A Distributed Fixed-Step Power Control Algorithm with Quantization and Active Link Quality Protection

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***Abstract*—A distributed fixed-step power control algorithm is presented. It is a simple feedback adjustment algorithm using only local information. In the ideal case where there is no power constraint, it is guaranteed that existing users will not be dropped due to admission of new users. If it is infeasible to accommodate all of them, the new user will be blocked. When the constraint on maximum power is imposed, it is shown by simulation that blocking a new call is more probable than dropping any existing calls, if the capacity is exceeded. Besides, its convergence property is demonstrated. The convergence rate, which depends on the step size, is studied through simulation. In addition, the issue of power quantization is addressed.**

I. Introduction

In order to achieve a high-capacity cellular communication system, efficient spectrum usage is of paramount importance. For a time-division multiple-access (TDMA), frequency-division multiple-access (FDMA), or hybrid TDMA/FDMA-based architecture, this implies the channels must be reused as compactly as possible so that the system capacity can be maximized. However, the extent of channel reuse is constrained by the effect of cochannel interference. Therefore, one way to achieve a high system capacity is by employment of an efficient channel allocation scheme. In addition, transmitter power control can also be used to further reduce cochannel interference. The latter approach will be addressed in this paper.

In radio communication systems, the quality of a communication link is usually measured by means of the *carrier-to-interference ratio* (CIR). Early work by Aein [1] introduced the concept of *CIR balancing* for power control. The basic idea is drive all the users to the same CIR. This solution is optimal in the sense that the minimum CIR of all communication links is maximized [14].

For practical considerations, CIR balancing should be achieved by means of distributed algorithms, such as those proposed by Zander [15] and Grandhi *et al.* [7]. However, in these algorithms, a global normalization factor is needed to scale individual transmitter power to a desired range. This requirement weakens the distributed nature of these algorithms as the factor must be computed based on global information. Another class of distributed algorithms is proposed by Wong and Lam [12]. In these schemes, centralized control is removed, but some sort of communication between neighboring cells is still necessary. Recently, an algorithm employing fuzzy control is proposed [11]. It is fully distributed, and the transmitter power can be kept within a finite dynamic range. However, the CIR's do not necessarily converge to the optimal value. It seems that optimal CIR balancing cannot be achieved in a completely distributed manner.

Apart from CIR balancing, another approach is adopted by Foschini and Miljanic [5]. The objective is no longer to maximize the minimum CIR, but to maintain the CIR's of all links above a target value. In this model, receiver noise is included in the definition of interference. This modification provides a more realistic model of the system and avoids the issue of relative power scaling. In this paper, we will follow this approach.

One major drawback of the CIR balancing approach is that when a channel becomes heavily loaded, the balanced CIR of all links may drop below the *protection ratio*, rendering the channel unusable to all users. This problem also occurs in Foschini's and Miljanic's algorithm. To deal with this situation, some removal algorithms have been proposed to drop users in order to maintain an acceptable CIR for some remaining links [9], [14]. These algorithms aim at maximizing the total system capacity without regard to other quality of service (QOS) requirements. In practice, it is more important to protect the quality of on-going calls than originating calls. In another words, if admitting an originating call would cause certain links to change from an *active* (i.e., CIR greater than the protection ratio) state to *inactive*, the originating call should not be admitted. This issue is addressed in this paper.

Another issue that is considered is concerned with the pragmatic issues of power level quantization. In practical systems, the transmitter power outputs are usually quantized into discrete levels. The effect of quantization is studied by Lin *et al.* by using simulation [10]. It is shown that for a protection ratio smaller than 20 dB, 32 levels are needed when the power range is 30 dB. A different approach is proposed here, and analytical results will be presented.

In this paper, we propose a new distributed fixed-step power control algorithm. The proposed algorithm bears some resemblance to the algorithms investigated by Ariyavisitakul [2] and Chuang and Sollenberger [4]. Our study shows that

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this algorithm can perform nearly as well as CIR balancing. Moreover, it can track short-term fading when the power control sampling rate substantially exceeds ten times the maximum Doppler frequency. In this paper, we consider only the effect of long-term fading, which is caused by path loss and shadowing. Convergence property of the algorithm will be discussed. System-level performances in terms of call blocking and dropping probability will be evaluated.

II. System Model

In this section, the system model is presented along with some relevant background results. Consider a cellular radio system. To each communication link, we allocate a pair of orthogonal channels (time slots or frequencies) for mobile-to-base (uplink) and base-to-mobile (downlink) communication. Since there is no interference between the uplink and downlink channels, we consider power control for only the uplink channels in this paper. However, the results can be applied to the downlink channels as well.

Consider a set of cells in which a particular channel is used at a particular instant. Let M be the cardinality of this set. Let P_i be the power transmitted by the ith mobile. The link gain from mobile j to base station i is denoted by G_{ij}. The matrix $\mathbf{G} = \{G_{ij}\}$ is known as the uplink gain matrix. Let η_i be the receiver noise at base station i. In our model, the effect of adjacent channel interference is ignored. Thus, the carrier-to-interference ratio (CIR) at base station i, Γ_i can be written as

$$\Gamma_i = \frac{G_{ii}P_i}{\sum_{j\neq i} G_{ij}P_j + \eta_i}. \tag{1}$$

If Γ_i is greater than or equal to a prespecified value γ_{pi}, link i is defined as *active*. The value γ_{pi} is called the *protection ratio* of link i.

In mobile cellular systems, the link gains change constantly in time. Thus, the link gain matrix $\mathbf{G}$ is actually a stochastic process. In our model, we consider a snapshot of the system so that $\mathbf{G}$ is treated as an $M \times M$ matrix of random variables.

III. Quantization of Power Level

In [5], [14], and [15], the transmission power level can take on any positive real values. In practice, however, some restrictions are inevitable. For example, the power level used cannot be infinitely large [8]. Besides, power levels usually quantized into discrete values in real systems [10]. In this section, the effect of power level quantization is studied.

As in [10], we assume that the power level is quantized in logarithmic scale. The difference between two consecutive power levels is $\delta^{(\mathrm{dB})}(>0)$ dB. (We use $x^{(\mathrm{dB})}$ to denote the decibel value of x, i.e., $x^{(\mathrm{dB})} = 10\log_{10} x$.)

Theorem 1: If there exists a power vector $\mathbf{P}^*$ such that $\Gamma_i(\mathbf{P}^*) = \gamma_i$ for all i, then there exists a quantized power vector $\hat{\mathbf{P}}$ such that $\delta^{-1}\gamma_i \leq \Gamma_i(\hat{\mathbf{P}}) \leq \delta\gamma_i$ for all i.

