The Join-Biased-Queue Rule and Its Application to Routing in Computer Communication Networks

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(Invited Paper)

Abstract—**A routing rule similar in nature to delta-routing [8] is studied in this paper. The approach is to superimpose local adaptivity on top of a fixed traffic flow distribution. The fixed flow distribution we choose is obtained from the best stochastic (BS) rule [3]. The adaptive part is called the join-biased-queue (JBQ) rule. The resultant JBQ-BS rule is analyzed on small networks and is shown to provide 10–27 percent delay improvement over the BS rule.**

I. INTRODUCTION

ROUTING in computer communication networks involves sending each incoming message to its destination intelligently via a set of paths that may be available to each node [10]. Consider a computer communication network with all its L links and N nodes always reliable and with fixed input trafic rates. We define the state of the network as the L-tuple $(q_1, q_2, \cdots, q_L)$ with q_i being the queue size of the output buffer of link i. A routing rule used in such a network is described as either fixed or adaptive depending on whether its routing decisions are independent of or dependent on the network state $(q_1, q_2, \cdots, q_L)$.

The simple shortest path rule [1] is classified as fixed. A more sophisticated one is the best stochastic (BS) rule which allocates traffic flows *stochastically* (i.e., by fixed probability assignment) throughout the network so as to minimize the overall average time delay, subject to constraints due to flow consevation. Poisson arrivals, exponential message lengths, and message independence assumptions are made in the analysis so as to force the queueing model to be the $M/M/1$ type. This is referred to as the optimum routing rule in [2]. Numerical techniques such as the flow deviation method [3], gradient projection method [4], and others [5], [6] have been used to solve for the optimum flow distribution. Better overall delay performance, however, can be obtained by bifurcating the flow *deterministically* (i.e., according to a predetermined routing sequence). This is called the best deterministic (BD) rule in [7] to contrast with the stochastic nature of the BS rule.

In this paper, we investigate an adaptive way of bifurcating the traffic flow. The approach is similar to the join-the-shortest-queue (JSQ) rule discussed in the literature [8], [12], [16], [17], [18], except that a biased term is introduced in comparing the queue lengths. We will show later that by adjusting the biased term in the so-called join-biased-queue (JBQ) rule, the proportions of traffic bifurcation can be regulated at will. Here in particular, we want to regulate the traffic flows in the network so that they are the same as the fixed BS rule introduced earlier. In other words, we want the same link utilization for these two rules. Now, what is different between the two rules is their message arrival processes. For the fixed BS rule, the arrivals remain Poisson distributed because random bifurcation of Poisson processes remains Poisson; and $M/M/1$ results can be used for queue length distribution. For the JBQ adaptive rule on the other hand, the message arrivals are state dependent because traffic bifurcation is based on the instantaneous queue lengths in the same network node; and the queue length distribution is not known analytically.

To study the JBQ adaptive rule and to compare its performance with the BS rule, we calculate the queue length distribution by solving a two-dimensional Markov chain numerically. This will be shown in the next section. Now, the queue length distributions we obtain from the examples in this paper, as well as those in [11], show that the average queue length for the queue with bifurcated arrivals is always smaller than that given by the $M/M/1$ queue with the same utilization. Moreover, for the special case where the biased term in the JBQ rule is zero [the familiar join-the-shortest-queue rule (JSQ)] and when there is no other traffic except the adaptive stream to be bifurcated, Foschini and Salz [16] have shown, by diffusion approximation, that the average queue length can be reduced to $1/k$ for k parallel queues, Flatto and McKean [17] obtained similar results with an exact formula for two parallel queues. Thus, comparing the queueing behavior for these two routing rules on a queue-by-queue basis, there is reason to believe that the use of the JBQ adaptive rule will result in average queue size no larger than the $M/M/1$ queue given by the BS rule. Since this argument can be used on all the queues in the network, Little's result applied to a network [10] would indicate that the overall average delay of the adaptive rule will be no worse than that given by the BS rule. The equality of the two delays holds only when there is no traffic bifurcation since, in that case, the JBQ adaptive rule degenerates to the BS rule.

We verify the above argument by several three and four node network examples in Section III. A rigorous proof of the above argument, however, requires the determination of the departure process from each queue and the adaptively bifurcated arrival process to each queue. The determination of both of these processes appears to lead to difficult queueing problems (except for the average rates) and we have not gone this far in our work.

