Spatial Decompositions, Modeling and Mapping Service Regions to Predict Access to Social Programs

John Radke and Lan Mu

Departments of Landscape Architecture and Environmental Planning
GISC, 102 Wheeler Hall, University of California, Berkeley, CA 94720-2000

Abstract
Although social programs intend to provide equal access for all, in the final evaluation, fairness of the distribution of services is usually dictated by location. Measuring and predicting access to social services can help these programs adjust and better accommodate under-served regions. A method is proposed which delineates the service area of providers delivering social services and produces a probability metric that maps the equity of the program of services for each household. We begin with a computationally trivial method for delineating service areas, map the probability of households being served, and propose an adjustment process, an allocation, to level access to services. We argue such methods can serve to better locate service providers and insure equity when implementing social programs.

I. INTRODUCTION

The relatively young field of Geographic Information Science is making it possible to build solutions to problems that until now were considered insurmountable due to their spatial complexity. Although the amount of data captured and catalogued might be considered extremely voluminous, it is the spatial interaction of that data that is the real impediment. Governments have always faced problems such as these but with their recent large investments in geographic data and information technologies, they are coming under more pressure to produce solutions.

Generally governments serve their people by enacting legislation and championing programs that level the playing field of its citizens. Many of these programs surface as social services and rebate schemes which provide incentives for citizens to improve their quality of life, often effecting the rest of the populace in a positive way. For example, an energy rebate scheme offered through government regulated utilities to help the disadvantaged insulate their homes, reduces their annual heating costs and contributes to a general reduction in energy needs and pollution. Although it is easy to propose such a scheme, it can be extremely difficult to implement, regulate and insure equal opportunity for all citizens where their spatial distribution is highly variable. It is likely impossible to provide equal access to services in a heterogeneous landscape and although many private service entities may not even wish to, a government providing social services to it citizens must attempt to. Geographic information and related technologies can be employed to reduce and possibly eliminate spatial uncertainties faced by governments and others in delivering social services in a spatially variable environment.

II. OBJECTIVES

The central objective of this study is to develop a method which measures access to social services for each household and makes adjustments among service providers to better accommodate under-served regions. The method encodes the defined service area of providers delivering social services and produces a probability metric that maps the equity of the program of services for each household. After successfully measuring the state of the current distribution of providers to households, we move to level the playing field by altering the probability of a household's access to social service providers by moving providers to areas that are currently under-served from areas over-served when considering the region as a whole.

Although the approach developed here might serve to enlighten the spatial adjustment of social service providers in general, we focus our attention on the distribution of government authorized contractors who provide service for rebate schemes. Adjusting the spatial distribution of suppliers to more evenly service the needs of households can be classified as a locational or facility search problem (Church 1999). An extensive literature on location-allocation problems exists (Beaumont 1981; Church 1999; Eiselt 1992) yet the problem of generating equity for all demand points

1082-4006/00/0602-105$5.00

where there exists extensive overlapping of supply regions remains. The method proposed here addresses this problem under applied conditions that could not be immediately challenged: i) suppliers filed that they were not impeded by friction of distance while providing service, ii) it is assumed that all suppliers were equally capable, had the same capacity to deliver and offer a similar service, iii) the contractors already form a spatial pattern of supply and it is assumed that their numbers are fixed, and iv) regions not serviced by the original supply set are assumed to be unimportant and are not consciously targeted for supply. Although these constraints are imposed by the input data, the method developed here is adaptable and can be altered as assumptions are eliminated.

