Precise Localization with Smart Antennas in Ad-Hoc Networks

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***Abstract*—In this paper, we study precise localization using Angle of Arrival (AOA) estimations by smart-antenna equipped beacons in Ad-Hoc networks. The node to be localized sends a signal to its surrounding beacons. The beacons estimate the signal directions with high resolution AOA methods and feed them back to the node for position calculation. In other words, position calculation is not required at beacons. When AOA estimates from three or more beacons are received, ambiguity occurs. Three resolution methods, namely (a) simple averaging (b) Minimax and (c) Precision-weighted averaging are proposed and compared. As estimation bias is heavily dependent on antenna orientations the center-facing approach is found to give better performance in a square field.**

***Index Terms*—Ad-Hoc networks, localization, AOA, smart antennas.**

I. INTRODUCTION

In recent years, the localization problem has received considerable attention in ad-hoc networks as the location information is typically useful for routing, environment monitoring, target tracking, and rescue. To obtain location, a node can use location determination hardware, such as a GPS receiver. But in a system where the wide spread deployment of GPS is not feasible, a small set of nodes with known positions, called beacons, can act as reference points for other nodes [1]. Localization methods can be classified as range-based, range-free and AOA-based in ad-hoc networks.

A. Range-based methods

The range-based methods use distance from a set of beacons and apply multilateration or triangulation techniques to find the coordinates [2-8]. The distance estimates may be obtained from time of arrival (TOA), time difference of arrival (TDOA), and received signal strength indicator (RSSI) information.

TOA is used to estimate the distance by measuring the propagation times of the signals. GPS uses such a method. TDOA is a special case of TOA. It estimates the distance from propagation times through different media, such as radio and ultrasound. To do this, additional hardware is required at the nodes to receive a signal. Currently, this technique is limited by the short range of ultrasound (up to 3m) [2]. Systems designs based on TDOA includes such as Cricket [3] and AHLos [4,5].

RSSI technology such as RADAR [6] and SpotOn [7] uses the knowledge of the transmitter power, the path loss model, and the power of the received signal to determine the distance of the receiver from the transmitter [8]. A node estimates the distances from three of more beacons to compute its location. The advantage of the RSSI method is its ubiquitous availability in practically all available receivers on the market [1]. The major drawback of is the difficulty of choosing path loss model for different environments as it can result in errors up to 50% of the measured distance [2].

B. Range-free methods

The range-free methods cannot accomplish as high a precision as the range-based method as distance and/or angle information is not available [9-15]. In [9], a node detects signals from its neighboring beacons and takes the centroid of the beacons as its estimated position. Doherty [10] approached the positioning problem by measuring the centroid of a rectangular bound according to the range of the beacons. In [11], hop counts (DV-Hop) or distance (DV-distance) from multiple beacons is used to obtain the location of the node. Algorithms based on Multidimensional Scaling (MDS) can operate in both range-free and range-based scenarios. These accurate centralized algorithms require considerable communication and computation overhead [12]. In the APIT (Approximated Point-In-Triangulation) algorithm, each node determines the set of triangles formed by beacons it can hear. The center of the intersection of all these triangles is taken as the node's position [13]. In [14], ROCRSSI was proposed, where circles instead of triangles are used to get the intersection based on RSSI. In [15] the CAB algorithm was proposed. Here each beacon emits signals at different power levels. Nodes determine the annular ring they are located with respect to each beacon. The center of ring intersection is then taken as the estimated position.

