The Maximum Mean Time to Blocking Routing in Circuit-Switched Networks

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***Abstract*—The Maximum Mean Time to Blocking (*MTB*) Routing is a state- and time-dependent adaptive routing scheme. In this scheme, overflowed calls are routed to an alternate path having the longest *mean time to blocking*. The *mean time to blocking* of a link is a function of the trunk group size, the traffic rate, and the instantaneous trunk group occupancy and is a particularly suitable measure of the busy status of links in networks with nonuniform trunk group sizes and asymmetric traffic rates. The computation of the *mean time to blocking* of a path is very demanding and two approximations are proposed. A comparative performance evaluation through a call-by-call computer simulation shows that the MTB routing can give a superior throughput-blocking performance.**

I. INTRODUCTION

ONE of the most important measures of the quality of a telephone network is the end-to-end call blocking probability and one of the most important functions of network management is to ensure that the end-to-end blocking probability of the network is as small as possible. When a network overload occurs, various network management functions such as the reduction of operator traffic, selective trunk reservation, and traffic rerouting are performed to minimize the network congestion. A good routing rule can reduce the blocking probability by making use of spare resource in other parts of the network. A good routing rule is, therefore, essential for assuring the quality of telecommunication services.

The traditional method of routing in telephone networks worldwide is of thehierarchical type. Switching offices and exchanges are classified according to their levels in a hierarchy. Alternate routing has been widely used in these hierarchical circuit-switched networks. With the widespread installation of a sophisticated computer-controlled electronic switching system such as the 4ESS switch and the use of a new digital out-of-band signaling system, the CCITT SS7, the hierarchical structure of the network is no longer a requirement. Various advantages of the nonhierarchical network are discussed in detail in [1].

Krupp [2] has shown that alternate routing in a fully connected network exhibits instability under high load. An effective way to resolve this problem is by trunk reservation, i.e., to reserve a number of trunks for direct calls only. Another approach is the external blocking [3].

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There are, in general, two approaches to dynamic routing in circuit-switched networks. One is called state-dependent routing, where the routing pattern varies dynamically according to the state of the network. Two implementations based on this approach are British Telecom's Dynamic Alternative Routing (DAR) [4], which is a decentralized routing system, and Northern Telecom's Dynamically Controlled Routing (DCR) [5], which is a centralized routing system. In particular, DCR selects an alternate path on a probability basis. The path selection probabilities are computed at fixed intervals and depend on the estimated idle capacity of the paths. DAR is a simplified implementation of the learning automata routing method. New alternate routing path are selected from a set of alternate paths in a random manner. In [13], a version of the Maximum Free Circuit Routing (M Routing) [15] and DAR are compared and it is reported that the former performs better.

The second approach is called time-dependent routing, in which alternate calls are routed according to a different preplanned routing patterns in different time intervals of a day. AT&T's Dynamic Nonhierarchical Routing (DNHR), introduced in 1984 [6], is an example of this approach. The routing pattern is optimized by a central network planning system based upon periodic traffic measurements made by the exchanges. A high degree of flexibility can be maintained in the planning of routing patterns but has little ability to accommodate unplanned traffic fluctuations.

Trunk groups in networks need to be resized from time to time to meet the increase of traffic over time. In real-time operation, some factors may cause overload in parts of the network while other parts of it may have idle resources [7]. Moreover, the use of Common Channel Signaling (CCS) allows trunk status information to be obtained in real time. As a result, the design of adaptive dynamic routing schemes which performs real-time tracking of networks, idle resources in networks have received much concern in recent years.

Various dynamic adaptive routing schemes have been proposed with the aim toprovide adaptability, flexibility, and robustness in network routing. A well-implemented dynamic routing scheme is Real-Time Network Routing (RTNR) [8]. It uses a simple trunk status map to show the trunk status information and supports a new class-of-service routing capability for dynamic networks. Class of services freely share link bandwidth when there is sufficient idle capacity, but bandwidth is reserved for a particular class of service when needed to meet its specific performance objective. RTNR can be considered as a simplified version of the M routing in the sense that trunk group occupancies are aggregated into

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several levels. (A detail analysis on the performance of this aggregated scheme can be found in [16]). If the aggregation is designed appropriately, the performance is shown to be very close to the original nonaggregated scheme. In July 1991, AT&T announced that RTNR has completely replaced DNHR and the first year of operation showed that RTNR has improved the network performance, reliability, and the ability to cope with failures.

In [13], several centralized and distributed routing schemes are compared. In particular, a new congestion measure, namely the probability that no call overflows before the end of a specific time interval (called the reconfiguration period) is used as a measure of trunk group congestion. This measure can also be extended to multilink paths. The computation of this probability, however, involves the solution of a system of $N + 1$ (N is the trunk group size) differential equations.

The State- and Time-Dependent Routing (STR) proposed in [9] is characterized by a two-level dynamic control. First, a set of alternate routes for each node pair is determined for each time period of the day by a centralized control method once a week. Second, in each exchange, a near-optimal alternate route is determined according to only the network state information obtained through the connection processes.