III. LOCATION-ALLOCATION

Location models and those that attempt to improve or optimize conditions by including allocation, are now common in the literature (Bailey and Gatrell 1995; Beaumont 1981; Eiselt 1992) and are making inroads into GIS (Church 1999). These models all share a common objective, to locate supply so that demand is met in the most efficient manner. The simplest models encode supply, often representing some production, distribution or service facilities, as a set of points in space. Demand is also often represented by point locations which are then allocated to the closest supply point for an optimal solution, as illustrated in Figure 1 (a). When a single supply point p is involved, the simple Weber problem in industrial location (Wheeler and Muller 1986) is solved, while adding one or more supply centers increases complexity and the *p-median* problem is presented. These location-allocation models range from spatially unconstrained, where demand and supply interact in straight lines and the friction or ease of travel remains constant (no distance-decay exists), to more discrete or constrained models that are embedded within a line network (such as a transportation network), where each segment and intersection contributes individual constraint parameters to the model.

In many instances demand data is not or cannot be reported at an actual point location. Here supply is known to serve a region and by sampling the region we can predict demand. The US Census Bureau, among others, follows this procedure to protect the identity of individuals, and to reduce costs in data gathering through sampling. This serves to complicate the location-allocation model by presenting a *point-polygon* location problem. Voronoi or Thiessen polygons (Okabe et al. 1992), now common in many commercially available software, are the best unconstrained allocation models to encode examples of supply points with demand regions, illustrated in Figure 1 (b). Here there are no line networks to alter travel direction or add friction. This model can be altered to solve variation in capacity of the supply points by adding weights (Aurenhammer and Edelsbrunner 1984) which impact the regions served, illustrated in Figure 1 (c).

Although most location-allocation models address supply and demand as discrete points (or at least supply as discrete), as more ancillary data is accessible through technological advances in GIS, a new geometric representation, *polygon-polygon* class of location problems, are emerging (Openshaw and Alvanides 1998; Miller 1996). Here supply is delivered in a region defined by some polygon (such as a cellular transmitting tower), and demand is dispersed throughout polygons (such as demographic classification units or census tracts).

Whatever the class of location problems presented, planners have been involved in developing the criteria and process to evaluate these models (Arentze 1996). In planning services, maximal distance or time is commonly used to determine if a demand is covered.

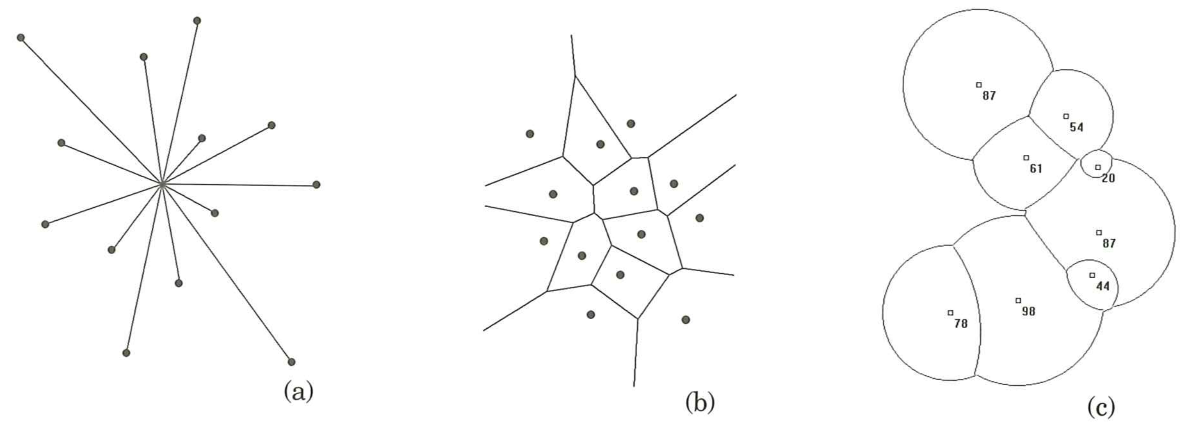

Figure 1. Unconstrained allocation models: (a) *point-point* class, (b) the Voronoi, a *point-polygon* class, and (c) the weighted Voronoi, a *point-polygon* class.

Even though planners intend to serve all demand, the focus is to provide a level of service which meets some minimal standard. Two types of location models share this coverage concept: *location set covering* models which minimize the required facilities to serve demand, and *maximal covering* models which maximize coverage with a fixed number of facilities (Church 1999).