Proof: Given P_i^*, we can always find one and only one quantized power level $\hat{P}_i$ such that $\delta^{-1/2}P_i^* \leq \hat{P}_i < \delta^{1/2}P_i^*$. Assume that P_i^* is quantized to $\hat{P}_i$. Let $\hat{\mathbf{P}}$ be the quantized power vector corresponding to the given vector $\mathbf{P}^*$

$$\begin{aligned}\Gamma_i(\hat{\mathbf{P}}) &= \frac{G_{ii}\hat{P}_i}{\sum_{j\neq i} G_{ij}\hat{P}_j + \eta_i}\\ &\geq \frac{G_{ii}\delta^{-1/2}P_i^*}{\sum_{j\neq i} G_{ij}\delta^{1/2}P_j^* + \eta_i}\\ &\geq \delta^{-1}\Gamma_i(\mathbf{P}^*)\\ &= \delta^{-1}\gamma_i.\end{aligned}$$

The upper bound can be derived similarly. □

In a quantized power level system, one can specify a target range with width $2\delta^{(\mathrm{dB})}$ dB for each user. If it is possible to find an unconstrained power vector such that the resulting CIR for each user is equal to the mid value (in decibels) of this specified range, then there exists a quantized power vector such that all the individual CIR's fall within the corresponding range. This observation motivates the design of the power control algorithm described in the next section.

Another implication of Theorem 1 is that power quantization reduces system capacity. For example, consider an unconstrained system with a given link gain matrix $\mathbf{G}$. Assume that there exists a power vector $\mathbf{P}$ such that $\Gamma_i \geq \gamma_i$ for all i. In a quantized power level system with the same link gain matrix $\mathbf{G}$, one can only guarantee that there exists a quantized power vector $\hat{\mathbf{P}}$ such that $\Gamma_i \geq \delta^{-1}\gamma_i$. It is possible to construct examples in which some users cannot be not accommodated in the quantized system. In other words, the system capacity is reduced. Although the amount of capacity reduction is difficult to quantify, it is clear that larger quantization levels will result in larger reduction in the capacity.

IV. Power Control Algorithm

In this section, the proposed fixed-step power control algorithm is presented. It is a discrete-time feedback adjustment algorithm. The only information needed to adjust the transmission power of a mobile terminal is the received CIR at the corresponding base station. Coordination among base stations is not required. A target window for the CIR is defined. If the received CIR is below the window, the base station will inform the mobile to raise its power to next level up. If the received CIR is above the window, the power will be adjusted downwards by one level. If it falls within the window, the power will remain unchanged. The following is a summary of the procedures in the algorithm.

A. Fixed-Step Power Control Algorithm

Each mobile unit adjusts its transmission power $P_i^{(n+1)}$ in the $(n+1)$th step according to the following rules:

$$P_i^{(n+1)} = \begin{cases}\delta P_i^{(n)}, & \text{if } \Gamma_i^{(n)} < \delta^{-1}\gamma_i\\ \delta^{-1}P_i^{(n)}, & \text{if } \Gamma_i^{(n)} > \delta\gamma_i\\ P_i^{(n)}, & \text{otherwise}\end{cases} \tag{2}$$

where $\delta > 1$.

One of the attractive features of the algorithm is its simplicity. Only two bits are needed for each power control command. Thus, bandwidth for control information can be saved. Besides, it is insensitive to CIR estimation error because the power change at each step depends only on a simple comparison rule. (This point will be justified in the section on numerical study.)

Although in our discussion the CIR is used as the quality measure of a communication link, it is easy to modify our algorithm so that other quality measures can be used instead. As an example, one can use the word-error-rate measure, (proportion of received codewords that are detected to be in error), which is readily available from the decoding process, as our quality indicator. With this modification, the qualitative behavior of the algorithm should remain unchanged while the implementation can be simplified.

V. Protection of Active Links

In this section, we will prove that the proposed algorithm possesses a property called *active link protection* [3], under the assumption that there is no maximum power constraint. Although this assumption is not valid in practical systems, one can nevertheless obtain valuable insight on the behavior of the algorithm. The issue of finite dynamic range will be addressed in the section on numerical study.

Consider the situation that an originating call is admitted. If there is no feasible power vector that satisfies the CIR requirement of all users, the CIR-balancing schemes and the Foschini–Miljanic scheme will force the CIR of all users to a value below their corresponding protection ratio γ_{pi}. All communication links become unreliable. Since these algorithms do not distinguish between originating and on-going calls, some on-going calls may be dropped.

In the fixed-step algorithm, the active links are protected. So, even if no feasible power vector exists, it is guaranteed that the CIR's of all active links will be kept above a certain level. After several iterations, the originating call will discover that it cannot be accommodated with the desired QOS. As a result, it will drop out of the highly contended channel out of its own accord.

To guarantee this important property, it is crucial that the originating call should use a low enough power level when it first joins the system, so that all existing links remain active at the beginning of the power control algorithm iteration cycle.

The convergence of the power control algorithm will be examined. To facilitate our discussion, we partition the M mobile terminals into four subsets $\mathcal{A}_n$, $\mathcal{B}_n$, $\mathcal{C}_n$, and $\mathcal{D}_n$ at step n. The membership relation is defined below

$$i \in \begin{cases} \mathcal{A}_n, & \text{if } \Gamma_i^{(n)} > \delta\gamma_i \\ \mathcal{B}_n, & \text{if } \gamma_i \le \Gamma_i^{(n)} \le \delta\gamma_i \\ \mathcal{C}_n, & \text{if } \delta^{-1}\gamma_i \le \Gamma_i^{(n)} < \gamma_i \\ \mathcal{D}_n, & \text{otherwise.} \end{cases} \tag{3}$$

Theorem 2: If $\Gamma_i^{(n)} \ge \delta^{-2}\gamma_i$, then $\Gamma_i^{(n+k)} \ge \delta^{-2}\gamma_i$ for all $k \ge 0$.

Proof: For $i \in \mathcal{A}_n$, see the equation given at the bottom of the page.

Similarly, for $i \in \mathcal{B}_n \cup \mathcal{C}_n$

$$\Gamma_i^{(n+1)} \ge \delta^{-1}\Gamma_i^{(n)} \ge \delta^{-2}\gamma_i.$$

For $i \in \mathcal{D}_n$ and $\Gamma_i^{(n)} \ge \delta^{-2}\gamma_i$

$$\Gamma_i^{(n+1)} \ge \Gamma_i^{(n)} \ge \delta^{-2}\gamma_i. \qquad \square$$

Theorem 2 states that a link with Γ_i originally greater than $\delta^{-2}\gamma_i$ will not fall below this value throughout the evolution of the power control algorithm. To ensure that all communication links remain active during the control process, one should set the value of the protection ratio of mobile i, γ_{pi} to $\delta^{-2}\gamma_i$. So, an additional $\delta^{(\mathrm{dB})}$ decibel margin is needed. As a consequence, there is a capacity loss due to the more stringent CIR requirement.