The State-Dependent Routing (*SDR*) [10], [11] is adecentralized routing scheme based on the Howard cost function computation on each link. The separable approximation technique computes multilink path cost as the sum of individual link costs. A modification of *SDR* is the Forward Looking Routing (*FLR*) [12].

Another adaptive routing technique is the so-called proportional routing in which a fixed proportion of calls of a node pair is carried along a specific route. These proportions are updated periodically in light of changing traffic conditions. One approach to determine the proportion is the ""implied cost" methodology [14]. This method can also be used in network dimensioning.

In this paper, we propose a new decentralized state- and time-dependent adaptive routing scheme called the Maximum Mean Time to Blocking (*MTB*) Routing. This scheme makes use of the *mean time to blocking* measure of links, which captures the loading and capacity information and reflects more accurately the busy status of a path. The rest of the paper is organized as follows: In Section II, we describe the network model and the *MTB* routing rule. In Section 3, we will discuss the implementation of the *MTB* routing algorithm. Two examples are presented in Section IV to illustrate the superiority of using *mean time to blocking* as a measure of the busy status in an asymmetric network. In Section V, extensive simulation results are presented. It is found that the *mean time to blocking* is a better measure of the busy status of a trunk group when compared with the number of free channels used in the Maximum Free Circuit Routing (*M* routing), and *MTB* routing gives superior blocking performance over both *SDR* and the *M* routing.

II. The Maximum Mean Time to Blocking Routing

Consider a nonhierarchical circuit-switched network of arbitrary topology. Each node pair in the network may be connected by a large number of paths. For each call on this node pair, the direct path (if available) is tried first. If the direct path is blocked, an alternate path satisfying the trunk reservation requirement is chosen to carry the call. Each trunk group can accommodate both direct and overflowed traffic. Trunk reservation technique is used at each trunk group to protect the direct traffic.[1] Let the arrival of calls to all node pairs be a Poisson process and the duration of calls be exponentially distributed. Furthermore, for the purpose of deriving the time to blocking statistics on a link, the overflowed traffic to a link is approximated by a Poisson process.

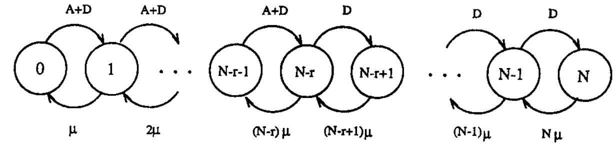

Fig. 1. An M/M/N/N queueing model.

With these assumptions, a trunk group of size N can be modeled as an M/M/N/N queue. Fig. 1 shows the state transition rate diagram of the queuing system where D is the direct traffic rate, A is the alternate traffic rate, μ^{-1} is the mean holding time (or service time), and r is the trunk reservation parameter.

Given that the trunk group has occupancy i, the time to blocking $\tau(i)$ is just the first passage time from state i to state N. Let $P_{i,j}$ be the transition probability from state i to state j and $h_{i,j}(x)$ be the probability density function of the one-step transition time from state i to state j. The integral formula of the probability density function of $\tau(i)$, denoted by $f_{\tau(i)}(t)$, is given by [18] as:

$$f_{\tau(i)}(t) = \sum_{j=0}^{N-1} P_{i,j} \int_0^t h_{i,j}(x) f_{\tau(j)}(t-x)\,dx + \delta(i-N+1) P_{i,N} h_{i,N}(t) \qquad (1)$$

where $\delta(n)$ is 1 when $n = 0$ and is zero otherwise.

Consider a path consisting of K trunk groups with occupancy $i_1, i_2, \cdots, i_K$. Let a new call be routed onto that path, then the time to blocking Γ on that path is simply:

$$\Gamma = \min\left[\tau_1(i_1+1), \tau_2(i_2+1), \cdots, \tau_K(i_K+1)\right]. \qquad (2)$$

For simplicity, we assume that the trunk groups are independent. The distribution function of $\Gamma, F_\Gamma(t)$, is then given by:

$$F_\Gamma(t) = 1 - \prod_{m=1}^{K} F_{\tau_m(i_m)}(t). \qquad (3)$$

[1] The trunk reservation parameter is determined by the direct and overflowed traffic rate [4]. For an aggregate state model of a link, another method can be used [16].

So, the expected value of Γ, denoted by $E[\Gamma]$, is then:

$$E[\Gamma] = \int_0^\infty 1 - F_\Gamma(t)\,dt = \int_0^\infty \prod_{m=1}^{K} F_{\tau_m(i_m)}(t)\,dt$$

and can be evaluated by numerical integration.

For a given node pair, there are many admissible alternate paths. To minimize future blocking, an intuitive way is to route the overflowed calls onto the path that gives the maximum *mean time to blocking* after the call is routed on it. Let $E[\Gamma_1], E[\Gamma_2], \cdots, E[\Gamma_J]$ be the expected *mean time to blocking* for the set of J admissible paths. The maximum *mean time to blocking* path j is given by:

$$\{j \mid \Gamma_j = \max[E[\Gamma_1], E[\Gamma_2], \cdots, E[\Gamma_J]]\}. \quad (4)$$

As the $E[\Gamma_i]$'s are real variables, in practice, j can be uniquely determined from (1)–(4) if the direct and alternate traffic rates on each link are known.