IV. DECOMPOSING SERVICE REGIONS, PREDICTING ACCESS AND GENERATING EQUITY

The State of California is experiencing unprecedented population and economic growth. By the year 2020 it is predicted the population will increase by 2.4 million and there will be an increase of 5.1 million jobs requiring 4.3 million housing units, transportation, telecommunications, gas and electric utility connections, fire and police protection, health care, education, and parks and recreation facilities among others (PG& E, 1999). During this massive expansion a major challenge for planners is how to allocate public facilities and services in order to optimally serve this growing population. Since the deregulation of the electric industry in the state, competition among the contractors is changing the utility business paradigm where government decision makers are now facing the problem of how to insure equal access to services. Complex problems demand robust approaches. We introduce one here that joins the family of *maximal covering* models and solves a *polygon-polygon* geometric representation location problem. Although the model introduced is applied here under several data constraints, we argue it is robust and will hold its validity as constraints are removed.

To develop and apply our functional model we could have used any data where social services or a rebate scheme identified contractors supplying services. With a Residential Contractor Program being implemented statewide in California, we choose a local utility company, Pacific Gas & Electric, to illustrate our model. Our study region, Figure 2, covers a large portion of Northern California and includes a household population exceeding 5.5 million.

Figure 2. The study region within the State of California.

We fuel our demand for services (or clients) side of our model with the number of *households* reported in the 1990 US Census, and following Miller (1996), geometrically represented them as polygons or Census Tracts in this instance. We built our *supplier of services* data base from a list of 53 contractors who are in the Residential Contractor Program and geocode them by applying an address matching algorithm using the 1996 US Census Bureau's TIGER data. Based on a survey of contractors, their business range is assessed at 60 miles for rural/suburban areas and 35 miles for San Francisco and Alameda counties. Although the survey forced assumptions such as: contractors being equally capable, and not being impeded by friction of distance; our model as constructed is robust, could ingest varied contractor service regions, and be altered to deal with friction of distance.

Decomposing service regions and predicting Access

Our *polygon-polygon* location problem involves a set of contractor buffers or polygons to serve, and a set of polygons containing a density of households to be served. Unlike the classic p-median location model (Church 1999) where the objective is to locate supply facilities to minimize the distance to serve all demand, we begin with 53 supply contractors already in place, and employ a heuristic to relocate some supply regions. This model, conceptually similar to a substitute model (Teitz and Bart 1968), balances service to demand over the entire study region. In order to assess the equality of those being served we generate the probability of all households being served by each supplier of services or contractor in this instance. During this process we generate a density surface of households per area, altering the reported census tract household data so it can be disaggregated when intersected with each contractor service area buffer. This process estimates the number of households that can be potentially serviced by each contractor.

$$Hd = nH/Area \quad (1)$$

where: Hd = the density of households per area within each census tract, nH = number of households in the census tract, Area = area of the census tract.

We generate service area buffers with a radius of 60 miles (35 for those contractors in San Francisco and Alameda counties), individually intersect them with the census tract polygons illustrated in Figure 3, and estimate the number of households serviced by each contractor buffer $n\hat{H}_{bj}$ by multiplying the area of each new intersected polygon by its density of household values and summing them for the entire buffer.

$$n\hat{H} = Hd * Area \qquad (2)$$

where: $n\hat{H}$ = estimate of number of households in each intersected polygon, Area = the area of each new intersected polygon.

$$n\hat{H}_{bj} = \sum_{i=1}^{n} n\hat{H}_i \qquad (3)$$

where: $n\hat{H}_{bj}$ = the estimate of the number of households within the contractor buffer j, $n\hat{H}_i$ = estimate of number of households in each intersected polygon i, n = the number of intersected polygons within the contractor buffer j.

