Wavelet Transform Based Kalman Filtering Algorithm for Anti-SA Effect

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Abstract

A new mathematical approach, wavelet transformation, is introduced for GPS data analysis. Based on its good features on both time-domain and frequency-domain, one can obtain a true time-frequency representation of a signal. The so-called mutiresolution analysis (MRA) will be used for data analysis, such as for the removal of noises, and the detection and rejection of gross errors from signal. In this paper the authors present a wavelet-based algorithm for the modeling and prediction of SA effect by combining time series analysis methods. A new self-adaptive Kalman filtering algorithm is presented for anti-SA. Some preliminary test results from experimental data are also summarized.

I. INTRODUCTION

Global Positioning System (GPS) has been widely applied in the guidance of ships, cars and aircraft. For all these applications GPS is affected by both systematic errors or biases and random noise, including the clock error, ephemeris error, ionospheric delay, the refractive effect of the troposphere, multipath and especially the SA (selective availability). These biases have heavily blocked the further application of GPS in the precise approach and landing. The primary error of GPS point positioning comes from the SA effect, which mainly consists of low frequency components. Wavelet transformation is an efficient tool for distinguishing signal from noises. Positioning errors can be decomposed into various components (high frequencies and low frequencies) by the wavelet transformation. In recent years wavelet analysis has been developed very quickly and has been widely applied in image processing, fracture graphs, CT imaging, Radar, earthquake, cycle slip detection of phase measurements etc (Chatfield, 1984; Huang and Zhuo, 1997). Arbitrary details of a signal can be observed and analyzed by the aid of wavelet analysis. SA effects whose primary components are of low frequency, shows as a random process. Time series analysis method can be used for the modeling of SA effect. Practical data analysis manifests that SA effect has the characteristics of an AR (auto-regressive) random process and the one-step predictive results of AR (p) model are very precise. Since the parameters of AR model vary with time, a self-adaptive algorithm is used to obtain parameters in the model AR (p) and this in turn could be utilized for anti-SA filtering.

II. MALLAT DECOMPOSITION AND SIGNAL-NOISE SEPARATION

Discrete wavelet transform (DWT) is often for practical use, which provides sufficient information both for analysis and synthesis of the original signal with a significant reduction in computation time compared to the continue wavelet transform. For a given signal, one can compute the DWT and uses it in different ways. We can discard all the values in DWT that are less than a certain threshold, i.e. let these coefficients be zero, and save only those DWT coefficients that are above the threshold. When these coefficients are used to reconstruct the original signal, a significant data reduction can be done without losing too much of the information. We can also analyze the signal at different frequency bands, and reconstruct the original signal by using only the coefficients that are of a particular band. This makes it possible for us to obtain different components from the signal.

For a given multi-resolution analysis, the Mallat decomposition procedure can be written as:

$$f(x) = A_{J_2} f(x) + \sum_{j=J_1+1}^{J_2} D_j f(x) \qquad (1)$$

$$A_j f(x) = \sum_{k=-\infty}^{\infty} C_{j,k} \varphi_{j,k}(x) \qquad (2)$$

$$D_j f(x) = D_{j,k} \psi_{j,k}(x) \qquad (3)$$

$$C_{j+1} = H C_j \qquad (4)$$

$$D_{j+1} = G C_j \qquad (j = J_1, J_1+1 \cdots\cdots J_2-1)$$

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$$H=(H_{m,k}), \quad G=(G_{m,k}) \tag{5}$$

$$\overline{h}_{k-2m}=<\varphi_{J_1,K},\varphi_{J_1+1,m}>, \quad \overline{g}_{k-2m}=<\varphi_{J_1,K},\psi_{J_1+1,m}> \tag{6}$$

$$H_{m,k}=\overline{h}_{k-2m}, \quad G_{m,k}=\overline{g}_{k-2m} \tag{8}$$

where φ is a resolution function and ψ is a wavelet function, J_1 and J_2 are two given integers. A_jf is called the continuous approach of a signal with 2^j resolution and D_jf is called the continuous details of a signal with 2^j resolution. A_jf is understood as the components of a signal with frequency less than 2^{-j} and D_jf is understood as the components of a signal with frequencies from 2^{-j} to 2^{-j+1}.