This method of finding the distribution functions of $\tau(i)$ and Γ, although exact, is very computationally demanding. We study, in the following, two simple approximations and compare their effectiveness in choosing the best alternate path based on the maximum *mean time to blocking* criterion.

1) $E[\Gamma]$ is approximated by:

$$E[\Gamma] = \min(E[\tau_1(i_1+1)], E[\tau_2(i_2+1)], \cdots, E[\tau_K(i_K+1)]) \quad (5)$$

where the $E[\tau(i)]$'s are derived in the Appendix. The rationale behind this scheme is that the *mean time to blocking* of the busiest link should dominant the *mean time to blocking* of the path. It is easy to prove that (5) is an upper bound of $E[\Gamma]$.

2) Consider two 2-hops alternate paths, A and B, having *mean time to blocking* (MTB) values of 1.55 and 2s on A and 1.50 and 3s on B. Since the *mean time to blocking* of the busiest links of the two paths are very close, we may want to take a look at the less congested links in order to determine which path is less congested. More formally, we define a threshold $\epsilon \in [0, 1]$ and define two links i and j to be in the same congestion level if:

$$\frac{|E[\tau_i] - E[\tau_j]|}{E[\tau_i]} < \epsilon. \quad (6)$$

Thus, if there is a path with its busiest link having a *mean time to blocking* that is significantly longer than the others by the criterion of (6), choose that path. Otherwise, the second busiest links are compared by the same criterion. This process goes until a candidate path is identified.

To illustrate the difference from the first MTB routing rule, suppose that the direct link of a node pair is blocked and two candidate paths (A and B) are available. The MTB values of the busiest links of both paths are first compared by using the criterion of (6) with ϵ = 0.05. Since criterion (6) are satisfied, they are considered to be in the same congestion level. Next, the MTB values of the second busiest links are compared by the same criterion. Since the MTB value of the less busy link of path A is significantly larger than that of path B, path B is selected. If the first MTB routing rule is used, the routing path is determined by comparing the MTB values of the busiest links *only* and path A will be chosen.

For convenience, the two variations of *MTB* routings are denoted as $MTB^{(1)}$ and $MTB^{(2)}$, respectively.

III. The Implementation of *MTB* Routing

In real-time operation, traffic rates vary from time to time. Routing should, therefore, be based on actual load measurements obtained during network operation. The operation of *MTB* routing requires the knowledge of the direct and alternate call arrival rates. In practice, the direct call arrival rate D can be measured.[2] The alternate call arrival rate A can be computed through an on-line measurement of trunk group occupancy as follows.

In Fig. 1, let P_i be the probability that the link occupancy is i. To simplify presentation, assume $\mu = 1$. By conservation of flow, we have:

$$\begin{array}{rcl} (D+A)P_0 & = & P_1 \\ (D+A)P_1 & = & 2P_2 \\ \vdots & & \vdots \\ (D+A)P_{N-r-1} & = & (N-r)P_{N-r} \\ DP_{N-r} & = & (N-r+1)P_{N-r+1} \\ \vdots & & \vdots \\ DP_{N-1} & = & NP_N \end{array} \quad (7)$$

Adding these N equations and simplifying, we have:

$$A\left(1 - \sum_{i=N-r}^{N} P_i\right) + D(1 - P_N) = \sum_{i=0}^{N} iP_i. \quad (8)$$

Solving for A, we have (9) and (10)- see the bottom of next page. These quantities can be measured by scanning the trunk status map of all trunk groups at regular intervals. By the assumption of Poisson arrivals, the direct call blocking probability is the fraction of time all trunks are fully occupied, the alternate call blocking probability is the fraction of time the trunk group occupancy is greater than $N - r - 1$, and the carried load is simply the average trunk occupancy measured.

We now summarize the *MTB* routing algorithm as follows:

1) Initially, assume there is no alternate traffic. Assume a convenient set oftrunk reservation parameter $\{r\}$ and assume an initial set of nominal direct traffic rate $\{D\}$. Implement *MTB* routing according to the rules given in Section II.

2) At regular intervals, collect measurements of direct load, carried load,direct and alternate call blocking probabilities and calculate D, A, the first moments of all $\tau(i)$'s and the trunk reservation parameter[3] r for each trunk group and store them in tables.

3) For each call attempt on a node pair, collect the occupancies of all trunk groups in the admissible paths and compute

[2] For small and variable traffic stream, the sliding window method described in [17] should be used for such measurements. In practice, the smaller the update interval, the faster the response to the traffic variation, but the blocking probabilities (especially direct blocking, which is much smaller than alternate blocking) becomes difficult to measure if the window size is too small.

[3] The trunk reservation parameter can be chosen to minimize the direct-route blocking probability [4], [16].

the *mean time to blocking* of each admissible path. The call is routed onto the path having the maximum *mean time to blocking*.

Next, we estimate the amount of processor load this algorithm incurred during each update of D, A, and $E[\tau(i)]$'s. As shown in the Appendix, the update of all $E[\tau(i)]$'s needs at most $5N - 3$ operations.