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The concept of relegating portions of the routing decision to local nodes first appeared in Rudin's paper on delta-routing [8]. There, he showed by extensive simulation on small and medium size networks, that a routing rule generally performs better if 1) both local and global information are used in making routing decisions, 2) routing decisions can be made both centrally and in a distributed fashion, and 3) the strategy used can be partly fixed and partly adaptive. The parameter δ (hence delta-routing) determines 1) the amount of adaptivity in the strategy used, 2) the amount of decision power the Network Routing Center (NRC) relegates to the local nodes, and 3) the amount of local information to be used in routing decisions. In this perspective, we will classify the JBQ-BS rule (the JBQ rule with the BS rule flow pattern) as a version of delta-routing. The centralized portion of the rule is fixed; it collects the traffic rate information about the entire network (global information) and determines the amount of flow on each link. The local (or distributed) portion of the rule is adaptive, making use of the locally available queue length information and determines the instantaneous flow of messages in their local environment. The delay results we obtained from the analysis of three and four node networks in this paper agree with the simulation results given in [8].

Adaptive rules used in a real network are often complicated by the flow control schemes and other "special features" [13], [14] which also affect the delay performance. Hence, they can seldom agree exactly with the models of rules that can be described and analyzed mathematically. In order to fit the routing rules into models that can be analyzed, a number of assumptions, and a great deal of simplification, have to be made. This usually includes singling out the routing aspect of the network operation apart from the other interacting network operation functions such as flow control, error control, nodal buffer allocation, and scheduling (ways of allocating network delays among the various user classes [9]) strategies. Rudin and Mueller in their thought-provoking paper [9] cautioned that it is dangerous to draw conclusions from the study of flow control, routing, or scheduling as isolated mechanisms and infer that they are also valid when other network operation functions are incorporated. We would like to add to this that the isolated theoretical study can be of value (and thus necessary) provided the results obtained are interpreted with care. Thus, in general, we may interpret the results in the following ways.

1) If any of the network functions is shown not to perform well when studied in an isolated fashion, then it is not likely that it will work well with the incorporation of other network functions. In other words, its contribution to the overall network performance is likely to be minimum.

2) If any of these network functions is shown to work well in an isolated manner, then it is potentially well performing. The next step is to examine its "joint" performance with other potentially well performing network functions. The models and analyses of individual network functions in isolation would certainly be of value in providing insight in formulating the more complicated "joint" models.

In the above perspective we present the isolated, theoretical analysis of the JBQ rule in the following two sections.

II. THE JBQ RULE CONCEPT AND THE ANALYSIS

In this section, we study the JBQ rule in an isolated network node. Its extension to a network with the incorporation of the BS rule will be studied in the next section.

Consider the isolated node and its queueing model in Fig. 1. Let γ_1, γ_2 represent Poisson arrival rates of messages that are constrained to be sent out via link 1 and link 2, respectively. They are henceforth called fixed arrival rates. These message arrivals represent the demands on the network by many independent users and are quite accurately modeled as Poisson processes. The rates in practice would be obtained from real-time measurements. Let q_1, q_2 be the lengths of queue 1 (Q_1) and queue 2 (Q_2); and let λ_1, λ_2 be the actual input rates to Q_1 and Q_2. Let all queues be formed in finite buffers of size M and let γ represent the adaptvie Poisson arrival rates of messages that can be switched to either Q_1 or Q_2. The following is the JBQ rule of routing the γ messages:

$$1)\ \left.\begin{aligned}\lambda_1 &= \gamma_1 + \gamma\\ \lambda_2 &= \gamma_2\end{aligned}\right\}\quad \text{when } q_1 < q_2 + \Delta$$

$$2)\ \left.\begin{aligned}\lambda_1 &= \gamma_1\\ \lambda_2 &= \gamma_2 + \gamma\end{aligned}\right\}\quad \text{when } q_1 > q_2 + \Delta$$

$$3)\ \left.\begin{aligned}\lambda_1 &= \gamma_1 + \beta\gamma\\ \lambda_2 &= \gamma_2 + (1-\beta)\gamma\end{aligned}\right\}\quad \text{when } q_1 = q_2 + \Delta.$$