C. AOA (Direction) methods

AOA methods use special antennas to estimate directions of arrival. In the VOR/VORTAC system for aircraft navigation, the VORTAC stations transmit omni-directional signals, and a receiver determines its bearings with respect to the stations [16]. Niculescu proposed using the AOA of the signal and node orientation adjustment to find node locations, where node has at least 3 bearings to beacons that are not on the same line or on the same circle with this node [17]. In [18], the location estimation problem was solved by measuring the RSS from one or just two beacons in a 2D plane with directional antennas. In this scheme, the nodes are equipped with multiple directional antennas. In the near-field scenario, only one beacon is needed

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to get the nodes' location. When two beacons are used, the node's position can be obtained by calculating the RSS from the two beacons. In [19], RSS measurements from directional antenna arrays on each node were also used to estimate arrival angles, which are then used to estimate locations. Compared to distance-based estimators, sub-meter location accuracy is possible using 802.11 radio frequency communication signals in this scheme. However, antenna array mounting on each node may not be feasible in practice. In [8], each beacon transmits a unique RF signal on a narrow directional beam rotated continuously at a constant angular speed. A node measures its angular bearings according to the TDOA when it receives at least three fixed beacon signals. This scheme requires the directional beams of the beacons be rotated synchronously. A similar method [20] was presented for an indoor 802.11 positioning architecture requiring special basestations with revolving directional antenna for sending signals to the mobiles. The mobile finds the strongest signal from each basestation to get the AOA. In these two methods, the beam width is the major cause of direction ambiguity. In a 56m by 25m region, the median error is about 3m in [20]. In a square of 75m by 75m, the distance error is about 2m [8]. The methods we proposed can give much higher accuracy.

In this paper, we propose a new precise AOA localization scheme using multiple beacons equipped with smart antenna. After obtaining the AOA information of a node, the beacon sends the AOA information together with its identity (ie. the beacon location) back to the node. Using the position of the beacon and the estimated AOA a line can be drawn from the beacon towards the direction of the node to be localized. Repeating with a second beacon the node can be localized at the intersection of the two lines. But when many beacons are used ambiguity occurs when the lines intersect at multiple points. Three methods are proposed to resolve the ambiguity and their performance are compared. The proposed smart-antennas based localization method is presented in Section II. The three aggregation methods are given in Section III. The performance of the proposed methods is evaluated in Section IV. We conclude our paper in Section V.

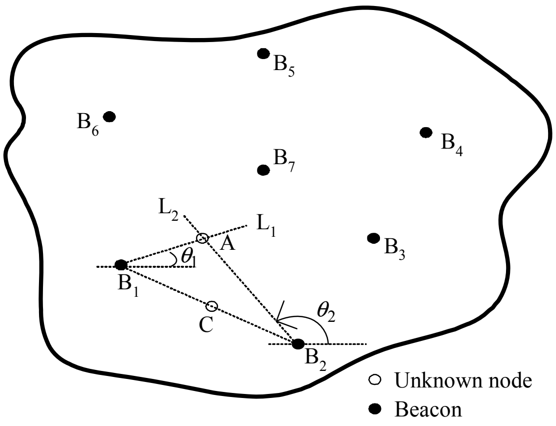

Figure 1. Node A can be located by B$_1$ and B$_2$ while node C, being collinear with B$_1$ and B$_2$, cannot.

II. SMART-ANTENNAS BASED LOCALIZATION

As shown in Fig. 1, node A can localize itself if it can receive the two signal directions θ_1 and θ_2 from beacon B$_1$ and beacon B$_2$ respectively. Let (x,y) be the location of node A, $(\hat{x},\hat{y})$ be the estimated location of node A and (a_m,b_m) be the coordinates of beacon B$_m$ $(m=1,2,...)$. The two lines B_1A and B_2A are respectively

$$\hat{y}-b_1=(\hat{x}-a_1)\tan\theta_1 \tag{1}$$

$$\hat{y}-b_2=(\hat{x}-a_2)\tan\theta_2 \tag{2}$$

From (1) and (2), $(\hat{x},\hat{y})$ can be solved as

$$\begin{cases}\hat{x}=\dfrac{a_1\tan\theta_1-a_2\tan\theta_2+b_2-b_1}{\tan\theta_1-\tan\theta_2}\\[2ex]\hat{y}=\dfrac{(a_1-a_2)\tan\theta_1\tan\theta_2+b_2\tan\theta_1-b_1\tan\theta_2}{\tan\theta_1-\tan\theta_2}\end{cases} \tag{3}$$

In the following, we introduce direction estimation methods from the literature.