MTB routing can be implemented either in a centralized way or a decentralized way. In the centralized way, a network routing controller located at one of the switch sites is responsible for collecting link status and traffic rate measurements from local switches and for periodically computing the MTB values of each trunk group. When a connection request is made, a signal is sent from the originating switch to this controller. The controller determines the optimal routing path and sends it back to the originating switch via the signaling network.

In the decentralized way, all switching nodes are responsible for collecting link measurements and computing the MTB values of their respective outgoing links. When a connection request is made, the direct link is attempted first. If the direct path is blocked, the switching node looks up a set of alternate paths from its topology database and sends out requests to the involved switching nodes for the MTB values of all nonlocal links of the alternate paths. Upon receiving them, it computes the routing path and routes the call over it. Alternatively, all switching nodes can broadcast the MTB values of their outgoing links periodically (e.g., five seconds). During a call setup, the originating switch can use the latest MTB values received to determine the routing path. In this case, the call setup procedure is similar to AT&T's Real Time Network Routing (RTNR), which exchanges the aggregated trunk group occupancies information instead of the MTB values. The decentralized implementation of MTB routing is particularly suitable for large networks, as a centralized network routing controller could easily become a bottleneck in that case.

IV. Illustrative Examples

The maximum free circuit routing (M routing) routes overflowed calls to an alternate path having the maximum number of free circuits. The number of free circuits is defined as the minimum of the number of free trunks in each link constituting the path. Consider two alternate paths between nodes 1 and 4 in Fig. 2. When the number of free circuits on the two paths are the same, the M routing would treat both paths as equally good. However, if we plot the *mean time to blocking* as a function of the number of remaining trunks (Fig. 3), we see that path 124 consisting of links A and B is obviously the better one. As another example, suppose that links A–D are in states 36, 55, 74, and 93 and the number of free trunks on them are 4, 5, 6, and 7, respectively. M routing will select path 134, as the number of free circuits is 6, which is larger than 4–the number of free circuits on path 124. It is obvious that path 124 has a lower trunk utilization (λ/N) and is less likely to get congested in the near future even though, instantaneously, it has fewer free circuits. This shows that the *mean time to blocking* is a better measure than the number of free trunks in an asymmetric network as it takes into account the trunk group size and traffic rate in addition to the number of free trunks in determining the best alternate path.

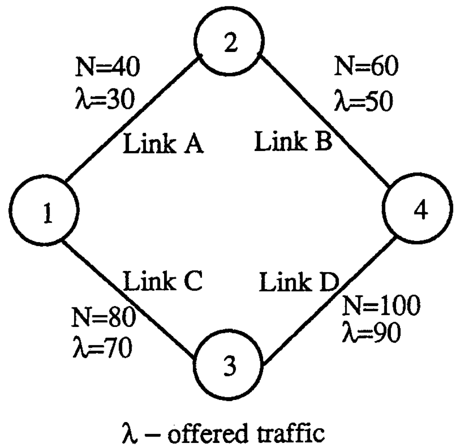

Fig. 2. A four-node network.

Considerb the two alternate paths a and b in Fig. 4. The probability density function of all $\tau(i)$'s is computed from (1) are shown in Fig. 5. By (3), the *mean time to blocking* under different occupancies for paths a and b can be computed. With that, the routing decisions under different occupancy combinations can be tabulated.

Table I shows part of the routing table obtained by *MTB* routing, $MTB^{(1)}$ and $MTB^{(2)}$. In the table, an "a" indicates the decision of routing a call to the upper path or through links A and B; and "b" indicates the lower path or through links C and D. The entry on row (5, 6) and column (2, 3) is "bba," meaning that when links C and D are in occupancies 5 and 6 and links A and B are in occupancies 2 and 3, the routing decisions for *MTB* routing, $MTB^{(1)}$ and $MTB^{(2)}$ are "b," "b," and "a," respectively. Simple counting shows that 92% of the $MTB^{(1)}$'s routing decisions are the same as *MTB* routing, while for $MTB^{(2)}$ 94% of the decisions are the same with ϵ being set at 0.08.

V. Simulation Results and Discussions

To evaluate the performance of the *MTB* routing rule, we simulate it on an eight-node fully connected network. The

$$A = \frac{\sum_{i=0}^{N} iP_i - D(1 - P_N)}{1 - \sum_{i=N-r}^{N} P_i} \tag{9}$$

$$= \frac{[\text{carried load}] - [\text{direct load}]\,(1 - [\text{direct call blocking prob.}])}{1 - [\text{alternate call blocking prob.}]} \tag{10}$$

