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Fixed Channel Assignment Optimization for Cellular Mobile Networks

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SUMMARY The optimization of channel assignment in cellular mobile networks is an NP-complete combinatorial optimization problem. For any reasonable size network, only sub-optimal solutions can be obtained by heuristic algorithms. In this paper, six channel assignment heuristic algorithms are proposed and evaluated. They are the combinations of three channel assignment strategies and two cell ordering methods. What we found are (i) the *node-color ordering* of cells is a more efficient ordering method than the *node-degree ordering*; (ii) the *frequency exhaustive strategy* is more suitable for systems with highly non-uniformly distributed traffic, and the *requirement exhaustive* strategy is more suitable for systems with less non-uniformly distributed traffic; and (iii) the *combined frequency and requirement exhaustive* strategy with *node-color re-ordering* is the most efficient algorithm. The frequency spans obtained using the proposed algorithms are much lower than that reported in the literature, and in many cases are equal to the theoretical lower bounds.

key words: *fixed channel assignment, heuristics, hotspot, cell re-ordering*

1. Introduction

The demand for cellular mobile services is increasing at a very high rate each year and in a lot of metropolitan areas the demand has already far exceeded the capacity. Different techniques can be used to increase the capacity. This includes cell splitting, allocation of new spectrum, alternative multiple access schemes (TDMA, CDMA) and dynamic channel assignments. For a given cellular system with a fixed spectrum assigned and a specific multiplexing technology used, the traffic-carrying capacity of a system depends on the efficiency of the channel assignment strategy used. Despite various proposals on dynamic channel assignment strategies, all existing cellular systems employ the fixed channel assignment because of its cost effectiveness and predictable performance.

The problem of assigning a set of compatible channels to each cell in a spectrum efficient way is called channel assignment problem. It is very important in cellular mobile network planning because an efficient channel assignment gives an efficient use of the available spectrum [1]–[3]. However, the channel assignment problem has been shown to belong to the class of NP-complete combinatorial optimization problems [4]. Its solution is in general not feasible for any reasonable size cellular mobile networks. A number of more practical approaches to this problem were proposed in the literature. This includes the use of traditional graph theoretic approach [5], the use of simulated annealing [6] approach and the use of neural networks [7] approach. Each of these approaches was shown to have its own limitations.

The neural network approach [7], [8] was shown to be inappropriate for channel assignment as it generates poor solutions even in simple cases. The use of simulated annealing [6] can avoid being trapped by the local minimum solutions but at the expense of very high running time complexity. Besides, the solution quality is very difficult to control. Further research in this direction is needed.

The graph theoretic approach has been extensively studied and a lot of research results have been reported. In the following the most important ones are summarized. Based on the heuristic of assigning channels to the cell with the highest assignment difficulty first, Box [1] proposed an iterative algorithm with an initial set of randomly generated numbers to represent the assignment difficulties of individual cells. This algorithm was shown to have a slow convergence rate and a high running time complexity especially when the system size is large.

In [2], a heuristic measure of the assignment difficulty was proposed and cells are ordered into a list by either node-color ordering or node-degree ordering. Based on the list, channels are assigned by either frequency exhaustive (F) or requirement exhaustive (R) strategies. Later in [5], an improved heuristic measure for channel assignment difficulty was proposed and a new cell ordering method called column-wise cell ordering was also introduced. It has been shown that algorithms proposed in [5] give the best performance over all existing algorithms on the 21-cell landmark examples adopted.

Since the heuristic algorithms give no information on the quality of the solution, some lower bounds on channel assignment problems were derived in [3] by considering a decoupled sub-network of the original system. Recently, a tighter lower bound under certain conditions was derived in [9], [10]. Two algorithms based on

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graph coloring approach were also proposed.

In this paper, we follow the basic idea of first ordering the cells into an ordered list, then performing channel assignment. Unlike conventional approaches, cells are re-ordered after each channel assignment according to their *modified* assignment difficulties (to be explained later). A total of six new channel assignment algorithms, namely algorithms F/CR, F/DR, R/CR, R/DR, FR/CR and FR/DR are proposed. The performances of these algorithms are studied and compared with previously reported results as well as the theoretical lower bounds reported. What we found are (i) the node-color ordering is a more efficient ordering method than the node-degree ordering; (ii) the frequency exnahustive strategy is more suitable for systems with highly non-uniformly distributed traffic, and requirement exhaustive strategy is more suitable for systems with less non-uniformly distributed traffic; and (iii) strategy FR with node-color re-ordering, or algorithm FR/CR, is the most efficient algorithm. The frequency spans found using our proposed algorithms are significantly lower than that found by algorithms in [5] and are either equal or very close to the theoretical lower bounds.

