Measuring Uncertainty of Spatial Features in a Three-Dimensional Geographical Information System Based upon Numerical Analysis

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Abstract

This paper defines an uncertainty description – a discrepancy that is a deviation of a measured value from its 'true' value – for three-dimensional (3D) vector data and proposes an uncertainty measure to quantify the uncertainty description. In an existing research issue, uncertain 3D spatial features was simulated and compared with the 'true' location of the spatial features in order to estimate an average value of a discrepancy of the spatial features. However, this time-consuming method cannot provide a precise and highly accurate solution. This study further proposes the development of a numerical uncertainty model for 3D spatial features. The expected value of the discrepancy is expressed as a multiple integral and solved by a numerical integration technique. Some preliminary test results from experimental data are summarized and compared with those from the earlier simulation model.

I. INTRODUCTION

A geographical information system (GIS) is a software package for inputting, storing, analyzing, retrieving and transforming geographical data (Cassettari [5]). It is now widely applied in different areas including military applications, environmental studies and geological exploration (Burrough [2], Laurini and Thompson [9]). Unfortunately, geographical data stored in GISs contain errors (Heuvelink [8]). Consequently, the development of a GIS in a market is depending on how to handle uncertainty in GIS.

Different uncertainty models have been developed for describing uncertainty of linear features in two-dimensional (2D) GIS: (a) error-band models (Caspary and Scheuring [4], Dutton [6]), (b) confidence region models (Shi [11], Shi and Liu [15]) and (c) reliability models (Stanfel and Stanfel [16, 17], Easa [7]). However, few research issues discuss uncertainty modeling for higher dimensional spatial features. Shi [12, 13] derived a confidence region model for 3D and N-dimensional linear features based upon statistical theory. Later on, a reliability model for simple 3D spatial features (including linear features, area features and volumetric features) was developed using a simulation technique (Shi and Cheung [14]). A weakness of the simulation approach is time-consuming. Hence, this study further proposes a development of a numerical uncertainty model in order to describe uncertainty of 3D spatial features quickly.

In this paper, we develop a numerical uncertainty model for a 3D spatial feature in a vector-based GIS. Here, uncertainty of a 3D spatial feature is measured by a discrepancy, which is a deviation of the measured location of the spatial feature away from the 'true' location. An analytical quantity for the discrepancy with uncertainty is expressed as a multiple integral in this study. Theoretically, the exact solution of the multiple integral should be derived automatically in GIS given the measured location and the 'true' location of the spatial feature; then GIS users will be aware of the uncertainty of the spatial feature from the derived analytical quantity. A research problem in this analytical model is how to solve the multiple integral for the analytical quantity, which is unable to be solved analytically. This paper thus presents a numerical integration method to solve the multiple integral to describe uncertainty of 3D spatial features in GIS.

II. SOURCES OF UNCERTAINTY IN GIS

There are different uncertainties in GISs (Burrough and McDonnell [3]) that are classified into intrinsic uncertainty, contextual uncertainty and processing uncertainty in this study (see Figure 1). Intrinsic uncertainty denotes that raw data are of their own uncertainties. It may be caused by natural spatial variation because vector data are not capable of representing a continuous change from one class to another; hence a crisp boundary is usually defined in vector data to separate classes in the natural spatial variation. Furthermore, measurement uncertainty arises in a process of data capture by ground survey, photogrammetric of remote sensing survey, map digitizing or scanning, or others. The measurement uncertainty is of three types: mistakes, systematic errors and random errors. Mistakes may be introduced through failure in an automatic recording technique, failure of the equipment (such as reading the fraction on a tape of the wrong side of the zero mark), or mistakes made by data collector (such as misidentifying an object of intersect or taking the wrong reading of a scale). Systematic errors are caused by imperfections in instrument construction or adjustment, or changing conditions in the surrounding environment. Random errors occur after removing the mistakes and the systematic errors. Another kind of the intrinsic uncertainty is model uncertainty due to generaliza-