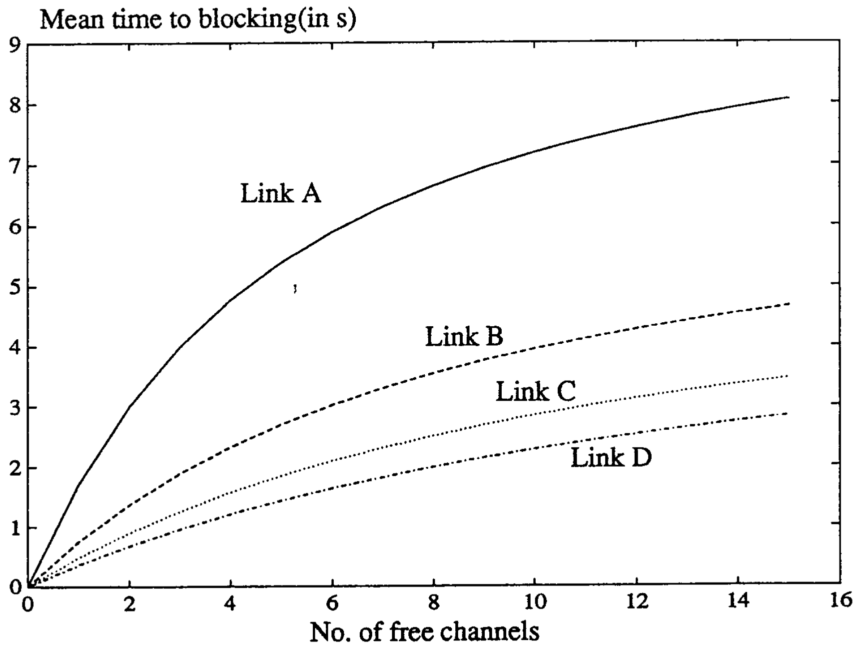

Fig. 3. Mean time to blocking of links versus the number of free channels.

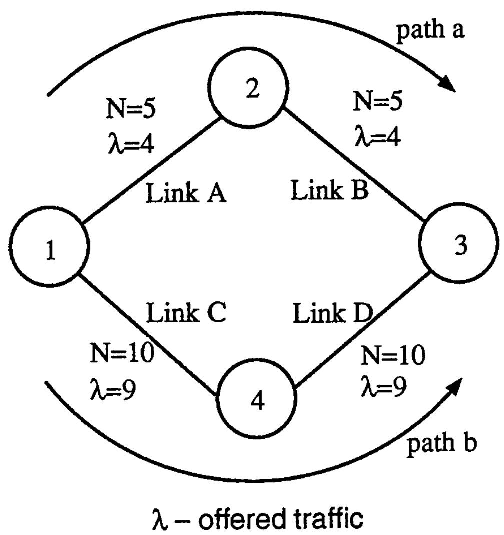

Fig. 4. A different four-node network.

trunk group size and the direct route offered load are shown in Table II. The following conditions are assumed.

- Calls generated in each origin destination node pair follow a Poisson process
- Call holding time obeys an exponential distribution with a mean of 180 seconds.
- Call setup and release time is negligible.
- Blocked calls are immediately cleared and not repeated.

All results are obtained by averaging 120 hours of simulation. We use a five-hour update interval. While the statistics can be more accurate when using a longer update period, there is also the shortcoming that the response to traffic pattern change is slower. One way to combat this shortcoming under varying traffic conditions is to use a sliding window mechanism with an appropriate weighting function that puts more emphasis on recent statistics.

Fig. 6 shows the trace of the end-to-end call blocking probability for $MTB^{(1)}$ as a function of time under different overload conditions. It shows that without *a priori* knowledge of the traffic rates, *MTB* routing can learn how to adjust its parameters through link measurements. It also shows that a simulation interval of 120 hours is very sufficient for an accurate estimation of blocking probabilities.

Fig. 7 shows the end-to-end call blocking probability for M, SDR,and *MTB* routings under the same range of overload conditions. For 0% to 7% overload, the trunk reservation parameter is 0.05 N while for an 8% to 20% overload, it is $0.1N$. Although SDR gives a better performance than M routing, it is not as good as that of the MTB routings.[4] It is found that the $MTB^{(2)}$ always gives the lowest blocking probability, provided a suitable ϵ value is used. In our simulation model, ϵ is 0.1. How ϵ depends on the other network parameters remains to be investigated.

Fig. 8 shows the relative blocking improvements of the SDR and the MTB routings over that of the M routing. As expected, the relative improvements decrease with increased overload.

Next, we compare the routing rules in a 15-node mesh network with 95 links. In this network, a total of 105 node pairs[5] are divided into two types, with type A node pairs having a direct link and type B node pairs do not. All links

[4] The performance of SDR depends on the network parameters as reported in [10].

[5] For a n node connected network, the total number of node pairs is $n(n-1)/2$.