In the next section, the channel assignment problem is formulated and our motivations are explained. In Sect. 3. we review some of the basic heuristics in channel assignments. In Sect. 4, algorithms F/CR, F/DR, R/CR and R/DR are proposed. Strategy FR, the combined frequency exhaustive and requirement exhaustive strategy, is then studied in Sect. 5 with emphasis on reasons behind the heuristics. In Sect. 6, the performances of the six proposed channel assignment algorithms are evaluated and compared with those proposed in [5] as well as the theoretical lower bounds. Finally, conclusions are presented in Sect. 7.

2. Problem Formulation

Three types of constraints are usually considered in channel assignment problems:

- Co-channel interference constraints: a channel assigned to one cell cannot be reused in its nearby cells that are within its co-channel interference range;
- Adjacent channel interference constraints: channels assigned to adjacent cells must maintain a minimum separation of a channels;
- Co-site channel interference constraints: channels assigned to the same cell must maintain a minimum separation of s channels.

For an N-cell cellular system, an $N \times N$ channel compatibility matrix $\mathbf{C} = [c_{ij}]$ can be used to represent these three types of constraints, where c_{ij} denotes the minimum channel separation between channels assigned to cell i and cell j. It is easy to see that $c_{ij} = 0$ means a channel assigned to cell i can be reused at cell j. $c_{ii} = s$ means the co-site channel interference constraint is s channels. $c_{ij} = a$ means the adjacent channel interference constraint is a channels. In practice, the compatability matrix is usually obtained by measurements since a real system does not have an idealized regular hexagonal structure.

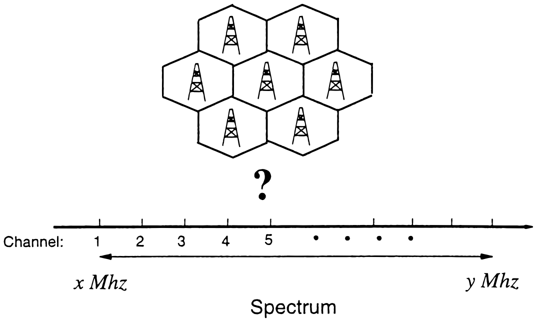

Fig. 1 Fixed channel assignment in a cellular mobile system.

Let vector $\mathbf{M} = (m_1, m_2, \cdots, m_N)$ denote the channel requirements of an N-cell network. Let the set of frequency channels be ranked by a set of positive integers $\{1, 2, 3, \cdots\}$ according to their carrier frequencies (Fig. 1). Let f_{ik} be the channel assigned to the k-th call in cell i. An admissible channel assignment [2] is a collection of integers $F = (f_{ik})$, where $i = 1, 2, \cdots, N$ and $k = 1, 2, \cdots, m_i$, such that

$$|f_{ik} - f_{jl}| \geq c_{ij}$$

for all i, j, k and l (except for $i = j$ and $k = l$). An efficient channel assignment algorithm should have an admissible channel assignment F^* with $N(F^*) = \max_{i,k} f_{ik}$ as small as possible. $N(F^*)$ is known as the frequency span of an assignment.

Traffic rates in a cellular network vary from cell to cell. Those areas with substantially higher traffic rates are called hotspots. Normally, a hotspot involves several interfering cells with each cell having a high channel requirement. Cells inside a hotspot are called hotspot cells. The frequency span of a system is usually determined by the span of the "hottest" hotspot. The span of a hotspot depends on how channels are assigned to *its* hotspot cells. In other words, if channels assigned to the hotspot cells (belonging to the same hotspot) are packed efficiently, a small frequency span for the overall system is likely to be obtained. Therefore, an efficient channel assignment algorithm should concentrate on satisfying the requirements of each group of hotspot cells.

Many previous studies [2], [5], [11] focus on assigning channels to the cell with the highest assignment difficulty first. Heuristic measures of the assignment difficulties are defined and cells are then ordered into a list. Based on the list, channels are assigned. We find two problems on this approach. The first problem is on the ordering of cells. As channels are assigned to the cells, the assignment difficulty of individual cell changes. It is therefore necessary to re-order the cells after each channel assignment. The second problem is that this approach only concentrates on assigning channels to the *cell* with the highest assignment difficulty. It fails to identify *hotspot* areas and thus can cause very inefficient channel allocation when consecutive cells in the ordered list do not belong to the same hotspot.