AOA Estimation

A number of AOA estimation methods can be used, including MUSIC [21], ESPRIT [22], CA-MUSIC [23] etc. In this paper, we choose MUSIC for its simplicity. The use of other more accurate and hence more complicated methods is similar. MUSIC is a relatively simple eigen-structure method of AOA estimation. The estimation model is

$$\mathbf{X}(t)=[x_1(t),...,x_M(t)]^T=\mathbf{AS}(t)+\mathbf{n}(t) \tag{4}$$

where $\mathbf{X}(t)$ is the observed data vector from M antennas, $\mathbf{S}(t)=[s_1(t),...,s_D(t)]^T$ is an unknown vector from D source signals, $\mathbf{n}(t)=[n_1(t),...,n_M(t)]^T$ is an additive noise vector, and $\mathbf{A}$ is the steering matrix defined as

$$\mathbf{A}=[\mathbf{a}(\theta_1),...,\mathbf{a}(\theta_D)] \tag{5}$$

where $\mathbf{a}(\theta_i)$ is the steering vector associated with the ith source signal. This method estimates the noise subspace and signal subspace from the estimated array correlation matrix $E\{\mathbf{X}(t)\mathbf{X}^H(t)\}$ using singular value decomposition. Once the noise subspace is estimated, the directions can be obtained by searching for peaks in the MUSIC spectrum given by

$$\Psi(\theta)=D_{\mathrm{MU}}^{-1}(\theta)=\left(v^H(\theta)(I_M-\mathbf{E}_s\mathbf{E}_s^H)v(\theta)\right)^{-1} \tag{6}$$

where I_M is the $M\times M$ identity matrix, $v(\theta)=\mathbf{a}(\theta)/\|\mathbf{a}(\theta)\|$, $\mathbf{E}_s=span\{e_1,...,e_D\}$ is the signal subspace with its D columns being the eigenvectors corresponding to the largest D eigenvalues, $\lambda_1,\lambda_2,...,\lambda_D$, of the matrix $E\{\mathbf{X}(t)\mathbf{X}^H(t)\}$. When the number of snapshots and the SNR value increase infinitely, the MUSIC estimator can approach the Cramer-Rao bound.

In Ad-hoc network localization with a limited number of snapshots, let θ_d and $\hat{\theta}_d$ be the d-th source angle and its estimate respectively and let $\Delta\theta_d=\hat{\theta}_d-\theta_d$ be the estimation error. Let $\dot{Q},\ddot{Q},\dddot{Q}$ denote the first three derivatives of Q with respect to θ . The mean of $\Delta\theta_d$ is derived in [24] as

$$\Delta \equiv E\{\Delta\theta_d\} \approx$$

$$-\frac{1}{N}\frac{2\sum_{k=1}^{D}\frac{(M-D-1)\lambda_k\sigma_n^2}{(\lambda_k-\sigma_n^2)^2}\mathrm{Re}[\dot{v}^H(\theta_d)e_k e_k^H v(\theta_d)]}{\ddot{D}_{\mathrm{MU}}(\theta_d,E_s)} \tag{7}$$

$$-\frac{\dddot{D}_{\mathrm{MU}}(\theta_d,E_s)}{6\ddot{D}_{\mathrm{MU}}(\theta_d,E_s)}\mathrm{Var}\{\Delta\theta_d\}$$

where N is the number of the independent snapshots, σ_n^2 is the noise power level and

$$\mathrm{Var}\{\Delta\theta_d\} \approx \frac{1}{N}\frac{\sum_{k=1}^{D}\frac{\lambda_k\sigma_n^2}{(\lambda_k-\sigma_n^2)^2}\left|v^H(\theta_d)e_k\right|^2}{\ddot{D}_{\mathrm{MU}}(\theta_d,E_s)} \tag{8}$$

III. AGGREGATION OF ESTIMATION

Two non-parallel lines are sufficient to locate a position on a plane. How accurate the position is depends on θ_1 and θ_2. If direction information from n beacons are received by node A, a maximum of $n(n-1)/2$ intersection points are available. In general, the set of usable points is much less and we denote them as $\{(x_i, y_i), i = 1,2,..., K\}$. We now propose three methods for estimating (x, y) from $\{(x_i, y_i)\}$.

