched off," and there will not be an HN problem if we set CSRange to $2.7d_0$.

4.2 Experiments and Discussions

To conserve space, this paper will not go into the details of the formulation of the HFP problem and the HFP experimental methodology. For the interested readers, such details can be found in the Appendix of our technical report [12]. In a nutshell, our approach extends that in [13] by additionally taking into consideration the effects of carrier sensing and HFD requirements. We also provide a branch-and-bound heuristic algorithm for the resulting integer linear program (ILP). Here, we only present the performance results of experiments on schemes 1, 2, and 3 and their implications. Solving the ILP of scheme 3 is computationally intensive. The experimental results of scheme 3 in this section are therefore obtained using our branch-and-bound heuristic. Schemes 1 and 2 are still solved in an optimal manner. As will be seen, even with a suboptimal heuristic, scheme 3 still yields better results.

In our experiments, we put the nodes inside a disk of radius one. A sink node is placed at the center of the disk, and six source nodes are placed at the boundary of the disk spaced evenly apart. For each source node, a node is randomly generated within the transmission range of 0.4. More nodes are generated similarly with reference to the newly created node until a node is within the transmission range from the sink node. In this way, we could ensure that there is a path from any source node to the sink node. By setting the transmission range to 0.4, the data from the source nodes will need at least three hops to reach the sink node.

Table 6 shows the experiment results for five randomly generated networks, $\mathrm{Net}_1, \mathrm{Net}_2, \ldots, \mathrm{Net}_5$. ρ_1, ρ_2, and ρ_3 are the throughputs of the three schemes. In obtaining ρ_i, $i = 1$, 2, 3, we vary the offered load at the source nodes until the highest throughput is obtained [8]. Similar to the simulations in Section 3.2, in the experiments here, we also use constant-bit-rate application in NS2 and randomly vary the starting times of different sessions by tenths of a millisecond. In Table 6, scheme 3 has improvements of 4.8 percent to 43.8 percent over scheme 1 and 4.8 percent to 23.2 percent over scheme 2. As related earlier, we did not solve scheme 3 optimally but rather used a heuristic. Therefore, the CSRange (CS_3) found for HFP in the experiments may not be the shortest possible CSRange. In addition, as shown in Fig. 16, the result shows that the solutions of scheme 3 exhibit some properties similar to the canonical network in Fig. 11. We discuss the similarities in the following paragraph.

TABLE 6
Result for the Throughput of Random Networks

	ρ_1	ρ_2	ρ_3	ρ_3/ρ_1	ρ_3/ρ_2	CS_3	CS_3/TX
Net_1	0.4	0.5	0.575	1.438	1.15	1.253	3.133
Net_2	0.412	0.439	0.541	1.313	1.232	1.265	3.162
Net_3	0.429	0.451	0.536	1.25	1.189	1.265	3.163
Net_4	0.429	0.5	0.6	1.4	1.2	1.205	3.012
Net_5	0.5	0.5	0.524	1.048	1.048	1.287	3.216

Net_i: Network *i*
TX: Transmission range, set to 0.4 in experiments
ρ_1: Throughput when CSRange=3.78 TX (Scheme 1)
ρ_2: Throughput when CSRange is minimized with respect to links in the network (Scheme 2).
ρ_3: Throughput when only some links in the network are activated (HFP) (Scheme 3)
CS_3: CSRange for Scheme 3

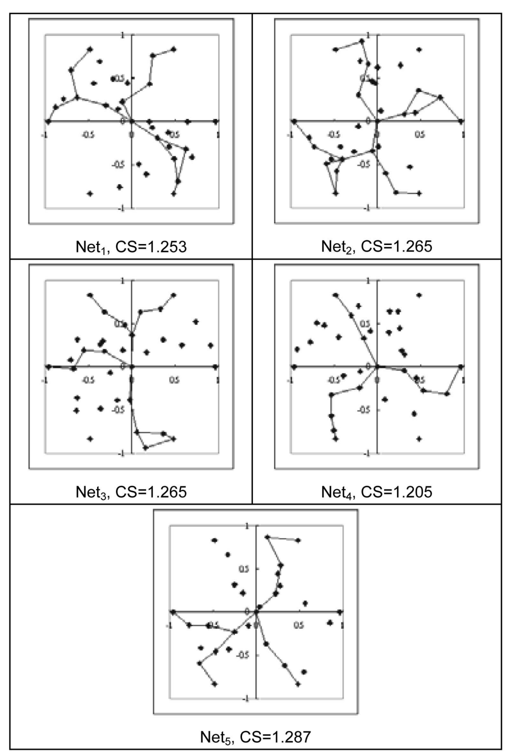

Fig. 16. Randomly generated networks and HFP.