3. Basic Heuristics

3.1 Two Methods for Cell Ordering

Let the degree d_i be a measure of the difficulty of assigning a channel to cell i. In [5], it was defined as

$$d_i = \sum_{j=1}^{N} m_j c_{ij} - c_{ii}, \quad 1 \le i \le N \tag{1}$$

if $m_i \neq 0$; otherwise, $d_i = 0$. Since the term c_{ii} on the right hand side of Eq. (1) is the same for all cells, we propose to remove it for a simpler form

$$d_i = \sum_{j=1}^{N} m_j c_{ij}, \quad 1 \le i \le N. \tag{2}$$

Using *node-degree ordering* [11], cells are ordered in descending values of d_i. The first cell therefore has the highest priority of getting a channel first.

Using *node-color ordering* [11], cells are first ordered by node-degree ordering. The last cell in the list is moved to an empty list, say list A. The degrees of the remaining $N-1$ cells are re-computed with the last cell's channel requirement eliminated. The remaining $N-1$ cells are then re-ordered by their modified node-degrees. Next, move the last cell in the reduced list to list A and place it immediately before the already listed ones. Continue this procedure until all N cells are moved to list A. This is known as the node-color ordering [11].

The above two cell ordering methods have been extensively used in the heuristic algorithm designs. It is however not obvious which ordering method provides a better performance. In Sect. 6, the efficiency of the two cell orderings will be studied.

3.2 Two Channel Assignment Strategies

Given an ordered list of cells (obtained by either node-degree ordering or node-color ordering), two channel assignment strategies, frequency exhaustive (F) strategy and requirement exhaustive (R) strategy [11], can be used to assign channels to cells. In frequency exhaustive strategy, starting with the first cell in the ordered list, each cell with some unsatisfied channel requirement is assigned a channel with the lowest rank (all channels are ranked according to their carrier frequencies), and is consistent with all the previous assignments.

In requirement exhaustive strategy, channel 1 is assigned to the first cell in the ordered list with some unsatisfied channel requirements. Then try the same channel for the next cell in the list that has unsatisfied channel requirements. Continue this attempt until this channel cannot be accepted by any more cell. Then follow the same procedure to assign channel 2. Repeat this procedure until all channel requirements are satisfied (or exhausted).

4. Channel Assignments with Cell Re-Ordering

4.1 Four Channel Assignment Algorithms

The purpose of ordering cells is to identify the most difficult-to-assign cells. When channels are assigned to a cell, the cell's channel requirement is reduced and its associated assignment difficulty as well as that of its neighboring cells are also reduced. To truely reflect the *instantaneous* assignment difficulty, the ordered list should be re-ordered before assigning the next channel.

Based on the two basic cell ordering principles, two cell re-ordering methods, namely node-color re-ordering (CR) and node-degree re-ordering (DR), are designed. Combining each of these with frequency exhaustive strategy and requirement exhaustive strategy, four algorithms[†], namely F/CR, F/DR, R/CR and R/DR are obtained. They are described by the following pesudo-codes.