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tion. Model uncertainty exists during the transfer from both the measurement of reality to that in the digital database and the paper map to the digital database. It refers to the deviation between the 'true' function representing a shape of a spatial object and its approximate linear function in the database. Their existence is due to inherently complex of spatial objects and model generalization. In addition to model generalization, the intrinsic uncertainty is affected by cartographic generalization that is a process by which the presence of spatial objects on a map is reduced or modified in terms of their size, shape or numbers for display purposes. Existing paper maps are originally generalized cartographically for display purposes and their data have generalization effects embedded in them, however there is no information about their quality on the paper maps. Also, spatial databases from the paper maps introduce cartographic generalization effect in a GIS in order to improve the display quality of a map at a scale smaller than the one it was compiled from. Consequently, the cartographic generalization in GIS can potentially alter the topology of spatial objects undergoing unintended transformations of the spatial objects by shifting and distorting the spatial objects. Spatial data in GIS may contain different kinds of the intrinsic uncertainty.

Contextual uncertainty emphasizes the requirement that uncertainty of GIS data must be considered within the context of the task at hand. It includes age of data, map scale and resolution, and density of observations. Existing data in GIS may be out-dated and even be unsuitable to associate with other data collected at different time to complete a certain application. This leads to a fact that age of data is an important consideration in performing spatial analyses in GIS. The second requirement we should consider to complete the application is map scale and resolution. Many survey organizations provided geographical data in a form of paper maps or digital maps at a range of scales. A larger scale map is more completeness than a smaller scale one. Also, the representation of a larger scale map is more accurate than that of a smaller scale map (Quattrochi and Goodchild [10]). However, the large-scale map contains much information unrelated to the application and performing analyses on these maps in GIS increases computer-processing time to some extents. Thus, which scale map should be used depends on which is most appropriate to the application in hand. Furthermore, a spatial sample is a common way to investigate a continuous spatial pattern (such as temperature or soil type) in a study area in a form of a sample spatial distribution. Observations at a finite number of point samples that are believed to be representative of a distribution of the spatial pattern are made. Hence, the density of the point samples should be chosen to resolve the spatial pattern of interest. For an identical set of GIS data, we may accept the contextual uncertainty for a certain spatial analysis but reject for another spatial analysis. Thus, the contextual uncertainty plays an important role in data quality for the derived result from spatial analyses.

Processing uncertainty refers to those uncertainties of raw data propagated through a GIS process and those uncertainties arising from the process. When raw data is collected and stored in GIS, there exists the instinct uncertainty: the uncertainty due to natural spatial variation, the measurement uncertainty and the model uncertainty. During analyzing the raw data for decision-making, the intrinsic uncertainty would propagate through spatial processes (such as spatial queries, buffer analysis, overlay operations and others). In other words, the intrinsic uncertainty of raw data will be transferred to the resulted data via GIS operations. Uncertainties arising from GIS processes also exist. For example, we derive a mathematical model to describe a particular GIS process. A deviation between the true process and the mathematical model we derive for the process may occur. This deviation is secondary uncertainties in GIS. Since a mathematical model for an error propagation problem for a particular spatial analysis is derived according to an empirical relationship among different source data, it is