Denoting H^T and G^T as the dual operator of H and G respectively, Mallat reconstruction procedure can be written as

$$C_j=H^TC_{j+1}+G^TD_{j+1} \quad (j=J_2-1,\cdots\cdots,J_1) \tag{9}$$

The wavelet-based multi-resolution analysis decomposes a signal into $A_{J2}f(x)$ components with frequencies lower than 2^{-J_2} and components $D_jf(x)$ with frequencies from 2^{-j} to $2^{-(j-1)}$. Once the signal is decomposed into formulas (1) to (3) by means of Mallat algorithms, the signal without noises can be gained by Mallat reconstruction algorithms by means of setting the corresponding coefficients $C_{J2,k}$ and $D_{j,k}$ to zero according to the pre-knowledge to a given problem. The reconstruction algorithm follows that

$$f_0(x)=A_{J_1}f(x)=\sum_{k\in Z}C^T_{J_1,k}\varphi_{j,k}(x) \tag{10}$$

$$C^T_{j-1}=H^TC_j+G^TD_j \quad (j=J_2,J_2-1,\cdots,J_1+1) \tag{11}$$

III. WAVELET TRANSFORM BASED MODEL IDENTIFICATION AND PREDICTION OF SA EFFECTS

Three Basic Models for A Time Series

Auto regression model (AR)

$$x_t=\varphi_1x_{t-1}+\varphi_2x_{t-2}+\cdots\cdots+\varphi_px_{t-p}+a_t \tag{12}$$

with t = 1,2,3...,n

where $\{a_t\}$ is white noise

Moving average model (MA)

$$x_t=a_t-\vartheta_1a_{t-1}-\cdots\cdots-\vartheta_qa_{t-q} \tag{13}$$

with t = 1,2,3...,n

Auto regression moving average model (ARMA)

$$x_t=\varphi_1x_{t-1}+\varphi_2x_{t-2}+\cdots\cdots+\varphi_px_{t-p}-\vartheta_1a_{t-1}-\cdots\cdots-\vartheta_qa_{t-q}+a_t \tag{14}$$

with t = 1,2,3...,n

Wavelet Transform Based Error Modeling for GPS Point Positioning

Real time minim least squares algorithm is used to estimate the parameters of SA effect model, and F-test is utilized in the determination of model order, whose statistic reads

$$F=\frac{S_1-S_2}{S_2}\frac{N-n_2}{n_2-n_1} \tag{15}$$

where S_1 is the summation of low order residual squares, S_2 is the summation of high order Residual Squares and N is the number of the observations.

The prediction equation of SA effects

$$\hat{x}_{t+1}=\varphi_1\hat{x}_{t-1}+\varphi_2\hat{x}_{t-2}+\cdots\cdots+\varphi_p\hat{x}_{t-p}-\vartheta_1\hat{a}_{t-1}-\vartheta_2\hat{a}_{t-2}-\cdots\cdots\vartheta_q\hat{a}_{t-q} \tag{16}$$

Kalman Filtering Algorithm

Denoting the tree-dimensional coordinates as state vector, the discrete dynamic state equation can be written as

X(k+1)=fX(k)+W(k) (17)

where

$$\phi=\begin{bmatrix}1 & t & t^2/2\\ 0 & 1 & t\\ 0 & 0 & 1\end{bmatrix} \tag{18}$$

where W(k) is the dynamic noise vector with E[W(k)] =0 and $E[W(k),W^T(k)]=Q(k)$. The observation equation is

Y(k)=HX(k)+Z(k)+V(k) (19)

where $H=(1\ 0\ 0)^T$, V(k) is the observation noise vector with E[V(k)]=0 and E[V(k),VT(k)]=σ2, Z(k) is the prediction of SA effects given by

$$Z(k)=\varphi_1Z(k-1)+\varphi_2Z(k-2)+\cdots\varphi_pZ(k-p)+\varepsilon(k) \tag{20}$$

The main stages of wavelet based Kalman filtering algorithms are as follows:

(a) Estimate the parameters $\varphi_1\ldots\varphi_\pi$ of model (20) by timely least squares method

(b) Estimate the status vector X(k) by using Z(k) and Y(k).