A. Mean Aggregation

By averaging all of the intersection points we obtain

$$\begin{cases} \hat{x} = \sum_{i=1}^{K} x_i \Big/ K \\ \hat{y} = \sum_{i=1}^{K} y_i \Big/ K \end{cases} \tag{9}$$

B. Minimax Aggregation

Since multiple impinging signal from close by directions affect the resolution of MUSIC estimator, we assume here, for simplicity, only the signal from node A is received at beacon B_1 at direction θ_1 as shown in Fig. 2. Hence the bias expression in (7) can be reduced to [24]

$$\Delta_1 \equiv E\{\Delta\theta_1\} \approx -\frac{\dddot{D}_{\mathrm{MU}}(\theta_1,E_s)}{6\ddot{D}_{\mathrm{MU}}(\theta_1,E_s)}\mathrm{Var}\{\Delta\theta_1\} \tag{10}$$

As shown in Fig. 2, the lines using the estimated angles $\theta_2 \pm \Delta_2$ from beacon B_2 and that using the estimated angles $\theta_3 \pm \Delta_3$ from beacon B_3 form a quadrangle surrounding node D. Let the four corners of his quadrangle be denoted as p_1, p_2, p_3, p_4. We define ambiguity e_1 as the maximum distance error of this quadrangles, or

$$e_1 = \max\{\mathrm{d}(p_i, p_j), 1 \le \mathrm{i}, \mathrm{j} \le 4, \mathrm{i} \ne \mathrm{j}\} \tag{11}$$

where $\mathrm{d}(p_i, p_j)$ is the distance between points p_i and p_j . Similarly, quadrangles can also be formed from beacons B_1 and B_2 and beacons B_1 and B_3 surrounding node D. Let the corresponding ambiguity measures e_2 and e_3 also be computed. The Minimax Aggregator is the estimated location given by the two beacons with the minimum ambiguity $e_{\min} = \min(e_1, e_2, ...)$.

C. Precision-Weighted Aggregation

Since e is a measure of ambiguity, $f = 1/e$ can be interpreted as a measure of precision. Therefore the Precision-Weighted Aggregator is given by

$$\begin{cases} \hat{x} = f_s^{-1}\sum_{i=1}^{K} f_i x_i \\ \hat{y} = f_s^{-1}\sum_{i=1}^{K} f_i y_i \end{cases} \tag{12}$$

where $f_s = \sum_{i=1}^{K} f_i$.

D. MAC-Assisted Mechanism

MUSIC-like estimators can identify multiple signal directions at the same time depending on the number of array elements on the smart antenna. But the larger the number of signal elements to be estimated the lower the resolution. Therefore, for localization accuracy, multiple requests from nodes should be spread out by a distributed scheduling

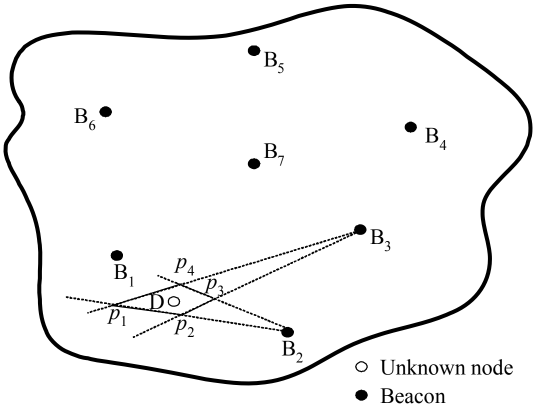

Figure 2. AOA estimation bias causes localization ambiguity.