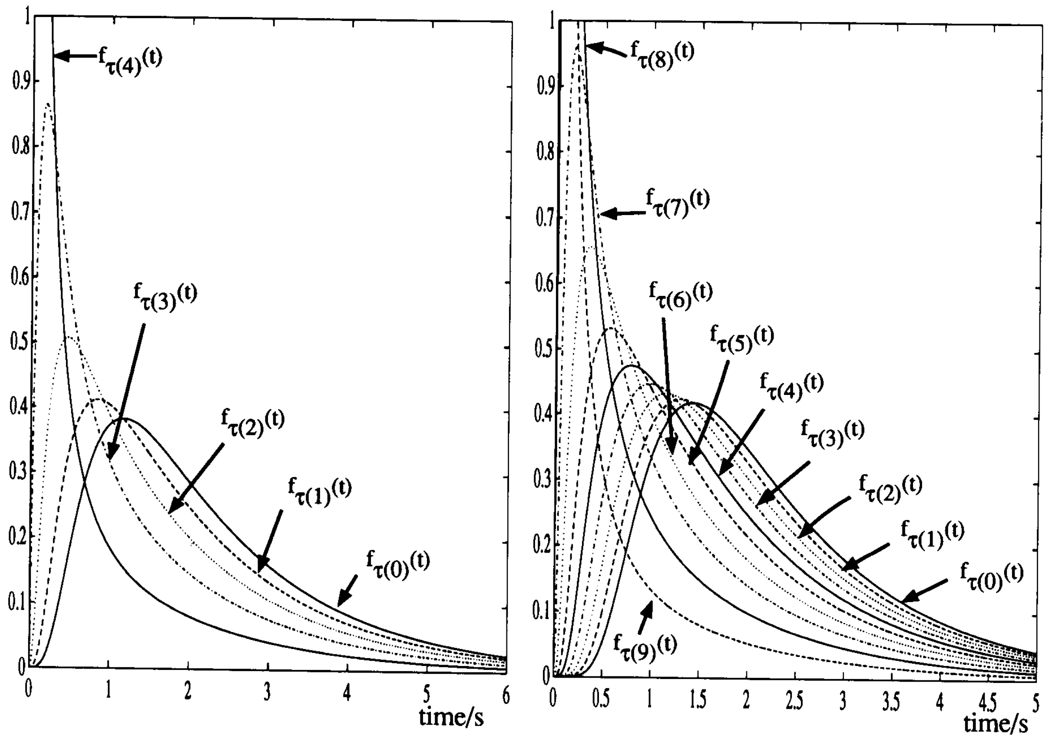

Fig. 5. Probability density functions of $\tau(i)$'s.

have 48 trunks. We assume a base traffic load of 40 Erlangs for all type A node pairs and 12 Erlangs for all type B node pairs. Three trunks are reserved on all links for direct traffic. The network is simulated for 48 hours[6] The results of the first six hours are discarded, and the remaining are divided into seven batches. A sliding window with a three-hour window size and a 30-minute update interval is employed for link statistics collection.

To study the robustness of the routing rules under increasingly asymmetric traffic conditions, we alter the network traffic load such that the traffic rate of 30 specific type A node pairs are increased by a factor of x while that of 20 other type A node pairs are reduced by the same factor. Fig. 9 shows the average call blocking probability as a function of x varies from 0 to 0.25 in steps of 0.05. The 95% confidence interval of all results are within 0.01%. It is found that *MTB* $^{(2)}$ routing outperforms the other routing schemes.

Fig. 8 shows the relative blocking improvements of the SDR and the MTB routings over that of the M routing. As expected, the relative improvements decrease with increased overload.

VI. Conclusions

The *mean time to blocking* is a robust measure of congestion on a path as it takes into consideration the offered traffic rate, the trunk group size, and the instantaneous trunk group occupancy. This measure is good for paths consisting of any number of links. Although the calculation of *mean time to blocking* on a path is very complicated, two very simple approximations with complexity O(N) (N is the trunk group size) are shown to perform very well. Extensive simulation shows that the *MTB* routing gives a better blocking performance than the *SDR* and M routings.

[6]The computer run time of SUN SPARC 10 is about 8 hours per data point.

TABLE I
ROUTING TABLES FOR MTB, $MTB^{(1)}$, AND $MTB^{(2)}$ ROUTINGS

states of links C and D	states of links A and B (0,3)	(0,4)	(1,1)	(1,2)	(1,3)	(1,4)	(2,2)	(2,3)	(2,4)	(3,3)	(3,4)	(4,4)
(5,0)	bbb	bbb	aaa	aaa	bbb	bbb	bab	bbb	bbb	bbb	bbb	bbb
(5,1)	bba	bbb	aaa	aaa	bba	bbb	baa	bbb	bbb	bbb	bbb	bbb
(5,2)	aaa	bbb	aaa	aaa	aaa	bbb	aaa	baa	bbb	baa	bbb	bbb
(5,3)	aaa	bbb	aaa	aaa	aaa	bbb	aaa	aaa	bbb	aaa	bbb	bbb
(5,4)	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	baa
(5,5)	bbb	bbb	aaa	aaa	bbb	bbb	baa	bbb	bbb	bbb	bbb	bbb
(5,6)	aba	bbb	aaa	aaa	bba	bbb	aaa	bba	bbb	bbb	bbb	bbb
(5,7)	aaa	bbb	aaa	aaa	aaa	bbb	aaa	aaa	bbb	baa	bbb	bbb
(5,8)	aaa	bbb	aaa	aaa	aaa	bbb	aaa	aaa	bbb	aaa	bbb	bbb
(5,9)	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	baa
(6,6)	aba	bbb	aaa	aaa	aba	bbb	aaa	bba	bbb	bbb	bbb	bbb
(6,7)	aaa	bbb	aaa	aaa	aaa	bbb	aaa	aaa	bbb	baa	bbb	bbb
(6,8)	aaa	bbb	aaa	aaa	aaa	bbb	aaa	aaa	bbb	aaa	bbb	bbb
(6,9)	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	baa
(7,7)	aaa	bbb	aaa	aaa	aaa	bbb	aaa	aaa	bbb	aaa	bbb	bbb
(7,8)	aaa	abb	aaa	aaa	aaa	bbb	aaa	aaa	bbb	aaa	bbb	bbb
(7,9)	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	baa
(8,8)	aaa	abb	aaa	aaa	aaa	abb	aaa	aaa	abb	aaa	abb	bbb
(8,9)	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa
(9,9)	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa	aaa