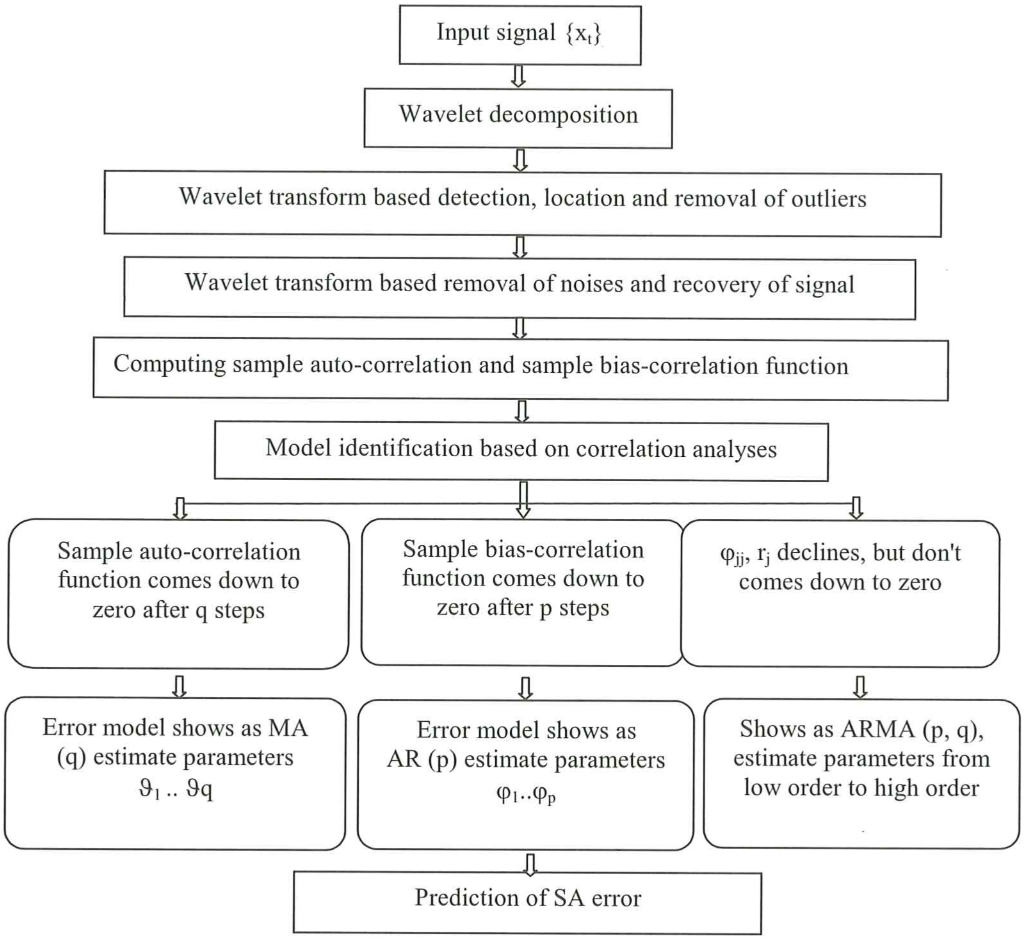

Figure1.

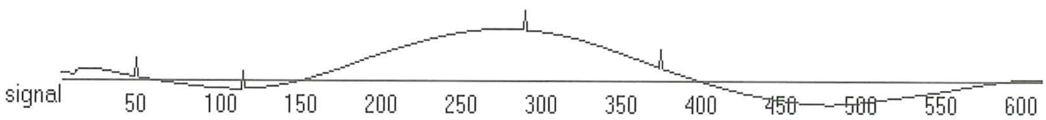

Figure 2. Error curve of GPS point positioning

The procedure is shown in Figure 1.

IV. ANALYSIS OF PRACTICAL DATA

The surveying mode is set as point positioning, and the sampling rate is 1s. The precise position of the receiver, by means of which the point positioning error series can be calculated, is obtained by static surveying mode. The point positioning error curve is shown in Figure 2.

As can be seen in Figure 2, the main part of point positioning error (it is mainly caused by SA effect) is a smooth curve. The abrupt points of the curve are called outliers, which are evoked by the signal blocking or loss of tracking, and the components of higher frequencies of the curve are caused by the other error sources apart from SA effect.