VI. Convergence Property

In [13], the convergence properties of a general class of power control algorithms are proved. However, the fixed-step algorithm does not fall into that class. In this section, we will prove that the algorithm will converge if a feasible solution exists. Since the proof is complicated, some intermediate results are stated as lemmas and placed in the Appendix in order to keep the main idea of the proof simple to follow.

First, it will be established that the power level of each user has a lower bound and an upper bound. Then, it will be shown that the power levels do not oscillate. As a consequence, the power vector must converge to a fixed point.

Proposition 1: For each mobile terminal, the power level at each iteration stage of the fixed-step power control algorithm is lower bounded by a positive constant that depends only on the gain matrix.

Proof: For any mobile j, if there exists N such that $\Gamma_j^{(N)} \ge \delta^{-2}\gamma_j$, by Theorem 2, $\Gamma_j^{(N+n)} \ge \delta^{-2}\gamma_j$ for all $n \ge 0$. It implies that $P_j^{(N+n)} \ge \delta^{-2}\gamma_j\eta_j/G_{jj}$ for $n \ge 0$.

If no such N exists, $P_j^{(n+1)} = \delta P_j^{(n)}$ for all n. Thus, $P_j^{(n)} \ge P_j^{(0)}$. $\square$

Proposition 2: If there exists a power vector $\mathbf{P}$ such that $\Gamma_i(\mathbf{P}) = \gamma_i$ for all i, then for each mobile terminal, its power level at each iteration stage under the fixed-step power control algorithm is upperbounded by a constant which depends only on the gain matrix.

$$\begin{aligned}\Gamma_i^{(n+1)} &= \frac{G_{ii}\delta^{-1}P_i^{(n)}}{\sum_{\mathcal{A}_n\setminus\{i\}} G_{ij}\delta^{-1}P_j^{(n)} + \sum_{\mathcal{B}_n\cup\mathcal{C}_n} G_{ij}P_j^{(n)} + \sum_{\mathcal{D}_n} G_{ij}\delta P_j^{(n)} + \eta_i} \\ &> \delta^{-2}\Gamma_i^{(n)} \\ &> \delta^{-1}\gamma_i.\end{aligned}$$

Proof: Assume that $P_i^{(n)}$ is unbounded from above. We claim that this implies $P_j^{(n)}$ is unbounded from above for all j. To prove this claim, suppose there exists a mobile j $(j \neq i)$ such that $P_j^{(n)} < c$ for all n. If there exists N such that $\Gamma_j^{(N)} \geq \delta^{-2}\gamma_j$, then by Theorem 2, $\Gamma_j^{(N+n)} \geq \delta^{-2}\gamma_j$ for $n \geq 0$. Since $P_i^{(n)}$ is unbounded while $P_j^{(n)}$ is bounded, $\Gamma_j^{(n)}$ can be arbitrarily small. It leads to a contradiction. If no such N exists, $P_j^{(n)}$ will go to infinity. Therefore, for any mobile j $(j \neq i)$, $P_j^{(n)}$ is also unbounded from above.

Since all the components of $\mathbf{P}$ are unbounded, for any mobile i, we can find $t_i > 0$ such that $P_i^{(t_i)} > P_i^{(0)}$. Let $T_0 > \max_i t_i$.

Define

$$N_0 = \max_i \left\{ \max_{0 \leq n \leq T_0} \log_\delta \left(\frac{P_i^{(0)}}{P_i^{(n)}} \right) \right\}.$$

Note that $N_0 \geq 0$. First, consider the case $N_0 \geq 1$.

Assume that $P_i^{(m_i)} \geq \delta^{N_0} P_i^{(n_i)}$, where $T_0 \leq m_i < n_i$. Since $P_i^{(t_i)} > P_i^{(0)}$, there exists r_i such that $i \in \mathcal{D}_{r_i}$, where $0 \leq r_i < t_i$. By Lemma 4, there exists $j \neq i$ such that $P_j^{(m_j)} \geq \delta^{N_0+1} P_j^{(n_j)}$, where $m_j < m_i \leq n_j < n_i$.

If there exists $r_j < m_j$ such that $j \in D_{r_j}$, the argument can be repeated, and, in general, we have

$$P_k^{(m_k)} \geq \delta^{N_0+x} P_k^{(n_k)} \tag{4}$$

for some $x \geq 1$ and $m_k < n_k < n_i$.

Now assume that $k \notin \mathcal{D}_{r_k}$ for $0 \leq r_k < m_k$. It follows that $m_k < t_k$ and $P_k^{(m_k)} \leq P_k^{(0)}$. By (4) and the definition of N_0, we have $n_k > T_0$. Therefore, $m_k < t_k < n_k$.

By the definition of t_k

$$P_k^{(t_k)} \geq P_k^{(m_k)} \geq \delta^{N_0+x} P_k^{(n_k)}.$$

Since there exists $r_k < t_k$ such that $k \in \mathcal{D}_{r_k}$, the argument can be repeated to show that x can be arbitrarily large. Hence, $N_0 + x$ can be made arbitrarily large also, which contradicts the fact that the amount of decrement in the power level is limited by the number of iterations, that is, $N_0 + x \leq n_i$.

As a result, it is impossible that $P_i^{(m_i)} \geq \delta^{N_0} P_i^{(n_i)}$,where $T_0 \leq m_i < n_i$. Therefore, we have

$$\max_i \left\{ \max_{n \geq T_0 + c} \log_\delta \left(\frac{P_i^{(T_0+c)}}{P_i^{(n)}} \right) \right\} < N_0 \tag{5}$$

for $c \geq 0$.

Now, one can treat $\mathbf{P}^{(T_0)}$ as the initial vector, with the additional constraint given in (5).

For any mobile i, again one can find $t_i > T_0$ such that $P_i^{(t_i)} > P_i^{(T_0)}$. Let $T_1 > \max_i t_i$.

Define

$$N_1 = \max_i \left\{ \max_{T_0 \leq n \leq T_1} \log_\delta \left(\frac{P_i^{(T_0)}}{P_i^{(n)}} \right) \right\}.$$

Note that $N_1 < N_0$ by (5).

Applying the same argument, one can prove that

$$\max_i \left\{ \max_{n \geq T_1 + c} \log_\delta \left(\frac{P_i^{(T_1+c)}}{P_i^{(n)}} \right) \right\} < N_1$$

for $c \geq 0$.