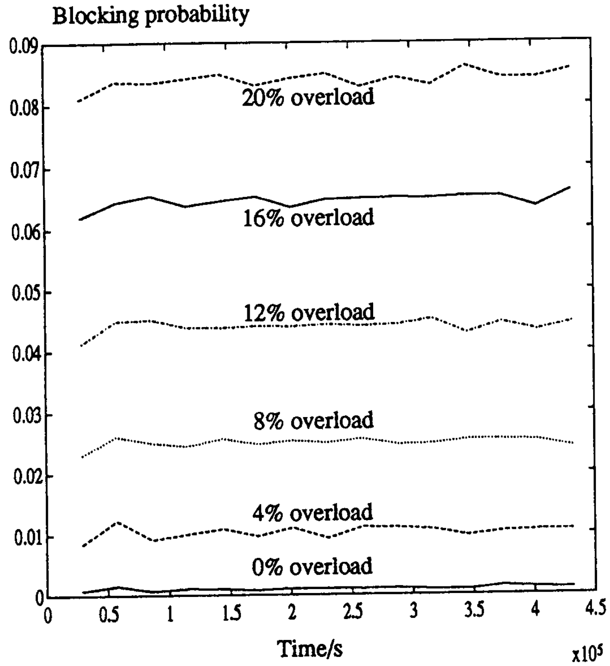

Fig. 6. Blocking probability versus time with $MTB(1)$.

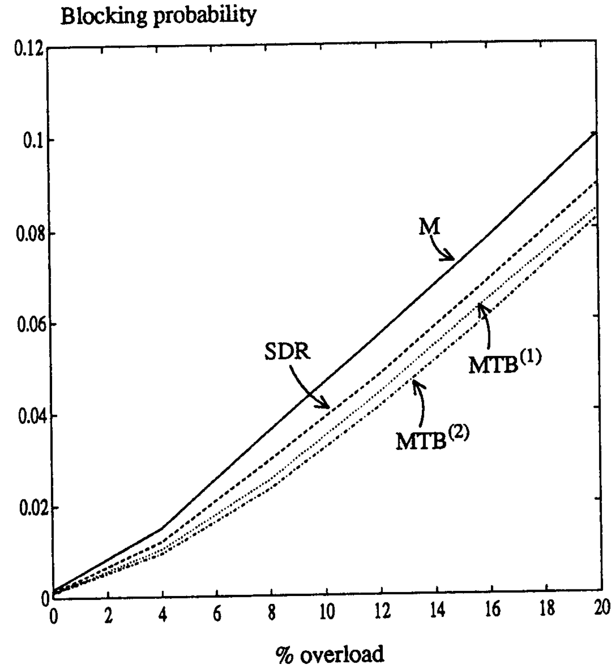

Fig. 7. Blocking probability of different routing schemes.

APPENDIX
COMPUTING THE MEAN TIME TO BLOCKING ON A LINK

Consider a particular trunk group consists of N trunks, letting the direct and alternate arrival rates of this trunk group be D and A. The trunk reservation parameter is r. For convenience, let $C = A + D$ and denote the first passage time from state i to state N be t_i. From [18], the first moment α_i of t_i is given by the following recursive equations:

$$\alpha_i = \begin{cases} \overline{\tau}_i + \sum_{j=0}^{N} P_{i,j}\alpha_j & 0 \le i \le N-1 \\ 0 & i = N \end{cases} \tag{A.1}$$

$$\Phi = \begin{bmatrix} C & -C & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ -\mu & C+\mu & -C & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & -2\mu & C+2\mu & -C & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & -(N-r-1)\mu & C+(N-r-1)\mu & -C & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -(N-r)\mu & D+(N-r)\mu & -D & & 0 \\ \vdots & & \vdots & & \vdots & & \vdots & \\ 0 & & & & & -(N-2)\mu & D+(N-2)\mu & -D \\ 0 & & & & & 0 & -(N-1)\mu & D+(N-1)\mu \end{bmatrix}.$$

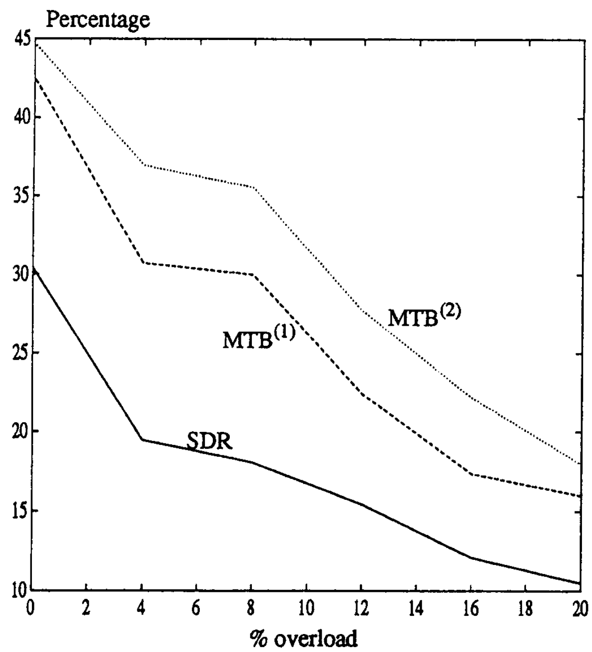

Fig. 8. Percentage of blocking improvement relative to M routing.