Algorithms F/CR or F/DR

```
Input: $\mathbf{C} = [c_{ij}]$ and $\mathbf{M} = (m_1, m_2, \cdots, m_N)$
Output: $N(F^*)$
1. For $i = 1$ to $N$ do
       $m'_i = m_i$;
2. For $i = 1$ to $N$ do
       if $m'_i = 0$, $d_i = 0$;
       else $d_i = \sum_{j=1}^{N} m'_j c_{ij}$;
3. Order cells into an ordered list using node-color
   OR node-degree ordering;
4. If the degree of the first cell in the list $d_i \neq 0$
       find $g$ the channel with the lowest rank such
       that the assignment of $g$ to cell $i$ is consistent
           with all previous assignments.
       $m'_i = m'_i - 1$, $k = m_i - m'_i$ and $f_{ik} = g$;
       goto Step 2;
5. Else $N(F^*) = \max_{i,k} f_{ik}$ and EXIT;
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Algorithms R/CR or R/DR

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Input: $\mathbf{C} = [c_{ij}]$ and $\mathbf{M} = (m_1, m_2, \cdots, m_N)$
Output: $N(F^*)$
1. For $i = 1$ to $N$ do
       $m'_i = m_i$;
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[†] The notation "channel-assignment-strategy/re-ordering-method" is used.

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2. $f = 1$;
3. For $i = 1$ to $N$ do
     if $m'_i = 0$, $d_i = 0$;
     else $d_i = \sum_{j=1}^{N} m'_j c_{ij}$;
4. Order cells into an ordered list using node-color
   **OR** node-degree ordering method.
5. Find $i$ the first cell in the list such that the
   assignment of channel $f$ to cell $i$ is consistent
     with all previous assignments;
6. If cell $i$ is found
     if $d_i \neq 0$
       $m'_i = m'_i - 1$, $k = m_i - m'_i$ and $f_{ik} = f$;
       goto Step 3;
     else $N(F^*) = \max_{i,k} f_{ik}$ and EXIT;
7. Else $f = f + 1$; goto Step 4.
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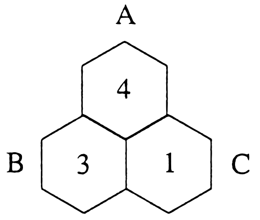
4.2 Complexity

Let the total number of channel requirements be $M = \sum_{i=1}^{N} m_i$. For algorithms F/CR or F/DR, Step 2 consists of N operations for finding d_i. Assume *bubble* sort is used, the number of operations involved in Step 3 for node-color ordering is $O(N^3)$ and that for node-degree ordering is $O(N^2)$. Steps 2 to 4 form a cycle and will excecute M times. The total number of comparisons for excecuting M times of Step 4 is $1+2+\cdots+M-1$, or $O(M^2)$. Therefore the overall time complexity of algorithm F/CR is $MO(N^3)+O(M^2) = O(MN^3+M^2)$. The overall time complexity of algorithm F/DR is $O(MN^2+M^2)$.

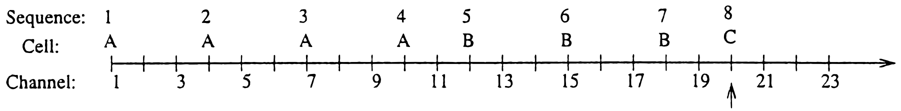
For algorithms R/CR or R/DR, Steps 3 to 7 perform M times for assigning M channels. Similar to algorithms F/CR and F/DR, the time complexity of Step 4 is $O(N^3)$ if node-color ordering is used and $O(N^2)$ if node-degree ordering is used. The total number of comparisons for excecuting M times of Step 5 is $O(M^2)$. The overall time compexity is found to be $O(MN^3+M^2)$ for R/CR and $O(MN^2+M^2)$ for R/DR.

4.3 An Example

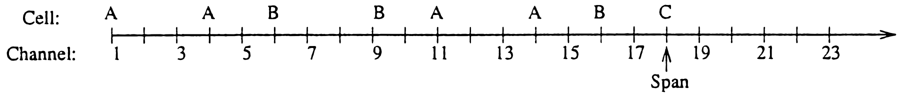
To demonstrate the effectiveness of using cell re-ordering, consider a simple 3-cell system shown in Fig. 2(a). Cells A, B and C have channel requirements 4, 3 and 1 respectively. Let the adjacent channel interference constraint and the co-site channel interference constraint be $a = 2$ and $s = 3$ respectively. The degrees of cells A, B and C are found to be 20, 19 and 14 from Eq. (2). For this particular example, it happens that the four algorithms proposed in the previous section perform the same. Therefore for convenience, we shall use "with cell re-ordering" and "without cell re-ordering" to differentiate these two types of channel assignments.



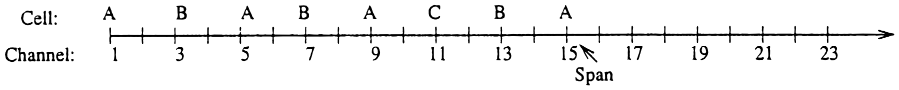
(a) A 3-cell system consists of cells A, B and C.



(b) Without cell re-ordering.



(c) With cell re-ordering and 1st tie resolution method.



(d) With cell re-ordering and 2nd tie resolution method.

Fig. 2 The channel assignment plans for a 3-cell system.

For channel assignments without cell re-ordering, the original ordered cell list (A, B, C) is used until all channel requirements are satisfied. The resulting channel assignment plan is shown in Fig. 2(b). The channel assignment sequence is also shown in the figure. The frequency span is found to be 20.