With the number of households known for each contractor service area buffer, estimating the probability of each household being served by the contractor is trivial:

$$\hat{P}_{bj} = \frac{1}{n\hat{H}_{bj}} \qquad (4)$$

where: $\hat{P}_{bj}$ = estimate of the probability of each household being served by the contractor j, $n\hat{H}_{bj}$ = estimate of the number of households within the contractor buffer j.

The complexity in our location model increases when we calculate the probability of each household being potentially served by more than one contractor, or where there are overlapping service area buffers. Here we spatially decompose the polygons further by overlaying them and producing many intersections of contractor service areas. Consider a set S of n buffers. Their intersections produce common polygons, each polygon i formed by intersecting a subset s of S buffers.

When buffers intersect, contractors compete to service households, increasing the probability of households in the overlapping areas being served. If the probability estimate $\hat{P}_{b1} = 1/n\hat{H}_{b1}$ for households within contractor buffer 1, and the probability estimate $\hat{P}_{b2} = 1/n\hat{H}_{b2}$ for households within contractor buffer 2 intersect, the probability estimate of households being served in a new polygon formed by their intersection is $\hat{P}_{b1} + \hat{P}_{b2}$, illustrated in Figure 4.

For each polygon i (formed by intersecting a set of buffers s_i) the estimate of the probability of households being served is calculated as:

$$\hat{P}_i = \sum_{j \in s_i} \hat{p}_{bj} = \sum_{j \in s_i} \frac{1}{n\hat{H}_{bj}} \qquad (5)$$

Where: $n\hat{H}_{bj}$ = estimate of the number of households within buffer j, $\hat{P}_i$ = estimate of the probability of each household being served in polygon i, $\hat{P}_{bj}$ = estimate of the probability of each household being served by the contractor j.

The estimate of the probability of households being served in polygon i can be mapped for all polygons formed by overlaying all the buffers in a study and the standard deviation mapped (Figure 5). The result delineates a spatial pattern of households under-served by contractors and those that enjoy an advantage.

Generating equity

The *substitution model* (Teitz and Bart 1968) has received wide recognition as a simple supply point substitution process. Here, an existing supply site is replaced with a candidate site to determine if an improvement in the objective results. If the candidate site yields improvement, a switch is made. This method is thought to be a robust heuristic (Rosing et al. 1979) for solving *covering models*, yet it can be impacted by local optima (Church and Sorensen 1994)

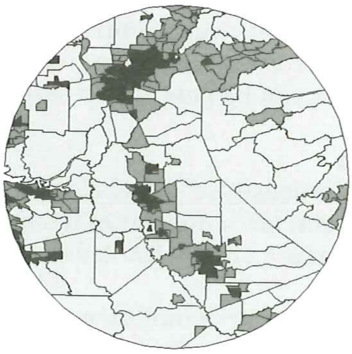

Figure 3. A contractor service area buffer intersected with Census tracts

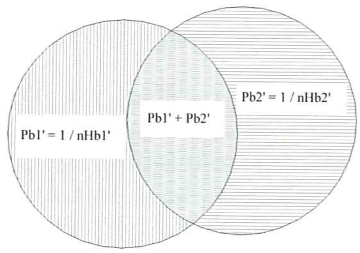

Figure 4. Estimate of the probability of households being served in polygons formed by intersecting two contractor buffers.

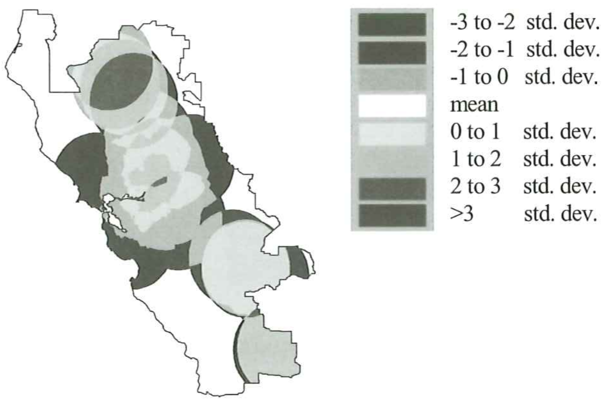

Figure 5. Maps the standard deviation of households served by the existing 53 contractors in the study.