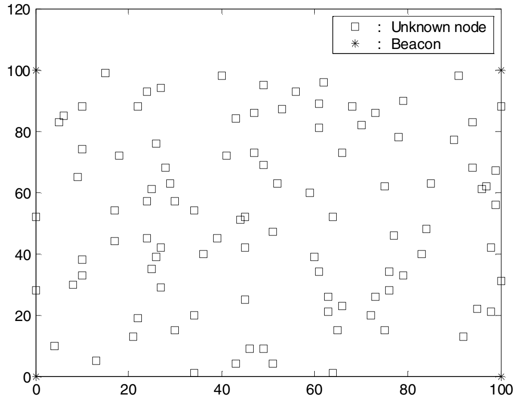

Figure 3. The 100 nodes to be localized.

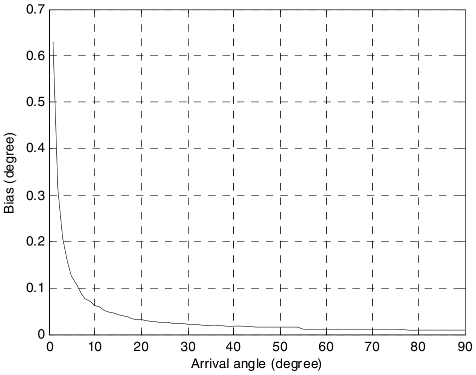

Figure 4. Estimate bias of impinged angle for a single source using an ULA, N=20, SNR=15dB, M=2.

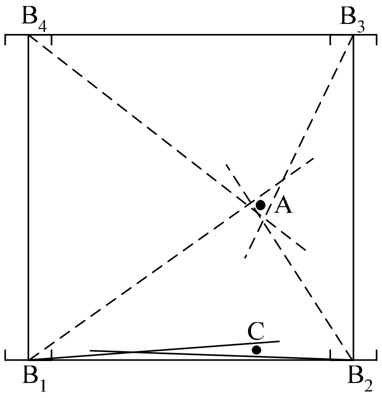

Figure 5(a). Localization with horizontal ULA.

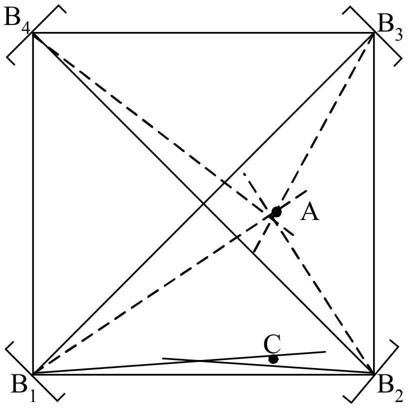

Figure 5(b). Localization with center-facing ULA.

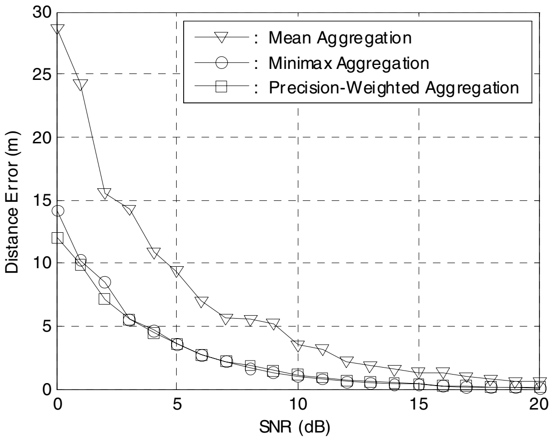

Figure 6. Average estimation error, horizontal ULA, N=20.

algorithm, or a multi-access protocol. If node localization is performed one at a time, localization throughput is lower but accuracy is higher. The study of this tradeoff is beyond the scope of this paper.