Here, $\Delta\epsilon\{0, \pm 1, \pm 2, \cdots\}$ is an integer representing the bias level and $\beta\epsilon[0, 1)$ is the *a priori* probability for routing the γ messages to Q_1 when $q_1 = q_2 + \Delta$. The parameter β can therefore be considered as a "fine tuning" of Δ. Thus, in words, the JBQ rule says: route a message in the adaptive category to Q_1 if the length of Q_1 is less than the length of Q_2 plus Δ; route it to Q_2 if greater. If the length of Q_1 equals the length of Q_2 plus Δ, then route to Q_1 and Q_2 with probability β and $(1-\beta)$, respectively. The special case $(\Delta, \beta) = (0, 0.5)$, $\gamma_1 = \gamma_2 = 0$ is merely the extensively studied JSQ rule [15]-[17]. The presence of fixed-path messages (i.e., $\gamma_1, \gamma_2 \neq 0$) in our way of modeling makes the JBQ rule particularly suitable for extension to the network case.

Let $P_{i,j} = \text{Prob}\ [q_1 = i, q_2 = j]$. Then the above routing rule can be embedded in a two-dimensional Markov chain with transition rates γ_1 and γ_2 taking on different values as listed above depending on the state (q_1, q_2). This is possible because Poisson arrival rates and exponential service times are assumed and the transition rates at each state are uniquely defined. The complete set of states for $\Delta = 2$ is represented by the two-dimensional sketch in Fig. 2. There are 19 regions in the sketch representing 19 groups of states. States in the same group have the same inward-outward transition rates. Therefore, the state equations associated with these states differ from each other by the indices i and j only. A typical

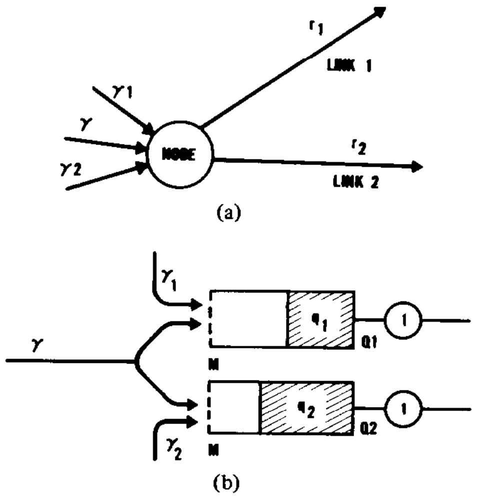


Fig. 1. An isolated network node and its queueing model. (a) An isolated network node. (b) Queueing model.

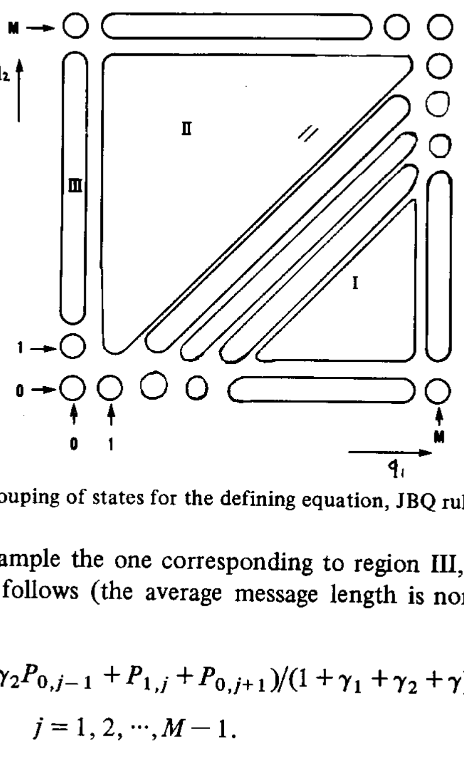


Fig. 2. Grouping of states for the defining equation, JBQ rule, $\Delta = 2$.

set, for example the one corresponding to region III, may be written as follows (the average message length is normalized to unity):

$$P_{0,j} = (\gamma_2 P_{0,j-1} + P_{1,j} + P_{0,j+1})/(1 + \gamma_1 + \gamma_2 + \gamma)$$
$$j = 1, 2, \cdots, M-1. \tag{1}$$

Nineteen such sets plus the normalizing equation

$$\sum_{j=0}^{M} \sum_{i=0}^{M} P_{i,j} = 1 \tag{2}$$

provide a total of $(M + 1)^2$ independent equations for the $(M + 1)^2$ unknowns $P_{i,j}$. A Fortran program was written for the solution of the equations with the set of input parameters $\{\gamma_1, \gamma_2, \gamma, \Delta, \beta, M\}$ using the Gauss-Seidel iteration method.