When channel assignments with cell re-ordering are used, the ordered list of cells is updated every time a channel is assigned. In case of a tie, i.e. more than one cell have the same assignment difficulty, two tie resolution methods are used: (1) choose the cell to which a channel is most recently assigned first, or (2) vice versa. (Note that any other tie resolution methods can be used.) Using the first tie resolution method, the resulting channel assignment plan is shown in Fig. 2(c) and the sequence of ordered list used is $(A,B,C) \to (A,B,C) \to (B,A,C) \to (B,A,C) \to (A,B,C) \to (A,B,C) \to (B,C,A) \to (C,B,A)$. The frequency span is 18. Using the second tie resolution method, the sequence of the ordered list used is $(A,B,C) \to (B,A,C) \to (A,B,C) \to (B,A,C) \to (A,C,B) \to (C,B,A) \to (B,A,C) \to (A,B,C)$ and the resulting frequency span is 15 as shown in Fig. 2(d). It can be shown by enumeration that the frequency span of 15 is optimal for this example. To summarize, the frequency span is reduced from 20 to 15 with the use of cell re-ordering.

5. Optimization of Channel Assignment at Hotspots

As mentioned in the Introduction, the conventional approach to heuristic channel assignment has two problems. In the previous section, we solved the first problem by re-ordering cells before assigning the next channel. In this section, we concentrate on the second problem: efficient assignment of channels to hotspots. Usually a hotspot involves a number of hotspot cells and the frequency span of a hotspot depends on how channels are assigned to *its* hotspot cells. Focusing on the optimization of channel assignments at hotspots, a new strategy called combined frequency exhaustive and requirement exhaustive (FR, in short) strategy is proposed. Strategy FR can then combine with the two cell re-ordering methods to produce two more channel assignment algorithms, FR/CR and FR/DR.

Before we proceed, let us take a closer look at frequency exhaustive strategy (F) and requirement exhaustive strategy (R) which provide us some insights on why combining the two strategies to get strategy FR.

5.1 Strategy F vs. Strategy R

Our first observation is that *using strategy R to assign a channel, the number of cochannel cells*[†] *of the assigned channel tends to be larger than that can be obtained using strategy F.* Without loss of generality, let co-site channel interference constraint value is greater or equal to that of the adjacent channel interference constraint, i.e. $s \geq a$. When assigning channel f to a cell using strategy R, the assignment will be successful if the interference constraints posed by the previous assignments of channels from $(f-s+1)$ to f are not violated. Note that the channel with the highest rank assigned so far is the channel which is currently being assigned, i.e. channel f.

If strategy F is used, assigning channel f to a cell needs to check the constraints posed by the assignments of channels from $(f-s+1)$ to $(f+s-1)$. The number of channels to be checked almost doubles that for strategy R. As a result, channel f has a higher probability being rejected by a cell. This results in relatively loosely packed cochannel cells of channel f.

Our second observation is that *unlike strategy F, using strategy R channels are assigned not exactly following the assignment difficulties of individual cells.* Suppose strategy R is used to assign channel f to a system. If no constraint is violated, the first cell in the ordered list gets channel f. Then channel f is assigned to *all* possible cells (i.e. cells with unsatisfied requirement and do not violate any channel assignment constraints) in the list before considering channel $f+1$. The subsequent assignments of channel f to the remaining cells, however, do not exactly follow the assignment difficulties.