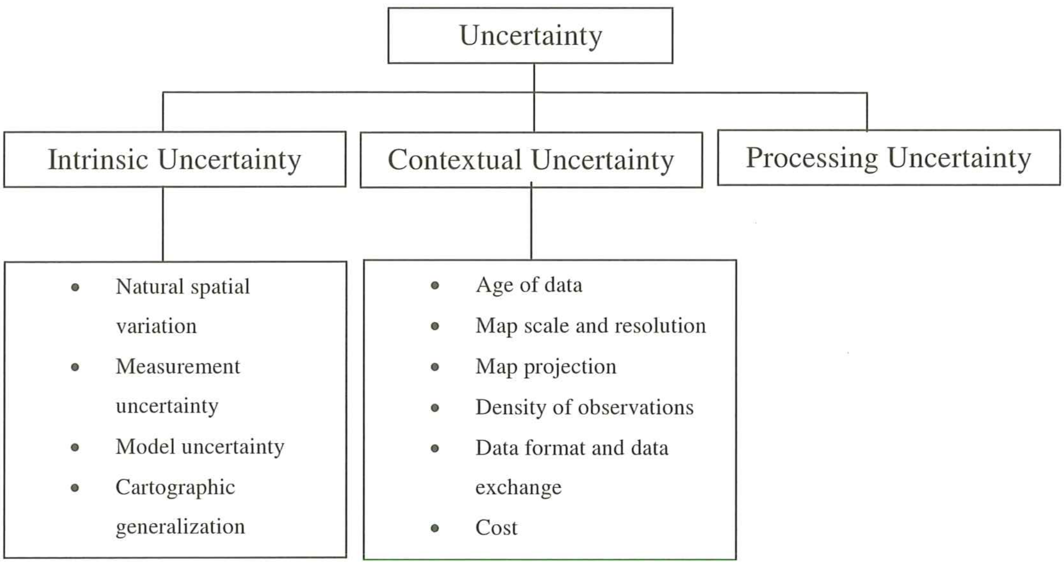

Figure 1. Types of uncertainties in GIS

difficult to provide a general model for all GIS analyses. Instead, each GIS analysis has its own error propagation model to describe the propagation of the intrinsic uncertainty and the secondary uncertainty.

Among different kinds of uncertainty, the measurement uncertainty commonly exists in GIS databases. In this paper, we study the measurement uncertainty of 3D spatial features. A discrepancy is defined as a measure of the uncertainty for different kinds of the 3D spatial features and elaborated in the next session.

III. MEASUREMENT UNCERTAINTY MODEL

Uncertainty of 3D spatial features is measured by a discrepancy, which is the deviation of the measured location of the spatial features away from the 'true' one. However, the 'true' location of the spatial features is not known. From a statistical point of view, a mean of any variable X is close to its actual value. Thus, the 'true' locations of the nodes of a spatial feature refer to their associated mean locations in this paper.

We will consider the discrepancy for different 3D spatial features including a linear feature, an areal feature and a volumetric feature.

Discrepancy of a linear feature

For a line segment of two nodes, its discrepancy is defined as the region bounded by the measured location of the line segment and its 'true' (or expected) location. This region is shaded in Figure 2.

In Figure 2, the solid line segment is the expected location of the line segment of expected nodes $(\mu_{x_1}, \mu_{y_1}, \mu_{z_1})$ and $(\mu_{x_2}, \mu_{y_2}, \mu_{z_2})$ while the dashed line segment is the corresponding measured location of measured nodes (x_1, y_1, z_1) and (x_2, y_2, z_2). Here, there is no intersection between the measured and the expected locations of the line segment. Figure 3 shows a case where the measured and the expected locations of the line segment intersect at a point (x_{12}, y_{12}, z_{12}).

An area of the discrepancy in a generic case is determined differently depending on the relationship between the measured nodes and the expected nodes of the line segment. There are three possible cases: (a) the measured locations of the nodes and their associated expected locations are on a flat plane but the measured and the expected locations of the line segment do not meet; (b) the measured and the expected locations of the line segment intersect with each other; and (c) neither case (a) or case (b) is a possibility. Figure 2 shows an example of case (a) or (c) while Figure 3 shows that of case (b).

In case (a), an area of the discrepancy (such as the shaded region of Figure 2 when the measured locations and the expected locations of the nodes of the line segment are on a flat plate) can be denoted as *area_quad*:

area_quad = 0.5 * (the magnitude of (***A*** x ***B***) + the magnitude of (***C*** x ***D***)) (1)

where, $\boldsymbol{A} = (x_1 - \mu_{x_1}, y_1 - \mu_{y_1}, z_1 - \mu_{z_1})$,

$\boldsymbol{B} = (\mu_{x_2} - \mu_{x_1}, \mu_{y_2} - \mu_{y_1}, \mu_{z_2} - \mu_{z_1})$,

$\boldsymbol{C} = (x_1 - \mu_{x_2}, y_1 - \mu_{y_2}, z_1 - \mu_{z_2})$,

$\boldsymbol{D} = (x_2 - \mu_{x_2}, y_2 - \mu_{y_2}, z_2 - \mu_{z_2})$, and

A x ***B*** and ***C*** x ***D*** are vector products of ***A*** and ***B***, as well as ***C*** and ***D*** respectively.