Therefore, there exists T^* such that

$$\max_i \left\{ \max_{n \geq T^*} \log_\delta \left(\frac{P_i^{(T^*)}}{P_i^{(n)}} \right) \right\} < 1. \tag{6}$$

Note that if $N_0 = 0$, (6) is still valid. (In that case, $T^* = 0$.) Therefore, (6) is valid for $N_0 \geq 0$.

Equation (6) implies that $\mathbf{P}^{(T^*+c)} \geq \mathbf{P}^{(T^*)}$ for $c > 0$. It means that $\mathcal{A}_n = \phi$ for $n \geq T^*$. Then Lemma 3 applies and $\mathbf{P}$ will converge to a fixed point $\mathbf{P}^*$. It contradicts to the assumption that $\mathbf{P}$ is unbounded. □

The states of the fixed-step algorithm are represented by the sequence $\mathbf{P}^{(n)}$. The sequence is said to be *asymptotically periodic* if there exist integers, $N > 0$ and $T > 1$ such that for all $n > N$

$$\mathbf{P}^{(n)} = \mathbf{P}^{(n+T)}.$$

The transition of the fixed-step algorithm depends only on the current state and is deterministic. This implies that if the set of states is a finite set, then the power state sequence must be asymptotically periodic if it does not converge.

Proposition 3: If the algorithm does not converge, the power vector of the mobile terminals is not asymptotically periodic.

Proof: Assume that the power vector oscillates with period T, where $T > 1$, i.e., $\mathbf{P}^{(n)} = \mathbf{P}^{(n+T)}$ for large enough n.

Since the algorithm does not converge, one can find a mobile i such that $P_i^{(m)} = \delta P_i^{(n)}$, where $m < n$ and $n - m < T$.

Note that $P_i^{(m)} = \delta P_i^{(n-T)}$. Therefore, there exists r such that $i \in \mathcal{D}_r$, where $n - T \leq r < m$.

By Lemma 4, there exists a mobile j $(j \neq i)$ such that $P_j^{(s)} = \delta^2 P_j^{(t)}$, where $r \leq s < m \leq t < n$. Note that $t - s < T$.

By repeating the argument, one can find a mobile k such that $P_k^{(p)} = \delta^x P_k^{(q)}$ for any integer x, where $q > p$ and $q - p < T$. Since at each step, the power level can change by an amount bounded by δ, x is upperbounded by T. Hence, this leads to a contradiction. □

Theorem 3: If there exists a power vector $\mathbf{P}$ such that $\Gamma_i(\mathbf{P}) = \gamma_i$ for all i, then the fixed-step power control algorithm converges to a fixed point $\mathbf{P}^*$, where $\delta^{-1}\gamma_i \leq \Gamma_i(\mathbf{P}^*) \leq \delta\gamma_i$ for all i.

Proof: By Propositions 1 and 2, there is an upper and lower bound for the power vector $\mathbf{P}$. Since the power level changes in fixed step, there is only a finite number of possible values for the vector $\mathbf{P}$. If the algorithm does not converge, then it must be asymptotically periodic. This contradicts the previous proposition. Therefore, the algorithm converges to a fixed point $\mathbf{P}^*$. It happens if and only if $\delta^{-1}\gamma_i \leq \Gamma_i(\mathbf{P}^*) \leq \delta\gamma_i$ for all i. □

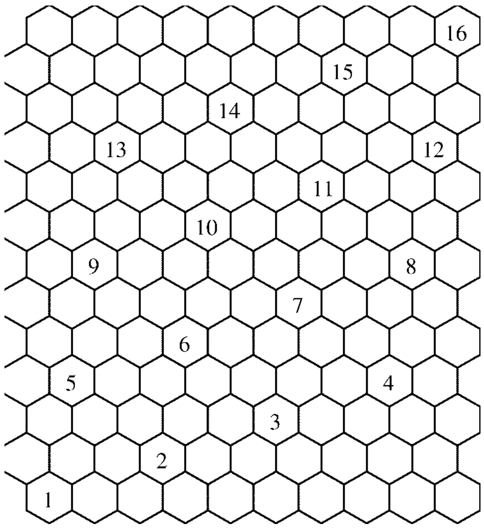

Fig. 1. Layout of interfering cells in the numerical study.

VII. NUMERICAL STUDIES

Some simulation studies on the fixed-step power control algorithm were conducted, assuming a standard hexagonal cellular layout with 16 cochannel cells (see Fig. 1) [12]. The geographical location of the cells corresponds to a reuse pattern of seven. We approximate each hexagonal cell by a circular cell of the same area. Within each cell, there is a mobile terminal communicating with the base station. The location of each mobile terminal is generated uniformly inside the cell. The link gain G_{ij} is defined as

$$G_{ij} = \frac{A_{ij}}{d_{ij}^4}$$

where d_{ij} is the distance between the ith base station and the jth mobile terminal and A_{ij} is the corresponding attenuation factor. In this study, we consider only lognormal fading. Hence, we assume A_{ij} is lognormal distributed with mean 0 dB and standard deviation 6 dB for all i and j. Each component of the initial power vector is generated uniformly between 0.001 and 1. The receiver noise η_i is assumed 10^{-6} for all i.

Figs. 2 and 3 show some typical results about the convergence of the fixed-step algorithm. We set $\gamma_i = \gamma_0$ for all i and equal to 17 dB. The two figures correspond to the cases where the step size equals 1 and 2 dB, respectively. The same link gain matrix and initial power vector are used. The maximum and minimum CIR of the 16 users are shown. It can be seen that the convergence rate is faster in the latter case, as expected. However, the tradeoff is that the target window is larger. It means that for the same γ_p, a larger value of γ_0 is needed. This incurs a capacity loss.

Next, we consider the case where there is no feasible power vector such that the CIR's of all mobiles can fall within the target window. This time we set γ_0 equal to 25 dB. The step size of the algorithm is 1 dB. Comparison is made with Foschini's and Miljanic's algorithm [5]. Their power control rule is shown below

$$P_i^{(n+1)} = \frac{\gamma_0}{\Gamma_i^{(n)}} P_i^{(n)}.$$

A typical result is shown in Fig. 4. As we have mentioned before, when Foschini's algorithm is used, the CIR's of all the users converge to an unacceptable value. Every link becomes unreliable. If the fixed-step algorithm is used, the CIR's do not converge to a single value. The CIR's of the links, if originally greater than $\delta^{-2}\gamma_0$ (i.e., 23 dB in this example), are kept above this value throughout the evolution. Thus, the link quality of them are protected. For other mobiles, if there is no significant improvement on CIR between successive iterations, they should realize that the channel is heavily loaded and they cannot be accommodated. In response, they should be dropped out of the channel contention.