We now investigate some properties of the JBQ rule, found from analysis, using the state probabilities calculated as indicated above. In Fig. 3, we show the average delay of the

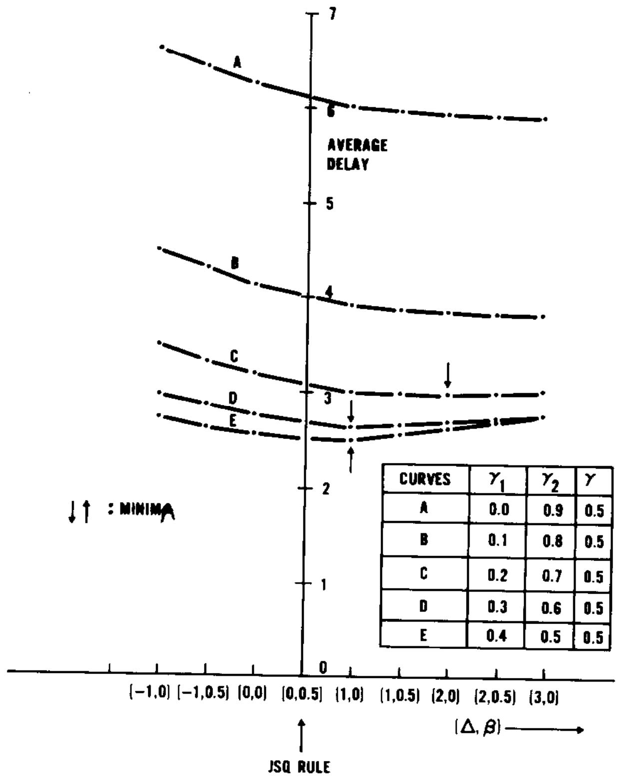


Fig. 3. Delay versus (Δ, β) parameter, JBQ rule.

JBQ rule as a function of (Δ, β) for the queueing system of Fig. 1. Five sets of arrival rates are assumed. In all cases, the total normalized nodal arrival rate is always 1.4 and the adaptive component γ is kept fixed at 0.5. The fixed components γ_1 and γ_2 vary over a selected range. The buffer size M has been taken to be 25 in all cases, so that the resultant average blocking probabilities P_B are all less than 10^{-5}. We then assume that this amount of blocking does not affect the average delay appreciably. Using Little's formula, the average delay T is calculated to be

$$T = \frac{E(q_1) + E(q_2)}{(\gamma_1 + \gamma_2 + \gamma)(1 - P_B)}$$
$$\doteq \frac{\sum_{i=1}^{M} i \sum_{j=0}^{M} P_{ij} + \sum_{j=1}^{M} j \sum_{i=0}^{M} P_{ij}}{\gamma_1 + \gamma_2 + \gamma}. \tag{3}$$

Now, it is clear that changing the (Δ, β) parameters will change the routing decisions defined in (1); and in turn changes also the queue length distribution and the average delay T. We denote the (Δ, β) that gives the minimum delay as the optimum (Δ, β) for that set of arrival rates. For curves C, D, and E in Fig. 3, the optimum (Δ, β) are located at (2, 0), (1, 0), and (1, 0), respectively. The optimum (Δ, β) for curves A and B are greater than (3, 0) and therefore are not shown. These five curves show that the delay is not sensitive to (Δ, β) over quite a large range. As we shall see later, this characteristic is important since (Δ, β) can be used to vary the rate of the output processes without affecting the delay appreciably. Note also that the delay curves agree with intuition: in all cases when $\gamma_2 > \gamma_1$, one would want to route more adaptive traffic to Q_1; hence $\Delta > 0$. As the disparity between γ_1 and γ_2 increases, the bias Δ increases accordingly.