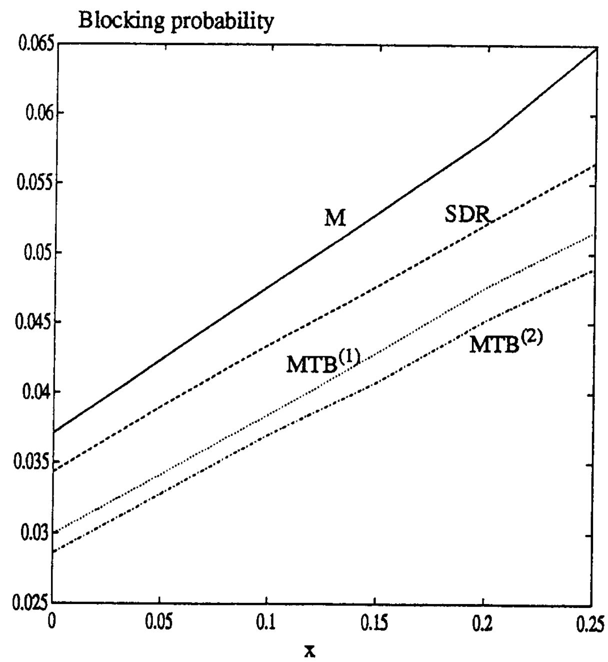

Fig. 9. Blocking probability of different schemes versus x.

where $P_{i,j}$ is the transition probability, and $\overline{\tau}_i$ is the mean sojourn time in state i.

It is easy to observe that τ_i is an exponentially distributed random variable with the first moment given by:

$$\overline{\tau}_i = \begin{cases} \dfrac{1}{C+i\mu} & 0 \le i \le N-r-1 \\ \dfrac{1}{D+i\mu} & N-r \le i \le N-1 \\ \dfrac{1}{N\mu} & i = N. \end{cases}$$

Substituting into (A.1), we have the following difference equation:

$$\begin{cases} C\alpha_i - C\alpha_{i+1} = 1 & i=0 \\ -i\mu\alpha_{i-1} + (C+i\mu)\alpha_i - C\alpha_{i+1} = 1 & 1 \le i \le N-r-1 \\ -i\mu\alpha_{i-1} + (D+i\mu)\alpha_i - D\alpha_{i+1} = 1 & N-r \le i \le N-1 \end{cases}$$

which, in matrix form, is:

$$\Phi \cdot \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{N-r-1} \\ \alpha_{N-r} \\ \vdots \\ \alpha_{N-2} \\ \alpha_{N-1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \tag{A.2}$$

where Φ is the transition rate matrix and is given by the unnumbered equation at the top of the page.

The α_i's can be calculated from A and D by the forward

TABLE II
NETWORK PARAMETERS FOR THE EIGHT-NODE NETWORK

Trunk group	no. of trunks	loading	Trunk group	no. of trunks	loading	Trunk group	no. of trunks	loading
1	100	89	11	100	89	21	30	19
2	120	109	12	40	29	22	80	69
3	30	19	13	100	89	23	90	79
4	50	39	14	80	69	24	100	89
5	80	69	15	100	89	25	60	49
6	100	89	16	70	59	26	120	109
7	40	29	17	40	29	27	40	29
8	60	49	18	90	79	28	110	99
9	50	39	19	60	49			
10	30	19	20	100	89			

and backward substitution [19] using (A.2) as follows:

$$
\begin{aligned}
Step\,1:\ C &= A + D \\
B_0 &= \frac{1}{C} \\
B_i &= \begin{cases} \dfrac{1 + i\mu B_{i-1}}{C} & 1 \le i \le N - r - 1 \\ \dfrac{1 + i\mu B_{i-1}}{D} & N - r \le i \le N - 1 \end{cases} \\
Step\,2:\ \alpha_{N-1} &= B_{N-1} \qquad\qquad i = N - 1 \\
\alpha_i &= B_i + \alpha_{i+1} \qquad\quad 0 \le i \le N - 2.
\end{aligned}
\tag{A.3}
$$

By simple counting, the number of basic operations required to update the mean first passage time by (A.3) is $5N - 3$.

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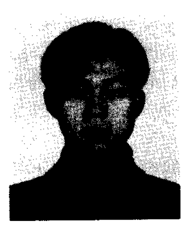

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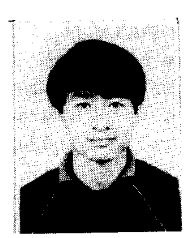

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