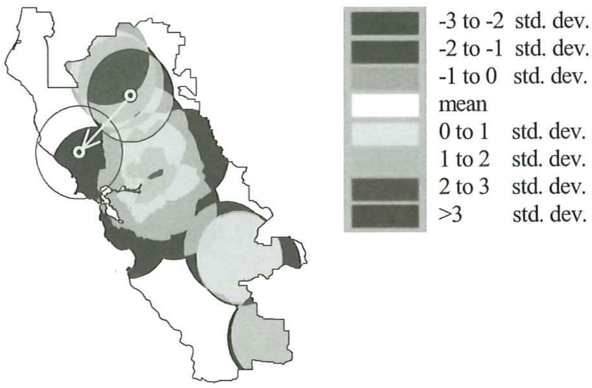

Figure 6. Substitution from the most over-served to the most under-served region.

yielding limitations such as its inability to always generate the same final solution. However, it is suggested that the value in these types of models lies in their ability to generate bench marks from which to compare solutions and provide input in the decision making process (Bailey and Gatrell 1995).

Our equity generation model is conceptually similar to a *substitution model* as we strive to balance service to demand over the entire study region. Our supply is polygon based with no evidence of distance impedance, so we substitute a region rather than a simple site in our heuristic. From the estimate of the probability of households being served we use standard deviations to threshold and map service. We observe from Figure 5 a considerable amount of variation, spatially mapped on the tails of the distribution. We seek to reduce this area by moving a supply polygon from the most over-served region to those areas most under served. We locate the spatial means (or centers of gravity) of the most over-served and most under-served regions. The supply polygon (a circle in this study) whose spatial mean is most closely aligned with the most over-served region is then moved to the spatial mean in the most under-served region as mapped in Figure 6. A new estimate of probability of households being served is generated and mapped. We continue the process through several iterations comparing the distribution's dispersion and variance.

V. RESULTS

We expect the *polygon-polygon* representation of our *substitution model* to yield less varied distribution patterns through iterations of the process. Kurtosis, a good metric for dispersion, moves from a *leptokurtic* (large tail) to a *platykurtic* (small tail) distribution over nine iterations of our model, seen in Figure 7. The large regions of over and under-served households in Figure 5 eventually give way to a peaked distribution where service is more evenly distributed and our expectation is reached. Skewness is symmetrical or non existent until the tenth iteration of the model.

Mapping the results in Figure 8 we can visually verify a marked change in the distribution of over and under-served regions. Here the dark red color represents the over-served regions at one end of a spectrum while dark blue polygons represent under-served households.

To determine whether this visible trend is significant we perform an analysis of variance (at a 95% confidence level) to test our null hypothesis:

H_0: $\mu_0 = \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7 = \mu_8 = \mu_9 = \mu_{10}$
where μ_i represents the mean of the distribution in iteration i.

From Table 1, since $F > F_{Critical}$, we reject H_0 and the mean values of the distributions resulting from the iterations of our model are significantly different. As the iterative process continues, we expect less variance among the service of households in different regions. We design an F-Test comparing the results of all the iterations with our original distribution, to determine whether the *substitution model* at least generates better results than no substitution at all. Also pair the current iteration of the model with the