Fig. 4 shows the fraction of adaptive traffic that joins Q_1 as a function of (Δ, β). We see that by an appropriate choice of (Δ, β) we can route (adaptively) a specific portion of the adaptive traffic γ to Q_1 and the remainder to Q_2. This is the essence of adaptive bifurcation we mentioned earlier; and curves like those shown in Fig. 4 will be used later for the design of the JBQ-BS rule.

Some other characteristics of the JBQ rule, such as the statistics of the JBQ rule departure process, the relation between the Δ and β parameters, the optimum (Δ, β) and average delay as functions of the fraction of adaptive traffic, etc., can be found in [11].

III. THE JBQ-BS ROUTING RULE AND ITS ANALYSIS IN A NETWORK

The JBQ-BS rule is essentially a rule where local JBQ adaptivity is superimposed on the fixed BS rule base. The BS rule specifies the flow distribution in the network and the JBQ rule, with its inherent bifurcation ability, regulates the traffic so as to achieve that flow distribution. In this section, we shall first work out an example in detail to illustrate the JBQ-BS rule concept and its analysis. We then quote results of three other examples to give an estimate of the actual delay improvement over the BS rule for small networks. Lastly, we discuss the reason for choosing the BS rule flow pattern. Consider the four node network with unidirectional flow in Fig. 5(a). The external traffic rates assumed are indicated in Fig. 5(b). The overall network utilization is calculated to be 0.7. We first calculate the BS flow rates and delay. Let x be the rate of the adaptive traffic that joins Q_1. We can achieve that rate by routing the adaptive messages randomly to Q_1 with probability $x/0.7$ and to Q_2 with the remaining probability. By using Little's formula and the $M/M/1$ results, we can write T, the total average time delay, as

$$T = \frac{1}{2.1}\left[\frac{0.4+x}{0.6-x} + \frac{1-x}{x} + \frac{0.2+x}{0.8-x} + \frac{1.2-x}{x-0.2}\right].$$

The value of x that minimizes T is 0.4. For that value, $T = 5.24$. The utilization factors of the four links and their average queue lengths are shown in the first row of Table I.

We now use the JBQ rule to split the adaptive traffic such that the utilization of each queue is the same as that given by the BS rule. The (Δ, β) needed is obtained by first quantizing β to ten values. Ten is chosen for convenience. Other levels of quantization can also be used. Then the Markov chain of the JBQ rule is solved repeatedly with different (Δ, β). Each set of solutions gives a set of link utilizations (ρ_1, ρ_2). The needed

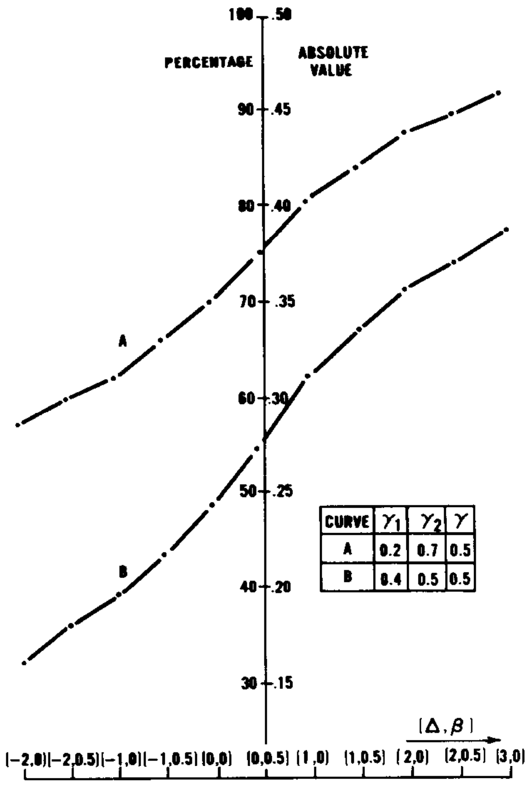


Fig. 4. Fraction of adaptive traffic that joins Q_1 as a function of (Δ, β).