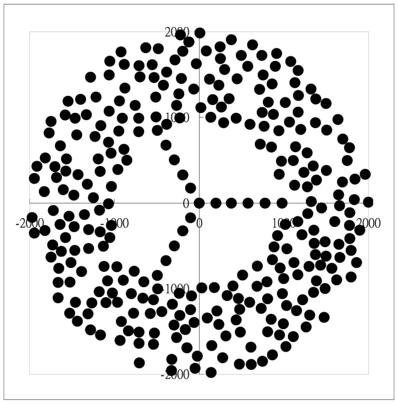

Fig. 17. Example of a centric canonical network.

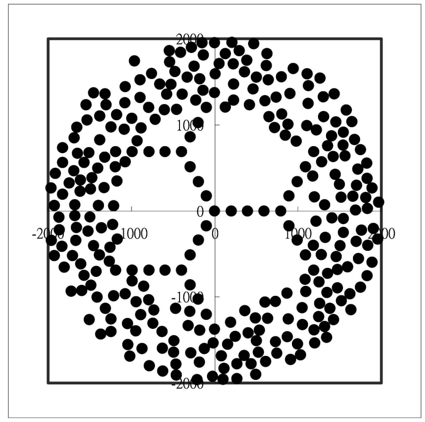

Fig. 18. Example of a manifold canonical network.

First, for scheme 3, CSRange/TxRange $(\mathrm{CS}_3/\mathrm{TX})$ for Net_1 to Net_5 is in the range of 2.354 to 3.303, which is the CSRange region we mentioned in Section 3.1 for achieving the capacity of $3L/4$ in a canonical network. Second, exactly three paths leading to the sink node are used, which is the same as the three-chain canonical network (Fig. 16). This gives us an intuition that the canonical network is in a sense optimal—that is, we may want to form a structure similar to the canonical network by turning on only some of the relay nodes.

4.3 Applying Canonical Network to General Networks

The preceding section shows that HFP outperforms other HNF schemes in terms of throughput. We also observe from the results that 1) HFP solutions for a random network exhibit structures similar to that of the three-chain canonical network near the center. Furthermore, from simulation results in Section 3.2 (see Fig. 12), we observe that 2) IEEE 802.11 scheduling in the *canonical network* achieves a throughput close to that of perfect scheduling. Observations 1 and 2 lead to the following general engineering principle:

4.3.1 Centric Canonical-Network Design Principle

- In a general multihop network densely populated with relay nodes, instead of solving the complex HFP optimization problem, as a heuristic, we may select routes near the center so that the structure looks like that of a three-chain canonical network.
- If we have the freedom for node placement near the center during the network design process, then the nodes around the center should be structured like a three-chain canonical network.

Note that there is no restriction on nodes far away from the center and that they can be randomly distributed (see Fig. 17 for illustration).

This section investigates the application of the Centric Canonical-Network Design Principle. For our simulations, we assume that there is a disk with a radius of 2,000 m. Within the disk, there is an inner circle with a radius of 980 m. As illustrated in Fig. 17, the inner circle is structured as a canonical network. The nodes outside the inner circle are placed randomly with the constraint that the smallest distance between any two of them is not shorter than 125 m. The nodes outside the inner circle act as source nodes and relay nodes at the same time, while the nodes inside the inner circle act merely as relay nodes. We refer to the network structure in Fig. 17 as a *centric canonical network*, alluding to the fact that only the vicinity of the center looks like a canonical network. Henceforth, we shall refer to vicinity of the center as the canonical network and the randomly structured part beyond that as the random network. The number of nodes beyond the inner circle is 284. We use the default setting in NS2, CSRange of 550 m and TXRange of 250 m, for performing the simulations. AODV routing is assumed. For the canonical network, with respect to Fig. 12, we set $d_0 = 200$ m. Since $550\ \mathrm{m}/200\ \mathrm{m} = 2.75$, which is within the range 2.354-3.303 (see Fig. 11), the canonical network is HNF. The random network, however, is not necessarily HNF in our experiments. The assumption is reasonable and corresponds to the real situation in which we only try to design the network architecture near the center judiciously by careful node placement.

For comparison, we have also conducted simulation experiments for a random network in which the inner circle is populated by 146 randomly placed nodes with no constraint on the node-to-node distance. We call this a pure random network. In all our simulations below, the offered load to the source nodes are varied until we find the largest throughput for each network structure [8]. Simulation of 802.11 with AODV yields a throughput of 1.16 Mbps for the pure random network and a throughput of 2.79 Mbps for the centric canonical network. That is, the throughput of the centric canonical network is more than 100 percent higher. This demonstrates that *a carefully designed structured network around the data center yields superior performance*.