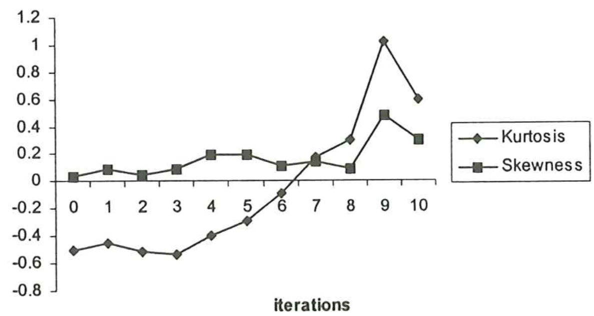

Figure 7. Graphs Kurtosis and Skewness over 10 iterations

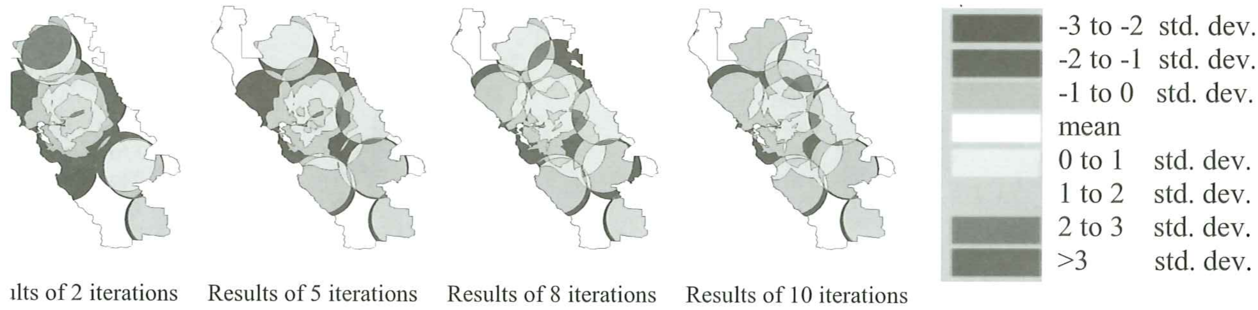

Figure 8. Observed change over iterations of the *substitution model*.

previous one to determine whether the variance continues to decrease.

H_0: $\sigma_i^2 = \sigma_j^2$ (i = 1,2,...10, and j = 0,1,2,...10)
H_1: $\sigma_i^2 > \sigma_j^2$ or $\sigma_i^2 < \sigma_j^2$
where, s^2 represents the variance of the distribution.

Once again, if $F > F_{Critical}$, we reject the H_0 and accept H_1. We expected σ_i^2 to always be less than σ_j^2, when $i > j$.

From Table 2 we observe:

- in most cases (10 out of 19), the variances become smaller (as expected);
- in seven instances no significant change is detected;
- in two out of 19 cases the variance is increasing in the opposite direction.

Overall, the *substitution model* appears to be working and improving the distribution of service to households.

Table 1. Results of ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.79E-09	10	1.78576E-10	5.454753782	3.97E-08	1.831296
Within Groups	5.22E-07	15931	3.27377E-11			
Total	5.23E-07	15941				

Table 2. Results of F-Tests comparing the results of all the iterations of our *substitution model*