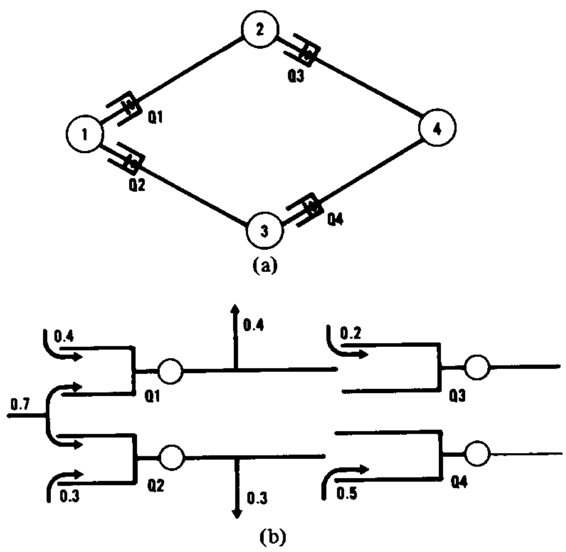


Fig. 5. Network and queueing model of example 1. (a) Four-node network with unidirectional flow, example 1. (b) Queueing model with input rates [network of Fig. 5(a)].

TABLE I
LINK UTILIZATION FACTORS AND AVERAGE QUEUE SIZES

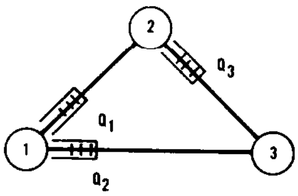
RULES	ρ_1	ρ_2	ρ_3	ρ_4	$E(q_1)$	$E(q_2)$	$E(q_3)$	$E(q_4)$
BS	.8	.7	.6	.8	4.00	2.33	1.50	4.00
JBQ-BS	.799	.701	.599	.801	2.22	1.30	1.49	4.03

set of (Δ, β) is that which gives (ρ_1, ρ_2) closest to $(0.8, 0.7)$. For the above example, the needed (Δ, β) is found to be (1, 0, 6). This gives the link utilization factors and average time delays as shown in the second row of Table I. Note first, the four link utilization factors agree closely with the desired BS rule flow pattern; and second, queues 1 and 2 are, on the average, considerably less occupied than by use of the BS rule alone. Using Little's result we obtain $T = 4.30$. Note that $E(q_3)$ and $E(q_4)$ are calculated as they were $M/M/1$ queues. This is based on the Poisson departure assumption which says that for local routing rules (i.e., nonfeedback rules) with exponentially distributed messages, the departure processes can be assumed as memoryless, or Poisson. This assumption has been used in [11], [15], [18] and possibly many other similar works in the analysis of queueing networks. It allows us to decouple queues at different nodes and analyze them separately. A discussion of this and its verification can be found in [11]. To verify the above delay result, 12 simulation runs of 10 000 messages each gave $T = 4.32 \pm 0.17$ with 95 percent confidence. Compared to the BS rule, this represents an 18 percent reduction of delay.

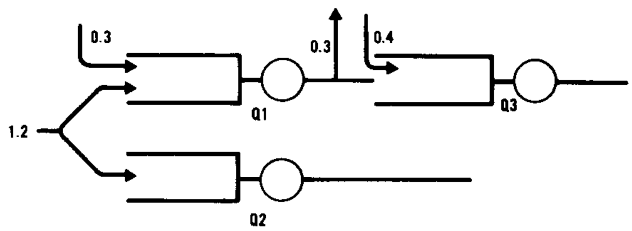
The networks, queueing models, and input traffic matrices for the second, third, and fourth examples are shown in Figs. 6 and 7. The analyses of the JBQ-BS rule on these networks are the same as that given by the first example. The delay results and the comparisons with the BS rule are shown in Table II. They show 10-27 percent reduction of delay.

In the above example, we have considered the case where each node has only two outgoing links and at most one adaptive traffic stream. Therefore, in regulating the amount of adaptive traffic that should join the two outgoing links, only one set of (Δ, β) per node needs to be set. In the network environment, however, there may generally be two or more adaptive traffic streams joining as many as three or more outgoing links at each node.