In case (b), an area of the discrepancy – the shaded area of Figure 3 – is denoted as *area_triangle* and given by

area_triangle = 0.5 * (the magnitude of (***A'*** x ***B'***) + the magnitude of (***C'*** x ***D'***)) (2)

where $\boldsymbol{A'} = (x_1 - \mu_{x_1}, y_1 - \mu_{y_1}, z_1 - \mu_{z_1})$,

$\boldsymbol{B'} = (x_{12} - \mu_{x_1}, y_{12} - \mu_{y_1}, z_{12} - \mu_{z_1})$,

$\boldsymbol{C'} = (x_{12} - \mu_{x_2}, y_{12} - \mu_{y_2}, z_{12} - \mu_{z_2})$, and

$\boldsymbol{D'} = (x_2 - \mu_{x_2}, y_2 - \mu_{y_2}, z_2 - \mu_{z_2})$.

It is also possible that both the measured and the expected

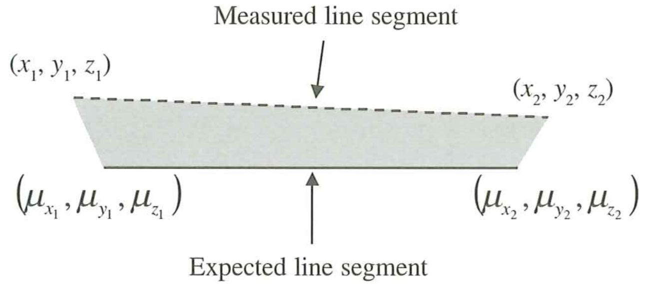

Figure 2. The discrepancy of a line segment in a case where the measured location of the line segment does not intersect with the associated expected location

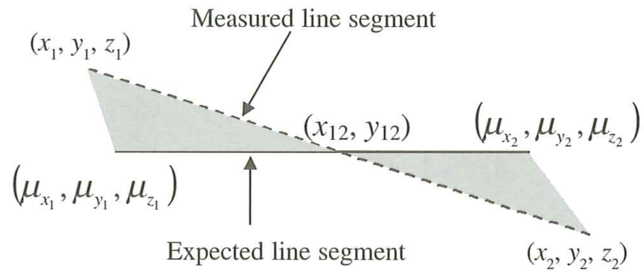

Figure 3. The discrepancy of a line segment in a case where the measured location of the line segment and the associated expected location intersect with each other

locations of the nodes are neither on a flat plane nor intersect (case c). Under these circumstances, an area of the discrepancy is unable to obtain exactly since we cannot define a plane containing the measured and expected nodes of the line segment without any further information about that plane. To simplify and quantify such a case, the approximate area of the shaded region in Figure 2 is described by Equation (1). Given $\mu_{x_1}, \mu_{y_1}, \mu_{z_1}, \mu_{x_2}, \mu_{y_2}$ and μ_{z_2}, *area_quad* and *area_triangle* become a function of x_1, y_1, z_1, x_2, y_2 and z_2. Then, the expected discrepant area *E(discrepancy)* of the uncertain line segment is computed as per Equation (3).