Wavelet Based Outlier Detection and Filtering

Apart from the main part of SA effects, there exist other errors, which are caused by the satellite signal blocking or loss of lock and multi-path effect among the GPS position errors. SA effect must be extracted from all the errors in order to set up a SA effect model. Since wavelet analyses is a strong tool for signal extraction, we will make use of the noise removal algorithm that has been given in the previous sections to capture the needed information from the raw signals. The key of wavelet transform is the choice of wavelet function. Because of its good features, such as compactly supporting, three-order spline wavelet

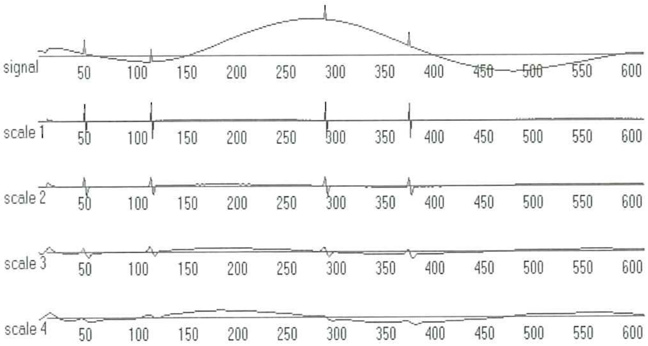

Figure 3. Results of wavelet decomposition

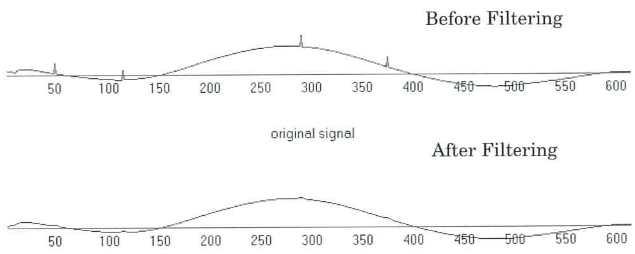

Figure 4. Error curve after filtering

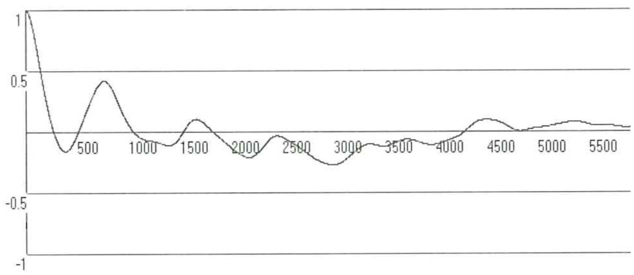

Figure 5. Auto-correlation function curve before filtering

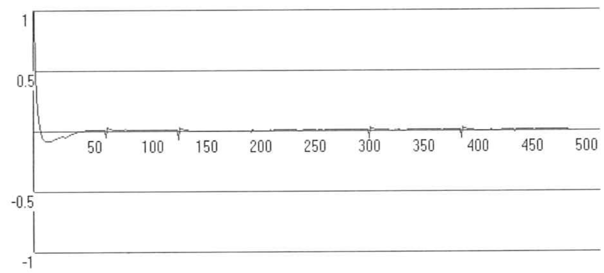

Figure 6. Bias-correlation function curve before filtering

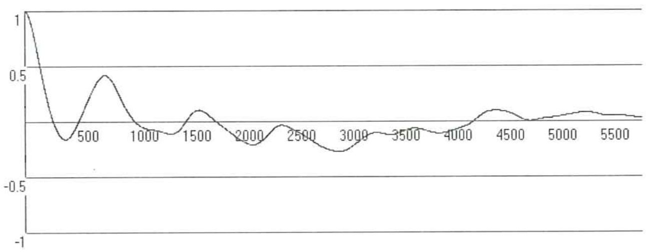

Figure 7. Auto-correlation function curve after filtering

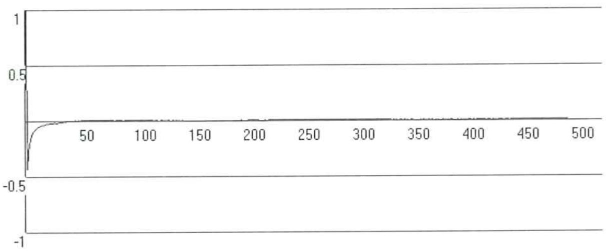

Figure 8. Bias-correlation function curve after filtering

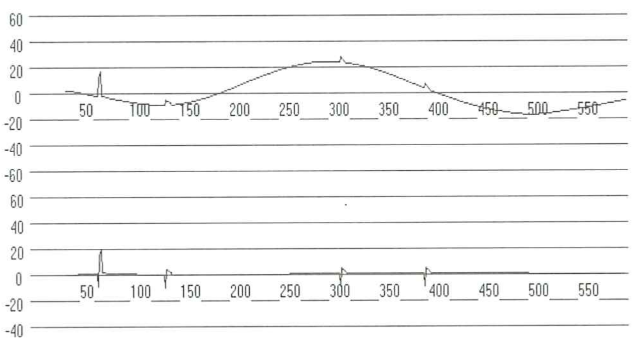

Figure 9. One step prediction and residuals (before filtering)

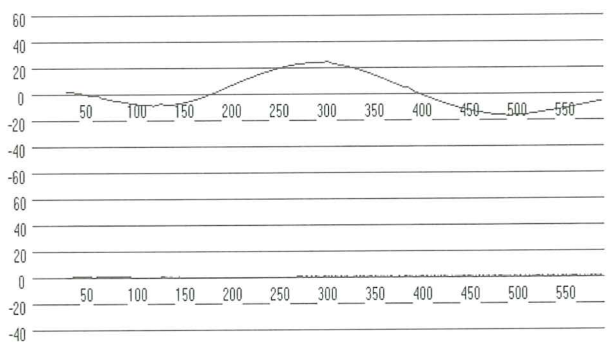

Figure 10. One step prediction and residuals (after filtering)

function is chosen as a basic wavelet function. The wavelet decomposition results of less than 4 scales are shown in Figure 3.

After the raw signals are decomposed into components of different frequencies, the location of signal abrupt can be detected easily. As discussed in the previous sections, if no outlier exists, GPS point position error curve is a continuous and smooth curve. If an outlier occurs in the signals, the smoothness of position error curve will be broken abruptly. Therefore the location, in which smooth degree changes, is the location of an outlier. Lipschitz exponent may be used to depict the signal singularity.

From the above discussions, we know that wavelet transform has a zooming nature, thus it is effective to detecting the location of the singular points and analyzing its degree of singularity by means of wavelet transform. As can be seen in Figure 3, the component of 2^1 scale mainly consists of outliers; moreover it is 1.5 as large as its original outlier. Therefor the location of an outlier can be detected easily.