First consider the generalization to three or more outgoing links. Fig. 8(a) shows a three-queue system with the fixed messages with rates $\gamma_1 \gamma_2 \gamma_3$ joining their respective queues while the messages with rate γ are to be split into three streams of rates a_1, a_2, a_3. The problem is how to set the biased level on each queue so as to achieve this specification bifurcation. We can generalize the approach in the last section by solving instead, a three-dimensional Markov chain for the three-queue system and partitioning the states into three regions for the three routing decisions: the incoming message should join Q_1, Q_2, or Q_3. No simple way of partitioning the decision regions so as to achieve the desired bifurcation is known. For the two-queue case in the examples, we just search for the desired partitioning. For three or more queues, the search procedure is prohibitively complex and time consuming and we have not continued our investigation. We suspect that simple approximate techniques exist, and regard this as a subject for future work. We would also like to point out the similarities between this decision space partitioning problem and the multihypothesis testing problem in mathematical statistics.



(a)

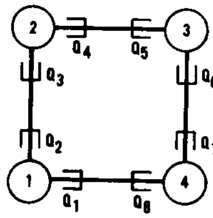


(b)

Δ	-1	-1	-1	-2	-2	-2	-2
β	0.2	0.1	0.0	0.9	0.8	0.7	0.6
T	3.95	3.90	3.84	3.82	3.80	3.79	3.82

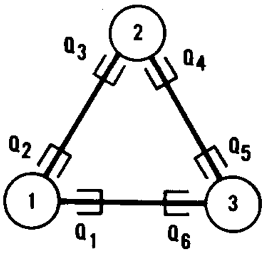
(c)

Fig. 6. Network and frequency model and delay results of example 2. (a) Three-node network with unidirectional flow, example 2. (b) Queueing model with input rates [network of Fig. 6(a)]. (c) Delay as a function of (Δ, β) parameter [queueing model of Fig. 6(b)].



$$\Gamma = [\gamma_{ij}] = \begin{bmatrix} - & 0.2 & 0.4 & 0.5 \\ 0.5 & - & 0.4 & 0.2 \\ 0.5 & 0.1 & - & 0.1 \\ 0.1 & 0.2 & 0.4 & - \end{bmatrix}$$

(a)



$$\Gamma = [\gamma_{ij}] = \begin{bmatrix} - & 0.6 & 0.3 \\ 0.4 & - & 0.7 \\ 0.8 & 0.2 & - \end{bmatrix}$$

(b)

Fig. 7. Networks and traffic matrices for examples 3 and 4. (a) Network of examples 3 and 4. (b) Traffic matrices, $\Gamma = [\gamma_{ij}]$, examples 3 and 4.

TABLE II
DELAY RESULTS AND COMPARISON

EXAMPLE	BS RULE	JBQ-BS RULE	% IMPROVEMENT
1	5.24	4.30	18%
2	5.38	3.95	27%
3	4.12	3.69	10%
4	2.60	2.1	17%

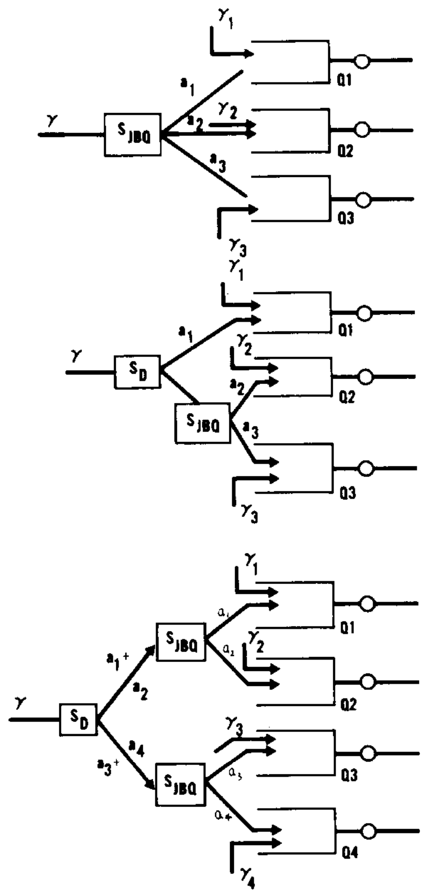


Fig. 8. The JBQ rule in a three or more outgoing link node. (a) A three-queue system with adaptive traffic, JBQ rule. (b) Traffic bifurcation using a deterministic and a JBQ switch. (c) Traffic bifurcation in a four-queue system.