The above discussion applies to the ideal case where there is no constraint on the maximum power used. In practice, it is not possible to transmit a signal with infinite power. Now we assume that there is such a constraint and we will show that the dropping probability can be reduced if the fixed-step algorithm is employed.

As before, we consider the same 16-cell system. When a channel is available in a particular cell, we assume that the arrival time of a call is geometrically distributed with mean equal to the duration of 100 power control iterations. The call holding time is also assumed to be geometric with mean equal to the duration of 500 power control iterations. This initial power used is set to $P_{\min} = 0.01$. Again, the receiver noise η_i is assumed 10^{-6} for all i. For acceptable quality, we require the CIR of each link be greater than 25 dB (i.e., $\gamma_p = 25$ dB).

We compare the fixed-step algorithm with Foschini's algorithm. In Foschini's algorithm, we set the target value $\gamma_0 = 26$ dB. A 1-dB margin is provided for protection. If the CIR of a link is less than γ_p and this situation sustains for a period of five iterations, we assume that the call is blocked or dropped.

In the fixed-step algorithm, we set $\gamma_0 = 27$ dB and the step size $\delta = 1$ dB. We set the target 2 dB higher than the requirement (i.e., $\gamma_0 = \delta^2\gamma_p$) so that we have 1-dB margin for protection and another 1 dB for the intrinsic quantization noise. A link with CIR originally greater than γ_p is called active. If its CIR falls below this value and this situation sustains for a period of five iterations, we assume that it is dropped. For a newly admitted call, we adopt the following rule. If at the nth iteration, the following two conditions hold:

1) $\Gamma_i^{(n)} < \gamma_p$ dB;
2) $\Gamma_i^{(n-1)} < \Gamma_i^{(n)} < \delta^{1/4}\Gamma_i^{(n-1)}$

we assume that the call originated from mobile i is blocked. Note that if no other user has power adjustment, $\Gamma_i^{(n)}$ should be increased by a factor of δ in successive steps. The second condition indicates that the improvement in CIR is much smaller than expected. It is likely that the call cannot be accommodated in the system. Therefore, the call should be blocked.

We run the simulation until 5000 calls had arrived. The number of blocked calls and dropped calls is recorded. We consider

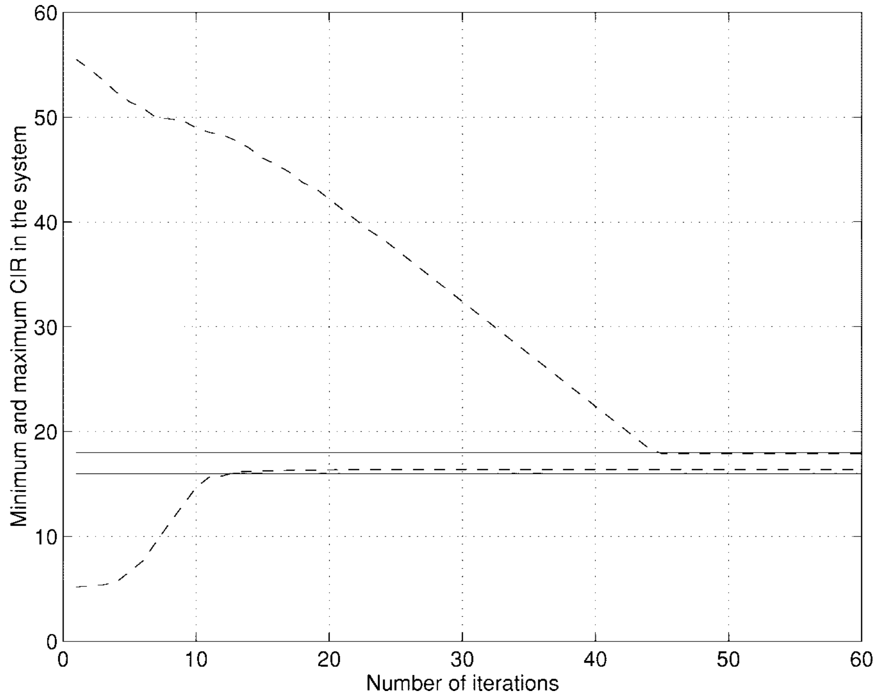

Fig. 2. Evolution of maximum and minimum CIR (decibels) with step size $\delta = 1$ dB. The target window is also shown.

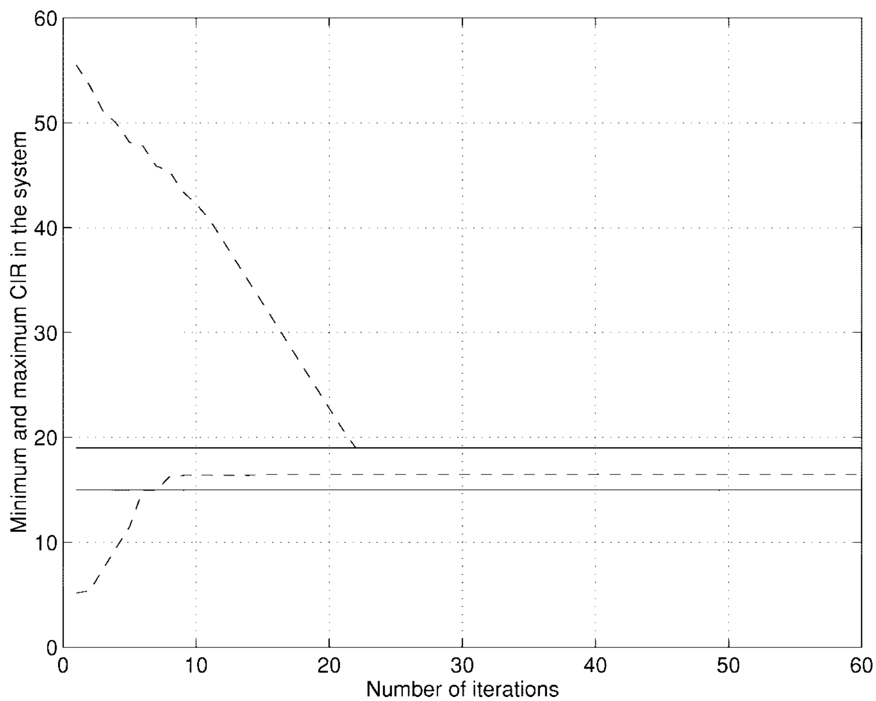

Fig. 3. Evolution of maximum and minimum CIR (decibels) with step size $\delta = 2$ dB. The target window is also shown.

three cases of the maximum power constraint: $P_{\max} = 1,\ 10,$ and 100. This corresponds to a dynamic range of 20, 30, and 40 dB. (We have set $P_{\min} = 0.01$.) In both algorithms, if a power greater than $P_{\max}$ is requested, we set the power to $P_{\max}$. Similarly, if a power less than $P_{\min}$ is requested, we set the power to $P_{\min}$.