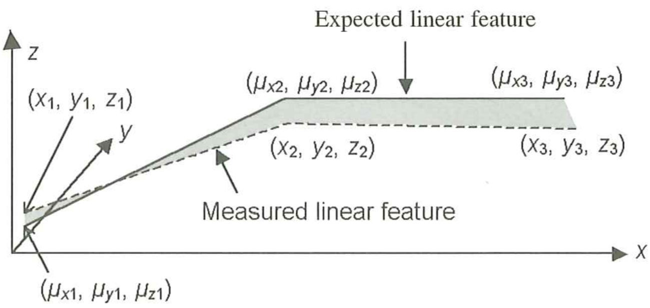

Figure 4. The discrepancy of a linear feature of three nodes

$$E(discrepancy) = \int_{R_1} f(x_1, y_1, z_1, x_2, y_2, z_2) \times area\_quad(x_1, y_1, z_1, x_2, y_2, z_2)\, dz_2 dy_2 dx_2 dz_1 dy_1 dx_1$$
$$+ \int_{R_2} f(x_1, y_1, z_1, x_2, y_2, z_2) \times area\_triangle(x_1, y_1, z_1, x_2, y_2, z_2)\, dz_2 dy_2 dx_2 dz_1 dy_1 dx_1 \qquad (3)$$

where, R_1 is an integral region so that cases (a) and (c) occur,
R_2 is an integral region for case (b), and
$f(x_1, y_1, z_1, x_2, y_2, z_2)$ is a multivariate probability density function of x_1, y_1, z_1, x_2, y_2 and z_2.

If $f(x_1, y_1, z_1, x_2, y_2, z_2)$ is multivariate normal distributed, its mathematical expression will become
$f(x_1, y_1, z_1, x_2, y_2, z_2, \ldots, x_N, y_N, z_N) =$

$$\frac{1}{(2\pi)^{N/2} |\Sigma|^{1/2}} \exp\left[\frac{-1}{2}(X - \mu_X)^T \Sigma^{-1} (X - \mu_X)\right] \qquad (4)$$

where, N is the total number of the expected nodes that is equal to two in a case of a line segment, Σ is the covariance matrix of x_1, y_1, z_1, ..., x_N, y_N, z_N in the form of

$$\begin{bmatrix} \sigma_{x_1x_1} & \sigma_{y_1x_1} & \sigma_{z_1x_1} & & \sigma_{x_Nx_1} & \sigma_{y_Nx_1} & \sigma_{z_Nx_1} \\ \sigma_{x_1y_1} & \sigma_{y_1y_1} & \sigma_{z_1y_1} & & \sigma_{x_Ny_1} & \sigma_{y_Ny_1} & \sigma_{z_Ny_1} \\ \sigma_{x_1z_1} & \sigma_{y_1z_1} & \sigma_{z_1z_1} & & \sigma_{x_Nz_1} & \sigma_{y_Nz_1} & \sigma_{z_Nz_1} \\ & & & & & & \\ \sigma_{x_1x_N} & \sigma_{y_1x_N} & \sigma_{z_1x_N} & & \sigma_{x_Nx_N} & \sigma_{y_Nx_N} & \sigma_{z_Nx_N} \\ \sigma_{x_1y_N} & \sigma_{y_1y_N} & \sigma_{z_1y_N} & & \sigma_{x_Ny_N} & \sigma_{y_Ny_N} & \sigma_{z_Ny_N} \\ \sigma_{x_1z_N} & \sigma_{y_1z_N} & \sigma_{z_1z_N} & & \sigma_{x_Nz_N} & \sigma_{y_Nz_N} & \sigma_{z_Nz_N} \end{bmatrix},$$

$X^T = [x_1, y_1, z_1, \ldots, x_N, y_N, z_N]$, and

$$\bar{X}^T = [\mu_{x_1}, \mu_{y_1}, \mu_{z_1}, \ldots, \mu_{x_N}, \mu_{y_N}, \mu_{z_N}].$$

Joining several line segments acyclically yields a polyline, which is a broad linear feature. Uncertainty of the polyline is measured by a discrepancy of the measured location of the polyline away from the corresponding expected location. Figure 4 shows an instance for the discrepancy of a linear feature of three nodes.

Let $area(x_1, y_1, z_1, \ldots, x_N, y_N, z_N)$ denote a function used to estimate the discrepant area of a linear feature of N nodes. Then, the expected discrepant area is presented in Equation (5) (see bottom).