IV. PERFORMANCE EVALUATION

In this section, we study the new localization scheme by computer simulation. The network is a square with side length of 100m as shown in Fig.3. All beacons are equipped with a two-element ULA (uniform linear array) with sensor separation $\lambda/2$ where λ is the wavelength of the signal carrier. Four beacons are placed at the corners of the square. N nodes are distributed randomly in the square. As before, $(\hat{x}_i, \hat{y}_i)$ is the estimated position and (x_i, y_i) is the actual position of node *i*. The average estimation error E of these N nodes is used for comparison where

$$\mathrm{E}=\frac{1}{N}\sum_{i=1}^{N}\sqrt{(\hat{x}_i-x_i)^2+(\hat{y}_i-y_i)^2} \tag{13}$$

A. Bias for one source using ULA

With ULA, we can use Root-MUSIC [25] estimator for its better performance. Its bias or rms deviation in the estimate, for one source AOA, is given by [26]

$$\Delta=\sqrt{\left(\frac{\lambda}{2\pi d\sin(\theta)}\right)^2\frac{6}{M^2}\left(\frac{\sigma_n^2}{MP_1N}\right)} \tag{14}$$

where d is the sensor separation and P_1 is the source power level. Fig. 4 shows the bias of impinged angle for a single source using an ULA antenna with N=20, SNR=15dB, M=2. It shows that the larger the impinged angle, the lower the bias.

B. Placement of the ULA antenna

In Fig. 5(a) the two-element antenna array is placed horizontally at the corner of the square and in Fig. 5(b) the arrays are placed facing the center of the square. When the arrays are facing the center, the incident angles are larger and hence the bias is smaller according to (14). This is illustrated by nodes A and C in the Fig. 5.

C. Simulation Results

Let the node distribution be distributed randomly in the network and the antenna orientation is either horizontal or center-facing as shown in Fig. 5. For each node, 10 position estimations are made. Fig. 6 shows the average estimation error E according to (13) for horizontal antenna. Fig. 7 shows the same for center-facing antenna arrangement. Both figures show

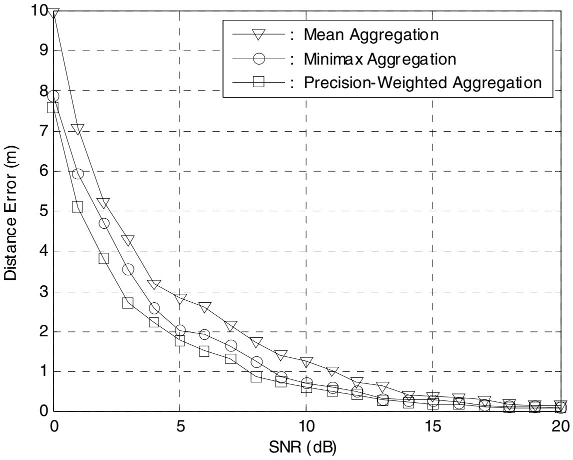

Figure 7. Average estimation error, center-facing ULA, N=20.

that Precision-weighted Aggregation gives the best performance. Comparing the two figures, it is confirmed that center-facing ULA is better than horizontal ULA position for a square field. The estimate error is about 3m in a 56m by 25m region [20] and 2m in a square of 75m by 75m [8] respectively. When N=20 and SNR=10, it can be seen from Fig. 7 that the estimate error is about 0.5m in a square of 100m by 100m. The larger snapshot and SNR, the smaller estimate error in our method. Therefore, the methods we proposed can give much higher accuracy. The study of beacon placement and array orientation for an arbitrary field is beyond the scope of this paper.

V. CONCLUSION

In this paper, we propose a new precise node localization scheme using Angle of Arrival (AOA) estimations of nodes in Ad-Hoc networks. The node sends signal to all the beacons in the entire networks, and then the beacons estimate the node's directions with high resolution AOA methods. After getting the directions of the node, the beacons send the signal direction and the beacon identity back to the node for localization. In a square field, center-facing antenna arrays and Precision-weighted position estimation can give uniformly better performance.

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