Although the improvement is significant, 2.79 Mbps is still a bit lower than the 4.30-Mbps simulated throughput of the three-chain canonical network in Section 3. It turns out that the centric canonical network actually fails to take another bottleneck into account. That is, in addition to the bottleneck around the center, there is also a bottleneck at the "confluence" of the random network and the canonical network, where the canonical network may branch off to many paths in the random network, and the nodes on these

TABLE 7
Comparison of Throughputs of Manifold Canonical Networks with and without Node Position Error

Throughput without position error (Mbps)	Throughput with position error (Mbps)	Ratio
3.44	3.45	1.003
3.35	3.11	0.928
3.32	3.18	0.958
3.29	2.94	0.894
3.37	2.96	0.878
3.36	2.82	0.839

branches may interfere with each other in a negative way to bring down the throughput.

To mitigate the bottleneck at the confluence, we modify the canonical network as in Fig. 18. As shown, each chain in the canonical network only branches out further into two chains before meeting the random network. We refer to this design as the manifold canonical network, in reference to the fact that there are actually two "layers" of canonical networks. The first one is at the center, with three more before meeting the random network. We refer to this design principle as the *Manifold Canonical-Network Design Principle*.

In our simulations, the manifold canonical network is placed inside an inner circle of radius 1,026 m. The nodes beyond the manifold canonical network are randomly generated with the same constraints as the nodes generated beyond the inner circle of the centric canonical network. As the inner circle is larger than previous networks and the number of nodes (which are relay nodes) in the manifold canonical network is 31, to keep the total number of nodes in the network constant, the number of randomly generated nodes (which are also the source nodes) outside the inner circle is decreased from 284 to 269. We set CSRange to 550 m and $d_0 = 200$ m in the manifold canonical network in our simulation (see Fig. 11). Simulation of 802.11 with AODV routing yields a throughput of 3.34 Mbps, which is 20 percent higher than that of the centric canonical network. For fair comparison, we again perform the simulation with the inner circle replaced by random node placements, but this time with the inner circle having a radius of 1,026 m, as in the manifold canonical network. The simulation of the pure random network yields a throughput of 1.31 Mbps. We find that the throughput of the manifold canonical network is more than 150 percent over that by the pure random network.

We have also investigated the robustness of the manifold canonical network with respect to node positioning. Simulations show that a 5 percent position error of the nodes in the two "layers" of the canonical network only decreases the throughput by 10 percent on the average, as summarized in Table 7.

5 Conclusion

In this paper, we have studied the throughput capacity of many-to-one multihop wireless networks based on the IEEE 802.11 MAC protocol. We focus on the nontrivial case where the source nodes are two or more hops away from the sink.

We have introduced a class of *canonical networks* whose throughput capacity can serve as a benchmark for general networks. We find that the throughput capacity of canonical networks under 802.11 is upper bounded by $3L/4$, where L is the link capacity at the sink allocated to the traffic. If we restrict our attention to networks in which all links have the same length, the upper bound for throughput capacity is further reduced to $2L/3$. The $2L/3$ upper bound is a general upper bound for all networks (not just canonical networks) in which links have the same length.

For validation of our analytical results, we have conducted 802.11 network simulations. The results yield throughputs of around $0.69L$ (for variable-link-length canonical networks) and $0.62L$ (for equal-link-length canonical networks) under the worse case scenario when all non-one-hop source nodes are very far away and their traffic needs to go through many hops before reaching the sink node. That is, the simulated throughputs are reasonably close to the theoretical upper bounds of $3L/4$ and $2L/3$, respectively. This is quite a positive result considering the fact that 802.11 schedules transmissions in a rather random manner, while the examples we gave in Section 3.1 to achieve throughputs of $3L/4$ and $2L/3$ require very specific transmission orders.

We have considered both canonical networks with and without HNs. Our results indicate that HFDs yield higher throughput capacity. This is in contrast to the many-to-many case where HFD may not yield better throughputs [6], [7] and may actually decrease the overall system throughput.

Building on our results on canonical networks, we have studied general networks: in particular, we examine how the results of general networks are related to those of canonical networks. For general networks, we put forth the concept of HFP for setting up routes to achieve a high throughput. Our experimental results indicate that the routes selected by the HFP algorithm *resemble the structure of the canonical network near the sink*. This gives rise to simple network design principles that attempt to approximate the canonical network structure. Specifically, we have shown that a *manifold canonical network structure* near the sink can yield a superior throughput that is more than 150 percent higher than that given by the popular AODV routing in a dense network. A key insight is that in a network densely populated with nodes, deliberating turning off some relay nodes in the area near the sink so as to approximate the canonical network structure can actually give rise to better throughput performance.

Yet another way to interpret our results is in terms of how to lay out relay nodes in a mesh network to relay traffic to the sink. In this scenario, relay nodes are part of the infrastructure of the mesh, and the source nodes are user nodes that can come and go. The relay structure is like a highway system, and it is desirable for it to resemble a canonical structure. The source traffic should get on the highway as soon as possible for further forwarding, perhaps using another frequency channel.

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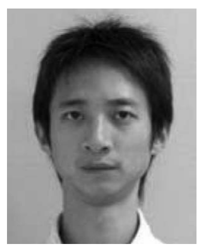

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