Consider a simple example. Assume when channel 1 is assigned to the first cell in an ordered list, the first cell *remains* to be the most difficult-to-assign cell after cell re-ordering. Since the same channel, i.e. channel 1, cannot be assigned to the same cell twice. As a result, the next assignment is assigning channel 1 to another cell which is *not* the most difficult-to-assign. On the contrary, strategy F does not have this problem.

From the above two observations, we can see that for systems with small variations in cell to cell channel requirements (or less non-uniformly distributed channel requirements), it is more important to maximize the number of cochannel cells. Therefore strategy R tends to give a better performance. For systems with highly non-uniformly distributed channel requirements, it is more critical to satisfy the channel requirements of the most difficult-to-assign cells first. Therefore strategy F tends to perform better.

5.2 Strategy FR

To combine the advantages of strategies R and F, strategy FR is proposed. It is a two level channel assignment strategy which consists of a Global Assignment using strategy R, and a Local Assignment using strategy F. The Global Assignment is to identify hotspots and to

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[†] Recall that cochannel cells are cells assigned with the same frequency channel.

use strategy R to maximize the number of cochannel cells. The Local Assignment focuses on assigning channels to an indentified hotspot and to pack those channels closely using strategy F.

When channel f is assigned to cell i (the first cell in the ordered list) by Global Assignment, a hotspot centered at cell i (or, hotspot i) is identified. It is very difficult to have a threshold-typed rule for telling which interfering cell of cell i belongs to the same hotspot and which does not. We take a heuristic approach by defining that hotspot i contains the cells which are within the interference range of cell i, and have *relatively high* channel requirements. Again to simplify the process of determine cells with relatively high channel requirements, we define a parameter Y. Let all interfering cells of cell i form an ordered list with descending assignment difficulties. The first Y cells in the ordered list, or hotspot cells, are chosen to participate in the subsequent Local Assignment. The value of Y can be varied for obtaining different performances. It should be noted that this is only a heuristic method for identifing a hotspot. It is simple but it is not optimal. There exists many other alternative ways.

Once the hotspot at cell i is identified using the above method, the Global Assignment branches out for performing Local Assignment. The Local Assignment uses strategy F to assign channels to hotspot cells of *hotspot* i (but not including cell i). In the Local Assignment, each hotspot cell is allowed to get at most one channel and that channel's rank must be less than or equal to $f + X$, where X is another integer parameter to be designed/tuned. In fact, it is impossible to find a channel with rank less than or equal to f (since the Global Assignment uses strategy R) and therefore, a channel can only be chosen from ranks $f+1$ to $f+X$ in the Local Assignment. The purpose of setting the limit $f + X$ is to minimize the interference that would be caused by the locally assigned channels on the subsequent channel assignments.

When the Local Assignment is finished, the Global Assignment resumes to assign channel f to the next possible cell in the ordered list (using strategy R). If no more cell can accept channel f, proceed with channel $f + 1$. Continue this procedure until all channel requirements are satisfied.

Combining strategy FR with the two cell reordering methods, two more channel assignment algorithms FR/CR and FR/DR are obtained. It should be noted that when cell re-ordering is used, cells are re-ordered after each channel assignment no matter the channel assignment is in Global Assignment or in Local Assignment. In the follwoing, we summarize algorithms FR/CR and FR/DR by pesudo-codes:

Global Assignment