Since the outliers in a time series have serious effect on the modeling and prediction of a time process, we must remove the outliers from the signals, and then reconstruct the signals. After the components with the high frequencies are set zero, by using the above wavelet transform based reconstruction algorithm, the reconstructed signal with the removal of noise can be obtained. The point position error curve after the wavelet based filtering is shown in Figure 4. Using the proposed algorithm the outlier can be removed effectively and the signal can be reconstructed accurately.

The Modeling and Prediction of SA Effects

In order to analyze the characteristics of the signals before and after the noises and outliers are subtracted from the signals, we calculate the corresponding auto-correlation function and bias-correlation function respectively. Figures 5~8 are the auto-correlation and bias-correlation function with and without the removal of noise and outliers from the signals. We can see that both the auto-correlation function and bias-correlation function show as a tailed curve. Therefore the error model can be depicted by ARMA (p, q) according to the above features. After the noises and outliers are removed from the signal, both the auto-correlation function and bias-correlation function come down to zero after p steps, thus the error model follows model AR (p). Since both the modeling and parameter estimation of AR (p) is easier than ARMA (p, q)[1], the wavelet transform based modeling algorithm is more feasible and practicable.

Using the above wavelet based modeling and prediction algorithms for GPS point position error, we can model and predict the SA effect. The one step predication results and residual curves are shown in Figure 9 and 10 for the cases with and without the removal of noises from the signal. We can know from the figures that the precision of model prediction is lower when there exist outlier in the signals; but after the noises and outliers are subtracted from the signals, the prediction precision of SA effect by the wavelet transform based filtering algorithms becomes very high.

V. CONCLUSIONS

Wavelet analysis is successfully used for the detection and removal of outliers. The SA effects can also be extracted by muti-resolution decomposition. When the outliers and the noises of high frequencies are subtracted, AR (p) can model SA effect. The practical data analysis has shown that the high prediction precision of SA effect can be achieved. Combining the error model with Kalman filter, a new wavelet transform based algorithm can efficiently be used for anti-SA filtering

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