Comparison	*F*	*F* Critical one-tail	H_1	Conclusion	Results with 95% confidence level
1 and 0	1.24	1.09	$\sigma_1^2 < \sigma_0^2$	reject H_0, and accept H_1	$\sigma_1^2 < \sigma_0^2$
2 and 0	1.18	1.09	$\sigma_2^2 < \sigma_0^2$	reject H_0, and accept H_1	$\sigma_2^2 < \sigma_0^2$
3 and 0	1.23	1.09	$\sigma_3^2 < \sigma_0^2$	reject H_0, and accept H_1	$\sigma_3^2 < \sigma_0^2$
4 and 0	1.02	1.09	$\sigma_4^2 < \sigma_0^2$	accept H_0	$\sigma_4^2 = \sigma_0^2$
5 and 0	1.12	1.09	$\sigma_5^2 < \sigma_0^2$	reject H_0, and accept H_1	$\sigma_5^2 < \sigma_0^2$
6 and 0	1.26	1.09	$\sigma_6^2 < \sigma_0^2$	reject H_0, and accept H_1	$\sigma_6^2 < \sigma_0^2$
7 and 0	1.19	1.09	$\sigma_7^2 < \sigma_0^2$	reject H_0, and accept H_1	$\sigma_7^2 < \sigma_0^2$
8 and 0	1.26	1.09	$\sigma_8^2 < \sigma_0^2$	reject H_0, and accept H_1	$\sigma_8^2 < \sigma_0^2$
9 and 0	1.07	1.09	$\sigma_9^2 < \sigma_0^2$	accept H_0	$\sigma_9^2 = \sigma_0^2$
10 and 0	1.17	1.09	$\sigma_{10}^2 < \sigma_0^2$	reject H_0, and accept H_1	$\sigma_{10}^2 < \sigma_0^2$
2 and 1	0.96	0.92	$\sigma_2^2 > \sigma_1^2$	reject H_0, and accept H_1	$\sigma_2^2 > \sigma_1^2$
3 and 2	1.04	1.09	$\sigma_3^2 < \sigma_2^2$	accept H_0	$\sigma_3^2 = \sigma_2^2$
4 and 3	0.83	0.92	$\sigma_4^2 > \sigma_3^2$	accept H_0	$\sigma_4^2 = \sigma_3^2$
5 and 4	1.10	1.09	$\sigma_5^2 < \sigma_4^2$	reject H_0, and accept H_1	$\sigma_5^2 < \sigma_4^2$
6 and 5	1.12	1.09	$\sigma_6^2 < \sigma_5^2$	reject H_0, and accept H_1	$\sigma_6^2 < \sigma_5^2$
7 and 6	0.94	0.92	$\sigma_7^2 > \sigma_6^2$	reject H_0, and accept H_1	$\sigma_7^2 > \sigma_6^2$
8 and 7	1.06	1.09	$\sigma_8^2 < \sigma_7^2$	accept H_0	$\sigma_8^2 = \sigma_7^2$
9 and 8	0.85	0.92	$\sigma_9^2 > \sigma_{10}^2$	accept H_0	$\sigma_9^2 = \sigma_{10}^2$
10 and 9	1.09	1.092	$\sigma_{10}^2 < \sigma_9^2$	accept H_0	$\sigma_{10}^2 = \sigma_9^2$

VI. CONCLUSIONS

The central objective of this study, to develop a method which measures access to social services for each household and makes adjustments among service providers to better accommodate under-served regions, is successfully accomplished. The results confirm and add further support for the creation of *polygon-polygon* representations of the location-allocation *substitution model*. The solution, although reaching an optimization is likely not possible, appears to improve and stabilize the allocation of supply to demand by the seventh iteration for this data set. For governments attempting to provide social services and insure equal opportunity for all citizens, especially where their spatial distribution is highly variable, allocation models such as this, embedded in a GIS, present a promising option.

VII. LIMITATIONS

A discussion of the uncertainty in the application of this model best begins with the constraints imposed by the data set. Although we generalize the process of generating the probability of households being served and adjusting contractor locations to level access to services, the application is heavily dependent on the delineation of service areas defined by the contractors. It is likely that the original disclosures regarding friction of distance, contractor capability and capacity to deliver services are over simplified and further investigation might produce evidence that would modify the service areas and weight the probability function for households. Although adding additional contractors would not alter the method, it would serve to alter the empirical outcome. Finally, regions not serviced by the original supply set were not consciously targeted for supply. More information on these regions would determine whether they should be targeted in the government rebate scheme. Although these constraints serve to increase uncertainty in the empirical study and globally optimal results may not be possible, the model as constructed here solves a problem that previously could not be solved, with results that are valuable to planner and decision makers.

ACKNOWLEDGMENTS

Robert Wirtshafter and Susan Lindell Radke provided support, comments and data. Vladimir Ulyashin provided comments and a program to automate the calculation of final households per service area.

REFERENCES

[1] Arentze, T.A., Borgers, A.W.J. and Timmerman, H.J.P., 1996, Integrating GIS into the Planning Process, in Fischer, M., Scholten, H., and Unwin, D.,(eds), *Spatial Analytical Perspectives on GIS*, Bristol, PA, Taylor & Francis Inc.