The simulation results are shown in Table I. When the fixed-step algorithm is employed, calls are more likely to be blocked than to be dropped. This effect becomes more obvious when the power constraint is less stringent. We have already shown in Section V that no existing calls will be dropped if $P_{\max}$ is infinitely large. However, if Foschini's algorithm is used, many users are treated the same and existing users are not specially protected.

We have mentioned before that the system capacity is reduced if the fixed-step algorithm is used. This is due to the

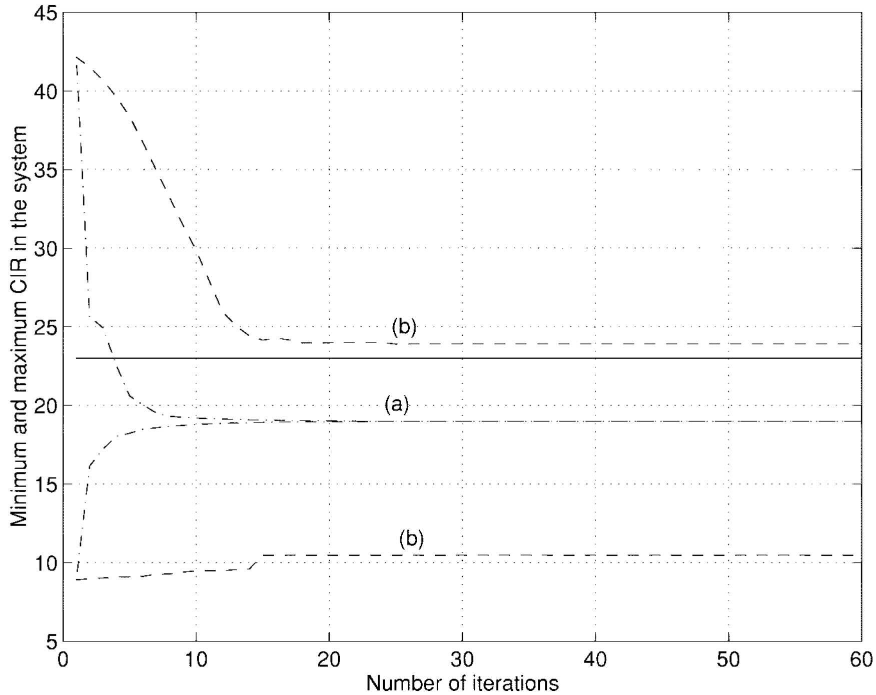

Fig. 4. Evolution of maximum and minimum CIR (decibels). (a) Foschini's algorithm. (b) Fixed-step algorithm with step size $\delta = 1$ dB. The protection line is also shown.

TABLE I
CALL BLOCKING AND DROPPING PROBABILITY OF TWO DIFFERENT POWER CONTROL ALGORITHMS WITH VARIOUS MAXIMUM POWER CONTRAINT

	20 dB dynamic range		30 dB dynamic range		40 dB dynamic range	
	Foschini's	Fixed-step	Foschini's	Fixed-step	Foschini's	Fixed-step
blocking probability	0.1964	0.2254	0.1622	0.2760	0.1374	0.2912
dropping probability	0.1076	0.0922	0.1730	0.0310	0.3226	0.0200
call loss probability	0.3040	0.3176	0.3352	0.3070	0.4600	0.3112

power quantization. However, if the dynamic range is large, more calls are lost if Foschini's algorithm is used. The reason is as follows. If the dynamic range is sufficiently large, every mobile unit can freely choose its power. When the channel is heavily loaded, many users may be dropped simultaneously because all of them strive to maintain the target value, but they fail to do so. As shown in Fig. 4, the CIR's of many users converge to a value below the protection ratio. It turns out that the theoretic capacity increase cannot compensate for the possibility that more than one user is dropped at the same time.

Finally, we investigate the effect of CIR estimation error on the performance of the algorithms. In practice, the estimated CIR will deviate from the actual CIR due to the effect of multipath fading or other measurement noise. The effect of estimation error is modeled as follows. Let $\Gamma_i^{(n)}$ and $\hat{\Gamma}_i^{(n)}$ be the actual and the estimated CIR of user i at iteration n, respectively. Assume that

$$\hat{\Gamma}_i^{(n)}(\text{dB}) = \Gamma_i^{(n)}(\text{dB}) + w_i^{(n)}(\text{dB})$$

where $w_i^{(n)}$ is a Gaussian random variable with mean zero and variance σ_w^2. We assume that the estimation noise of different users and at different iteration steps are all independent.

In our simulation, we set $\sigma_w = 1$ and 3 dB. The results are shown in Figs. 5 and 6. In the figures, both algorithms have reached the equilibrium state. It can be seen that the CIR variation of Foschini's algorithm is relatively larger in both cases. Comparatively, the fixed-step algorithm is more stable and less sensitive to CIR estimation error.

VIII. CONCLUSION

In this paper, we have investigated the performance of a fixed-step power control algorithm. The issue of quantization is addressed and its ability in active link quality protection is demonstrated. We have shown by simulation that the number of dropped calls can be significantly reduced, especially when the power constraint is not too stringent. This feature is highly desirable from a QOS viewpoint.

In addition, the fixed-step algorithm is easy to implement. It is insensitive to CIR estimation error because it relies only on a simple comparison rule. Besides, we have proved that the algorithm always converges if a feasible solution exists.

In this algorithm, there is a control parameter δ which needs a proper setting. A large δ implies faster convergence, but a more stringent requirement on CIR. Therefore, there is a tradeoff between the system capacity and the convergence rate.