One approximate technique that might be considered is the use of a deterministic switch[1] S_D [7]. In Fig. 8(b), S_D is used before the "JBQ" switch S_{JBQ}. S_D routes a_1 to Q_1 and $a_2 + a_3$ to S_{JBQ}. S_{JBQ} then sets the (Δ, β) parameter for Q_2 and Q_3 so that a rate of a_2 goes to Q_2 and a_3 goes to Q_3.[2] In the case of a four-queue system, we can use two "JBQ" switches as shown in Fig. 8(c). Generalization to five-, six-, ···, queue systems is similar.

The queueing behavior and the ways of determining (Δ, β) parameters when there are three or more adaptive streams is not really known. The problem we face is that of a multidimensional Markov chain with two or more sets of decision region partitionings (one for each adaptive stream) superimposed on each other. Again, this is a subject for future work. We would suggest an approximate solution as follows. Since different adaptive streams can be considered as independent, each adaptive stream can assume all other bifurcated adaptive streams as fixed Poisson arrivals and combine them with other fixed (as opposed to adaptive) streams. The model is then reduced to that described in the last paragraph, and (Δ, β) for that particular adaptive stream can be found as usual. The validity of this approach needs to be checked by simulation; again, we have not gone this far in our investigation.

Why do we choose the BS flow pattern and not others? Recall that the BS flow pattern is, by definition, the optimum flow pattern for stochastic fixed rules. But it is by no means the best flow pattern for adaptive rules. To clarify this point, we quote the delay results of the second example for different values of (Δ, β) in Fig. 6(c). It turns out that the JBQ-BS rule requires $(\Delta, \beta) = (-1, 0.2)$ with a corresponding delay of 3.95. Fig. 6(c), however, shows that the lowest delay given by this "biased-queue" method of routing is at $(\Delta, \beta) = (-2, 0.7)$. The corresponding delay is 3.79, which is 4 percent lower than the JBQ-BS delay. Thus the BS rule flow distribution is not the optimum flow distribution for the JBQ adaptive rule. In this simple network with only one set of (Δ, β) to optimize, we can get the optimum (Δ, β) with little effort, simply by searching the (Δ, β) around the particular value obtained by the JBQ-BS rule. In a general network, however, there are many sets of (Δ, β) to be optimized simultaneously. Altering a particular set will change the departure rates of the associated queues, and the flow pattern of the entire network is changed. Therefore the determination of the best flow pattern for the JBQ rule is still an open research problem. But it is safe to say that using the BS rule flow pattern as a substitute is satisfactory.

By now, it should be quite obvious that more traffic bifurcation in the network means more messages can be routed adaptively through different links. And as the fraction of adaptive traffic in a single node environment increases, the queueing delay decreases monotonically [11]. Therefore, the technique introduced in [7] that maximizes the amount of traffic bifurcation in a network while preserving the same utilization of each link can also be used on the JBQ-BS rule.

IV. SUMMARY AND CONCLUSION

After a brief review of the BS rule we introduced the JBQ rule, an adaptive way of bifurcating traffic in local nodes. Markov chain analysis was used on the JBQ rule to obtain some important properties and characteristics needed for later development. Then, in Section III, we introduced the JBQ-BS rule and worked out an example in detail to illustrate the concept. Three other examples together showed a 10-27 percent reduction of delay when compared to the BS rule. Finally, we discussed three problems that remain unsolved in the analysis of the JBQ-BS rule.

[1] The deterministic switch routes messages according to a predetermined sequence. In Fig. 8(b), the output of the deterministic switch becomes the arrival process to the queues. The interarrival time has an Erlangian distribution [7], which has a smaller coefficient of variation than the interarrival time of the Poisson process. Further, we have shown in [19] that this "deterministically bifurcated" process gives smaller queueing delay compared to that given by the Poisson process. Now, in the discussion of the JBQ-BS rule performance, we modeled the arrivals as Poisson processes. Hence, the use of the deterministic switch improves further the delay performance of the JBQ-BS rule.

Note also that a stochastic switch which uses fixed probability assignments to split the traffic can also be used in place of S_D. In that case, the output process will still be Poisson.

[2] Here we assume $a_1 \leqslant \min(a_2, a_3)$ so that more messages are routed adaptively through S_{JBQ}.

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Tak-Shing P. Yum (S'74–M'78), for a photograph and biography, see this issue, p. 504.

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