[2] Aronoff, S., 1989, *Geographic Information Systems: A Management Perspective,* WDL Publications, Ottawa, Canada.

[3] Aurenhammer, F. and Edelsbrunner, H., 1984, An optimal algorithm for constructing the Voronoi diagram in the plane, *Pattern Recognition*, 17:251-257.

[4] Bailey, T.C. and Gatrell, A.C., 1995, *Interactive Spatial Data Analysis,* Addison Wesley Longman, London, England.

[5] Beaumont, J.R., 1981, Location-allocation problems in the plane: a review of some models, *Socioeconomic Planning Sciences,* 15:217-29.

[6] Burrough, P.A., 1998, *Principles of Geographic Information Systems,* Oxford University Press, New York.

[7] Chrisman, N., 1997, *Exploring Geographic Information Systems,* John Wiley & Sons, New York.

[8] Church, R.L., 1999, Location Modelling and GIS, in P.A. Longley, M.F. Goodchild, D.J. Maguire and D.W. Rhind (eds.), *Geographical Information Systems: Principles, Techniques, Applications and Management*, 2nd Edition, John Wiley & Sons, New York.

[9] Church, R.L and Sorensen, P., 1994, *Integrating normative location models into GIS: problems and prospects with the p-median model,* Technical Report 94-5, National Center for Geographic Information and Analysis, University of California, Santa Barbara.

[10] Eiselt, H.A., 1992, Location modelling in practice, *American Journal of Mathematical and Management Sciences,* 12: 3-18.

[11] Longley, P.A., Goodchild, M.F., Maguire, D.J. and Rhind, D.W. (eds.), 1999, *Geographical Information Systems: Principles, Techniques, Applications and Management*, 2nd Edition, John Wiley & Sons, New York.

[12] Maguire, D.J., Goodchild, M.F. and Rhind, D.W. (eds.), 1991, *Geographic Information Systems: Principles and Application,* Harlow, Essex, England.

[13] Miller, H.J., 1996, GIS and geometric representation in facility location problems, *International Journal of Geographical Information System,* 10:791-816.

[14] Openshaw, S., 1977, A geographical solution to scale and aggregation problems in region-building, partitioning, and spatial modelling, *Transactions of the Institute of British Geographers,* 2:459-472.

[15] Openshaw, S., 1996, Developing GIS relevant zone based spatial analysis methods, in P.A. Longley and Batty, M. (eds.), *Spatial analysis: modelling in a GIS environment.* Cambridge, GeoInformation International, 55-73.

[16] Openshaw, S. and Openshaw, C., 1997, *Artificial Intelligence in Geography*, New York, NY: John Wiley & Sons.

[17] Openshaw, S. and Alvanides, S., 1998, Applying GeoComputation to the analysis of spatial

distributions, in P.A. Longley, Goodchild, M.F., Maguire, D.J. and Rhind, D.W. (eds.), *Geographical Information Systems: Principles, Techniques, Applications and Management*, 2nd Edition, John Wiley & Sons, New York.

[18] Okabe, A., Boots, B. and Sugihara, K., 1992, *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*, John Wiley & Sons, New York.

[19] PG&E, 1999, *California Economic Outlook And Key Issues - 1999 And Beyond*, (http://www.pge.com/customer_services/business/economic_outlook/#infrastructure), Pacific Gas and Electric, California.

[20] Rosing, K.E., Hillman, E.L. and Rosing-Vogelaar, H., 1979, A note comparing optimal and heuristic solutions to the p-median problem, *Geographical Analysis*, 11:86-89.

[21] Teitz, M.B. and Bart, P., 1968, Heuristic methods for estimating the generalized vertex median of a weighted graph, *Operations Research*, 16:953-61.

[22] Wheeler, J.O. and Muller, P.O., 1986, *Economic geography*, New York, John Wiley & Sons, New York.