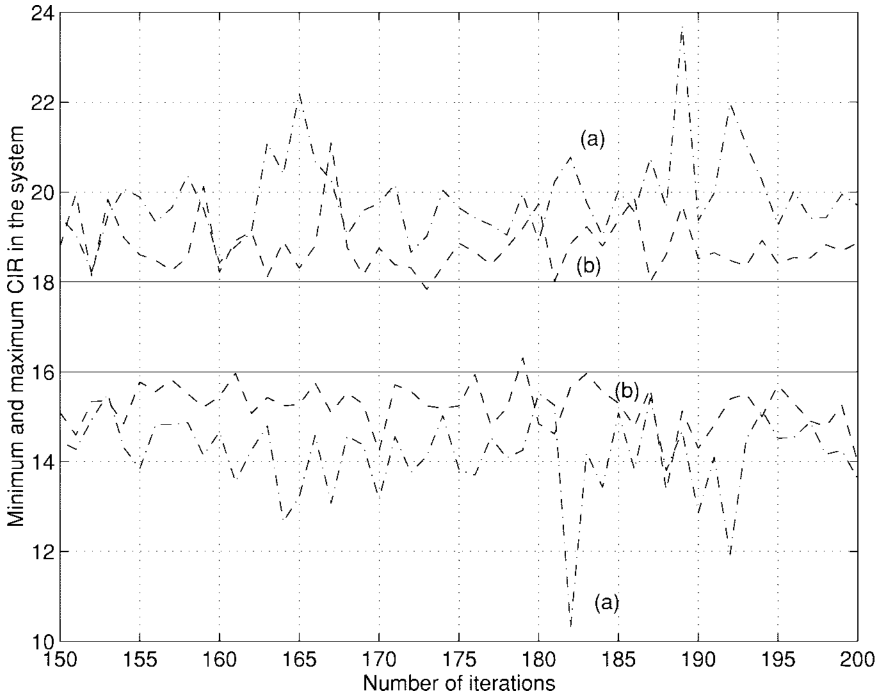

Fig. 5. Evolution of the maximum and minimum CIR (decibels) with CIR estimation error $\sigma_w = 1$ dB. (a) Foschini's algorithm. (b) Fixed-step algorithm with step size $\delta = 1$ dB. The target window of the fixed-step algorithm is also shown.

APPENDIX

In this Appendix, we present a sequence of lemmas which is part of the convergence proof of the fixed-step algorithm. Lemmas 1 and 2 are needed for the proof of Lemma 3. Lemma 3 says that if $\mathcal{A}_n$ is empty and a feasible power vector exists, the algorithm will converge. Lemma 4 says that if the power level of a mobile has increased before and has decreased x steps from iteration m to n, then there exists another mobile whose power level has decreased $x+1$ steps before. (Lemmas 3 and 4 are needed for the proof of Proposition 2 and Lemma 4 is needed for the proof of Proposition 3.)

Lemma 1: If $\Gamma_i^{(N)} \leq \delta\gamma_i$ for all i and the algorithm does not converge, then there exists $N' \geq N$ such that $\Gamma_i^{(n)} < \gamma_i$ for all i and all $n \geq N'$.

Proof: For $i \in \mathcal{D}_N$

$$\begin{aligned}\Gamma_i^{(N+1)} &= \frac{G_{ii}\delta P_i^{(N)}}{\sum_{\mathcal{B}_N \cup \mathcal{C}_N} G_{ij}P_j^{(N)} + \sum_{\mathcal{D}_N \setminus \{i\}} G_{ij}\delta P_j^{(N)} + \eta_i}\\ &\leq \delta\Gamma_i^{(N)}\\ &< \gamma_i.\end{aligned}$$

Similarly, for $i \in \mathcal{B}_N \cup \mathcal{C}_N$, one can prove

$$\Gamma_i^{(N+1)} \leq \Gamma_i^{(N)}.$$

The above results imply that $\mathcal{A}_n = \phi$ and $\mathcal{B}_{n+1} \subseteq \mathcal{B}_n$ for all $n \geq N$.

Assume $\exists i \in \mathcal{B}_n$ for all $n \geq N$. Thus, $P_i^{(n+k)} = P_i^{(n)}$ for $k \geq 0$. Since the algorithm does not converge, $\mathcal{D}_n \neq \phi$ for all $n \geq N$. As a result, there exists $P_j^{(n)}$ which grows without bound when n goes to infinity. (Note that it does not imply that $j \in \mathcal{D}_n$ for all $n \geq N$.) Then $\lim_{n\to\infty} \Gamma_i^{(n)} = 0$ which leads to a contradiction. Therefore, there exists N' such that $\mathcal{B}_n = \phi$ for all $n \geq N'$. □

Lemma 2: If the receiver noise of all users equal zero, i.e., $\eta_i = 0$ for all i, then given any two power vectors $\mathbf{P} = (P_1, P_2, \ldots, P_M)$ and $\mathbf{P'} = (P'_1, P'_2, \ldots, P'_M)$, it is impossible that $\Gamma_i(\mathbf{P}) < \Gamma_i(\mathbf{P'})$ for all i.

Proof: The lemma is obviously true if the number of mobile terminals M is equal to two. Now we assume that it is true for $M = k$. We need to prove that it is true for $M = k+1$.

Let $\mathbf{P} = (P_1, P_2, \ldots, P_{k+1})$ and $\mathbf{P'} = (P'_1, P'_2, \ldots, P'_{k+1})$ be two power vectors such that $\Gamma_i(\mathbf{P}) < \Gamma_i(\mathbf{P'})$ for all i.

Since $\eta_i = 0$ for all i, scaling the power vector has no effect on the resulting CIR, i.e., $\Gamma_i(\mathbf{P}) = \Gamma_i(c\mathbf{P})$ for all i and any constant c. Therefore, without loss of generality, we assume that

$$\sum_{j=1}^{k} G_{k+1,j}P_j = \sum_{j=1}^{k} G_{k+1,j}P'_j.$$

If $P_{k+1} = P'_{k+1} = 0$, then by the induction hypothesis, there exists i, where $1 \leq i \leq k$, such that $\Gamma_i(\mathbf{P}) \geq \Gamma_i(\mathbf{P'})$. This inequality still holds if $P_{k+1} \leq P'_{k+1}$. Therefore, we must have $P_{k+1} > P'_{k+1}$.

In consequence

$$\begin{aligned}\Gamma_{k+1}(\mathbf{P}) &= \frac{G_{k+1,k+1}P_{k+1}}{\sum_{j=1}^{k} G_{k+1,j}P_j}\\ &> \frac{G_{k+1,k+1}P'_{k+1}}{\sum_{j=1}^{k} G_{k+1,j}P'_j}\\ &= \Gamma_{k+1}(\mathbf{P'})\end{aligned}$$

which contradicts to the assumption that $\Gamma_i(\mathbf{P}) < \Gamma_i(\mathbf{P'})$ for all i.