```
Input: $\mathbf{C} = [c_{ij}]$ and $\mathbf{M} = (m_1, m_2, \cdots, m_N)$
Output: $N(F^*)$
1. For $i = 1$ to $N$ do
     $m'_i = m_i$;
   $f = 1$;
2. For $i = 1$ to $N$ do
     if $m'_i = 0$, $d_i = 0$;
     else $d_i = \sum_{j=1}^{N} m'_j c_{ij}$.
3. Order cells into an ordered list using node-color
   OR node-degree ordering.
4. Find $i$ the first cell in the list such that the
   assignment of channel $f$ to cell $i$ is consistent
     with all previous assignments.
5. If cell $i$ is found
     if $d_i \neq 0$
       $m'_i = m'_i - 1$, $k = m_i - m'_i$ and $f_{ik} = f$;
       goto Step 1 of Local Assignment;
     else $N(F^*) = \max_{i,k} f_{ik}$ and EXIT;
6. Else $f = f + 1$ and goto Step 4.
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Local Assignment

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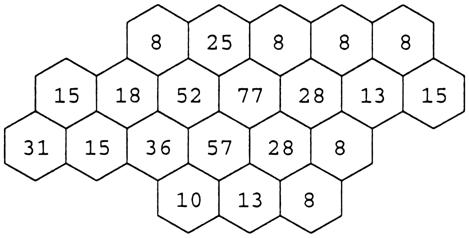
1. counter = 1.
2. While counter $\leq Y$ do
     for each cell $j$ with $c_{ij} \geq 1$ do
       if $m'_j = 0$, $d_j = 0$;
       else $d_j = \sum_{k=1}^{N} m'_k c_{jk}$;
     order cells with $c_{ij} \geq 1$ into an ordered list using
     node-color OR node-degree ordering;
     if the degree of the first cell in the list $d_j \neq 0$
       find $g$ the channel with the smallest rank such
       that the assignment of $g$ to cell $j$ is consistent
         with all previous assignments and
         $f < g \leq f + X$.
       if $g$ is found
         $m'_j = m'_j - 1$, $k = m_j - m'_j$ and $f_{jk} = g$;
       counter = counter + 1;
     else goto Step 2 of Global Assignment.
3. Goto Step 4 of Global Assignment.
```

The optimal values for X and Y such that the frequency span of a system is minimized are very difficult to obtain. In fact, their optimal values should be adjusted after each channel assignment just like performing cell re-ordering. For simplicity, we assume the values of X and Y are both fixed in this paper. Suppose channel f is assigned to cell k by Global Assignment. Some considerations for suitable values of X and Y are summarized below.

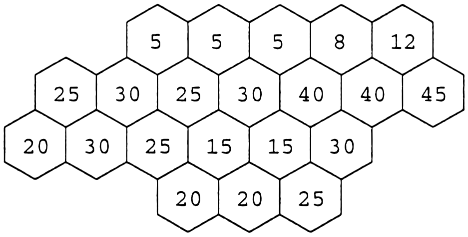
- If $X = 0$, Local Assignment is by-passed.
- If $X = 1$, only cells with $c_{kj} = 1$ can participate in Local Assignment.
- If $X \leq s - a$, the subsequent assignment of channel f to the rest of the system will not be affected by the interference introduced by the current Local Assignment.
- If $X \leq s + 1$, any interfering cell of cell k can get at most one channel in Local Assignment.
- $0 \leq Y \leq$ the total number of interfering cells of cell k.

- If $Y = 0$, Local Assignment is also by-passed.
- Y should take a value which is comparable to the number of hotspot cells in a hotspot centered at cell k.

It can be found that algorithms FR/CR and FR/DR have the same time complexity as that of F/CR and F/DR, i.e. $O(MN^3 + M^2)$ for FR/CR and $O(MN^2 + M^2)$ for FR/DR. This is because the Local Assignment only has the complexity of $O(M + N)$.



(a) Case I channel requirement.



(b) Case II channel requirement.

Fig. 3 A 21-cell hypothetical cellular network with two cases of channel requirements.

6. Performance Evaluations

We use the same 21-cell landmark examples as that used in [3], [5] for performance evaluations. For easy comparison, the theoretical channel assignment lower bounds for this system have also been obtained [3], [9]. Figure 3 shows two cases of channel requirements. The number shown inside each cell is the channel requirement of that cell. Cells are numbered from 1 to 21 in the order of left to right and top to bottom. In our program, when node-degree ordering is used, in case of a tie the cell with the smallest cell number is selected first. When node-color ordering is used, in case of a tie the cell with the largest cell number is selected first.

Let ordered triplet (N_c, a, s) denote a system with cluster size N_c, adjacent channel constraint a, and co-site channel cosntraint s. Channel assignment results are summarized in Tables 1, 2 and 3. Column SMK corresponds to the range of frequency spans obtained from the eight algorithms in [5]. Column LB corresponds to the theoretical lower bounds [3], [9]. Those bounds were obtained by considering a decoupled subnetwork of the original system. It should be noted that we do not know how tight those bounds are.

6.1 Performance of Algorithms F/CR, F/DR, R/CR and R/DR

For various network configurations, Table 1 summarizes the frequency spans obtained using algorithms F/CR, F/DR, R/CR and R/DR. The minimum span found

Table 1 Frequency spans obtained by F/CR, F/DR, R/CR and R/DR.

Case	Network config.	LB	F/CR	F/DR	R/CR	R/DR	SMK
I	(12,2,3)	427	435	472	427	431	436-554
I	(7,2,3)	427	433	475	442	439	442-554
I	(12,2,5)	427	431	448	489	481	460-543
I	(7,2,5)	427	432	476	468	496	447-543
I	(12,2,7)	533	533	533	568	568	536-565
I	(7,2,7)	533	533	533	557	557	533-566
II	(12,2,3)	258	286	339	262	278	272-327
II	(7,2,3)	253	265	309	263	271	265-340
II	(12,2,5)	258	293	289	267	291	283-360
II	(7,2,5)	258	264	269	264	277	269-347
II	(12,2,7)	309	309	315	318	321	310-384
II	(7,2,7)	309	309	315	325	337	310-358