Hence, it is true for $M = k+1$. The lemma then follows by the principle of mathematical induction. □

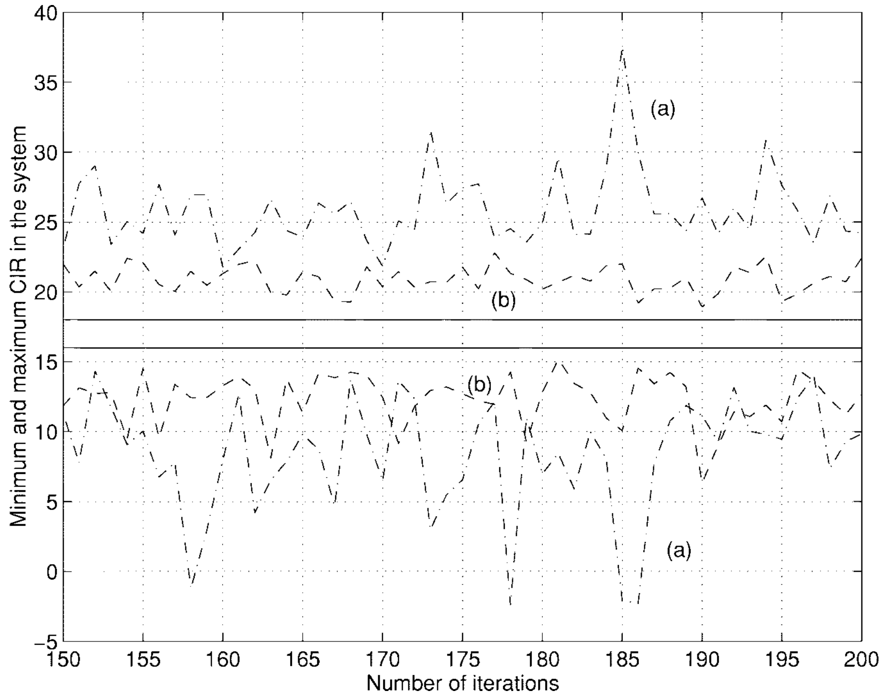

Fig. 6. Evolution of the maximum and minimum CIR (decibels) with CIR estimation error $\sigma_w = 3$ dB. (a) Foschini's algorithm. (b) Fixed-step algorithm with step size $\delta = 1$ dB. The target window of the fixed-step algorithm is also shown.

Lemma 3: If there exists a positive integer N such that $\Gamma_i^{(N)} \le \delta\gamma_i$ for all i and there exists a power vector $\mathbf{P}$ such that $\Gamma_i(\mathbf{P}) = \gamma_i$ for all i, the algorithm converges to a fixed point $\mathbf{P}^*$ where $\delta^{-1}\gamma_i \le \Gamma_i(\mathbf{P}^*) \le \delta\gamma_i$ for all i.

Proof: By the given condition, there exists a power vector $\mathbf{P}^*$ such that $\Gamma_i(\mathbf{P}^*) = \gamma_i$ for all i. By Lemma 1, for sufficiently large n, $\Gamma_i^{(n)} < \gamma_i$ for all i. Therefore, we have

$$\frac{G_{ii}P_i^{(n)}}{\sum_{j\neq i} G_{ij}P_j^{(n)} + \eta_i} < \frac{G_{ii}P_i^*}{\sum_{j\neq i} G_{ij}P_j^* + \eta_i}$$

which implies that

$$\frac{G_{ii}P_i^{(n)}}{\sum_{j\neq i} G_{ij}P_j^{(n)}} < \frac{G_{ii}P_i^*}{\sum_{j\neq i} G_{ij}P_j^*} + \frac{G_{ii}\left(P_i^* - P_i^{(n)}\right)\eta_i}{\sum_{j\neq i} G_{ij}P_j^{(n)} \sum_{j\neq i} G_{ij}P_j^*}.$$

Assume that the algorithm does not converge. From Lemma 1, there exists N' such that $i \in \mathcal{C}_n \cup \mathcal{D}_n$ for all i and all $n \ge N'$. As in the previous proof, when n tends to infinity, there exists j such that $P_j^{(n)}$ goes to infinity. So we cannot find mobile i where $i \in \mathcal{C}_n$ for all $n \ge N'$. Otherwise, $\lim_{n\to\infty} \Gamma_i^{(n)} = 0$, which is a contradiction. Therefore, $P_i^{(n)}$ becomes infinitely large for all i when n goes to infinity.

As a result, for all i and sufficiently large n

$$\frac{G_{ii}P_i^{(n)}}{\sum_{j\neq i} G_{ij}P_j^{(n)}} < \frac{G_{ii}P_i^*}{\sum_{j\neq i} G_{ij}P_j^*}.$$

This leads to a contradiction (see Lemma 2). □

Lemma 4: If $P_j^{(m)} \ge \delta^x P_j^{(n)}$ and $j \in \mathcal{D}_r$, where $r < m < n$ and $x \ge 1$, then there exists $k \neq j$ such that $P_k^{(s)} \ge \delta^{x+1} P_k^{(t)}$, where $r \le s < m \le t < n$.

Proof: If $P_j^{(r)} < \delta^{-1} P_j^{(m)}$, then there exists s such that $r < s < m$ and $P_j^{(s)} = \delta^{-1} P_j^{(m)}$ and $j \in \mathcal{D}_s$. If $P_j^{(r)} \ge \delta^{-1} P_j^{(m)}$, we let $s = r$.

Since $P_j^{(m)} \ge \delta^x P_j^{(n)}$, there exists t, where $m \le t < n$, such that $P_j^{(t)} = \delta P_j^{(n)}$ and $j \in \mathcal{A}_t$.

Therefore

$$\begin{aligned} P_j^{(s)} &\ge \delta^{-1} P_j^{(m)} \\ &\ge \delta^{x-1} P_j^{(n)} \\ &= \delta^{x-2} P_j^{(t)}. \end{aligned} \tag{7}$$

Denote the interference at base station j by $I_j^{(n)}$, i.e.,

$$I_j^{(n)} \equiv \sum_{k\neq j} G_{jk} P_k^{(n)} + \eta_j.$$

Since $j \in \mathcal{D}_s$ and $j \in \mathcal{A}_t$, we have

$$\frac{G_{jj}P_j^{(s)}}{I_j^{(s)}} < \frac{\gamma_j}{\delta} \tag{8}$$

and

$$\frac{G_{jj}P_j^{(t)}}{I_j^{(t)}} > \delta\gamma_j. \tag{9}$$

By (7), (8), and (9)

$$\begin{aligned}\delta\gamma_j &< \frac{G_{jj}P_j^{(t)}}{I_j^{(t)}}\\ &\le \frac{G_{jj}P_j^{(s)}}{I_j^{(t)}\delta^{x-2}}\\ &< \frac{I_j^{(s)}\gamma_j}{I_j^{(t)}\delta^{x-1}}\\ \delta^x I_j^{(t)} &< I_j^{(s)}.\end{aligned}$$

It implies that there exists k such that

$$P_k^{(s)} > \delta^x P_k^{(t)}.$$

Since the power level is quantized into discrete levels with step δ, we have

$$P_k^{(s)} \ge \delta^{x+1} P_k^{(t)}. \qquad \square$$

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