Table 2 Frequency spans obtained by FR/CR and FR/DR with (7,2,5) and Case I channel requirements.

Algorithm	Y	$X = 1$	$X = 2$	$X = 3$	$X = 4$	$X = 5$
FR/CR	$Y = 1$	488	450	446	445	446
	$Y = 2$	485	459	428	437	433
	$Y = 3$	487	466	445	438	437
FR/DR	$Y = 1$	508	451	457	456	458
	$Y = 2$	491	462	438	453	441
	$Y = 3$	493	472	446	448	444

Table 3 Frequency spans obtained by FR/CR and FR/DR with $0 \leq X \leq 5$ and $1 \leq Y \leq 3$.

Case	Network config.	LB	FR/CR	FR/DR	SMK
I	(12,2,3)	427	427 (0,Y) (1,1)-(1,3)	427 (1,2) (1,3)	436-554
I	(7,2,3)	427	430 (1,3)	428 (1,3)	442-554
I	(12,2,5)	427	428 (3,2) (4,2) (5,3)	431 (5,2) (3,3)	460-543
I	(7,2,5)	427	428 (3,2)	438 (3,2)	447-543
I	(12,2,7)	533	533 (3,1)-(5,1) (3,2) (5,2) (3,3)	533 (2,1)-(5,1)	536-565
I	(7,2,7)	533	533 (3,1)-(5,1)	533 (3,1)-(5,1)	533-566
II	(12,2,3)	258	262 (0,Y)	278 (0,Y)	272-327
II	(7,2,3)	253	257 (1,3)	265 (1,2)	265-340
II	(12,2,5)	258	263 (1,2)	272 (2,3)	283-360
II	(7,2,5)	258	263 (2,2) (4,3)	262 (1,3)	269-347
II	(12,2,7)	309	310 (4,1) (5,1)	316 (1,1) (4,3)	310-384
II	(7,2,7)	309	309 (5,2)	320 (3,1)	310-358

in each row is underlined. From the table, we can see that algorithm F/CR always outperforms F/DR. This shows the node-color cell ordering is more efficient than node-degree ordering. The same trend is however not so obvious for algorithms R/CR and R/DR. This is because channels are assigned not exactly following the assignment difficulties when strategy R is used (refer to Sect. 5.1). As what we predicted in Sect. 5.1, algorithm F/CR performs better in Case I channel requirements (which is highly non-uniformly distributed), and algorithm R/CR performs better in Case II channel requirements (which is more evenly distributed). For each case studied, the minimum span found by our algorithms is much lower than that found by algorithms in [5]. As an example, for Case I channel requirements and network configuration (12,2,5), the minimum span we found is 431 and that by algorithms in [5] is 460.

6.2 Effect of X and Y on Performance of Algorithms FR/CR and FR/DR

Next we study the performance of algorithms FR/CR and FR/DR with different values of X and Y. Consider a system with configuration (7,2,5) and case I channel requirements, Table 2 shows the resulting frequency spans for $1 \leq X \leq 5$ and $1 \leq Y \leq 3$. When $X = 3$ and $Y = 2$, the two algorithms both give the minimum frequency spans of 428 and 438 respectively. The best result obtained by algorithms in [5] is only 447 and that by a recent paper [12] is only 446. Besides, the values of X and Y meet our expections in the previous section.

6.3 Performance of Algorithms FR/CR and FR/DR

For $0 \leq X \leq 5$ and $1 \leq Y \leq 3$, Table 3 summarizes the minimum spans found using algorithms FR/CR and FR/DR. The ordered pair (X, Y) shown in the table denotes the values using which the minimum span is found. $(0, Y)$ means Y can be any value. Again, the minimum span of each row in the table is underlined. It can be seen that algorithm FR/CR always outperforms the algorithms in [5]. Besides, it produces a smaller span than algorithm FR/DR except for Case I with (7,2,3) and Case II with (7,2,5). Comparing algorithm FR/CR with algorithms F/CR and R/CR in Table 1, FR/CR gives the lowest span in every case except for Case II with (12,2,7) but the difference is only 1 channel.

In summary, for Case I channel requirements, the lowest spans found by our algorithms are at most one channel higher than the theoretical lower bounds. For Case II channel requirements, the lowest spans found by our algorithms are at most five channels higher than the theoretical lower bounds.

7. Conclusions

Two problems with the conventional heuristic channel assignment algorithms have been identified in this paper. Cell re-ordering and a channel assignment strategy focusing on hotspots were then proposed to solve these two problems. As a result, six new channel assignment algorithms which are the combinations of three channel assignment strategies and two cell re-ordering methods were proposed and studied. What we have found are (i) the node-color ordering of cells is a more efficient ordering method than the node-degree ordering; (ii) strategy R is more suitable for systems with highly non-uniformly distributed traffic, and strategy F is more suitable for systems with less non-uniformly distributed traffic; and (iii) algorithm FR/CR is the most efficient algorithm which gives the lowest frequency span in almost every case. Besides, the lowest spans found by our algorithms are much lower than that reported in the literature and are in many cases equal to the theoretical lower bounds.

Determining the optimal values for parameters X and Y in algorithms FR/CR and FR/DR is very important. In this paper, we have only studied the case that X and Y are fixed. It is worthwhile to further investigate how to improve the performance by dynamically adjusting the values of X and Y in response to the remaining channel requirements of the individual cells.

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