Discrepancy of an areal feature

Uncertainty of an areal feature (or polygon) can be appraised by the discrepancy of the measured location of the areal feature away from the associated expected location. However, it is distinct from the discrepancy of a linear feature. According to the definition of the discrepancy of a linear feature, the discrepancy of the boundary of an areal feature should be the surface area bounded by the measured and the expected locations of the boundary. On the other hand, the discrepancy of the areal feature refers to a volume of the region bounded by the measured and the expected locations of the areal feature. Figures 5 and 6 illustrate this difference in conformity with respect to surface area and volume.

Figure 5 shows a case where the discrepancy of the boundary of an areal feature based on surface area and Figure 6 a case where the discrepancy of the areal feature based on volume. The solid and the dashed lines are the boundary of the expected areal feature and of the measured areal feature respectively. Both an area of the shaded region of Figure 5 and a volume of the shaded region of Figure 6 represent the discrepancy related to the areal feature. Since discrepancy is the difference between reality and users' representation of reality, using the volume of the shaded region to describe the discrepancy of the areal feature is satisfactory.

$$E(discrepancy) = \int f(x_1, y_1, z_1, \ldots, x_N, y_N, z_N) \times area(x_1, y_1, z_1, \ldots, x_N, y_N, z_N)\, dz_N dy_N dx_N \cdots dz_1 dy_1 dx_1 \quad (5)$$

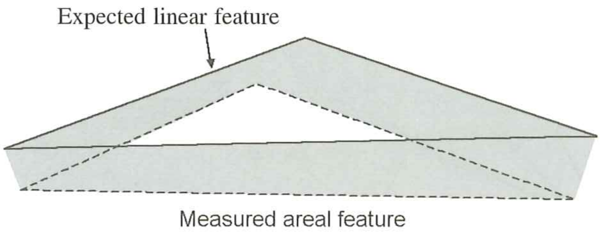

Figure 5. The discrepancy of the boundary of an areal feature of three nodes defined by surface area

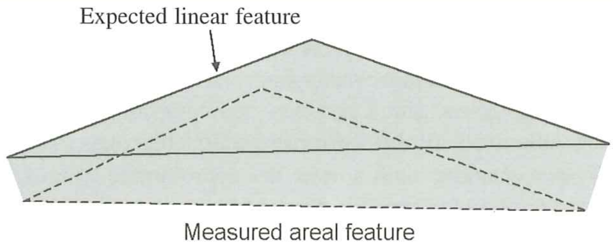

Figure 6. The discrepancy of an areal feature of three nodes defined by volume

Let $volume(x_1, y_1, z_1, \ldots, x_N, y_N, z_N)$ denote a function used to estimate the discrepant volume of an uncertain areal feature of N nodes nodes $(x_1, y_1, z_1), \ldots, (x_N, y_N, z_N)$. The expected discrepant volume of the areal feature can be given as Equation (6) (see bottom).

Discrepancy of a volumetric feature

In a 3D GIS, another important unit is volumetric features. The difference between the measured location and the expected location of a volumetric feature is a measure of the uncertainty of the volumetric feature. This discrepancy can be viewed as a union of surfaces' discrepancies. For instance, a volumetric feature contains five nodes and therefore five surfaces. For each surface, its corresponding discrepancy is computed. The discrepancy of the volumetric feature is computed by the union of all of the surfaces' discrepancies and is shaded in Figure 7.

Let $u\_volume(x_1, y_1, z_1, \ldots, x_N, y_N, z_N)$ denote a function used to estimate the discrepant volume of an uncertain volumetric feature of N nodes $(x_1, y_1, z_1), \ldots, (x_N, y_N, z_N)$. The expected discrepant volume of the volumetric feature is illustrated in Equation (7) (see bottom).

IV. NUMERICAL INTEGRATION METHOD

The analytical expression of the discrepancy for spatial features has been derived as multiple integrals, however it is difficult to get an exact solution of the multiple integrals, which is too complex, using a traditional integration technique. Gaussian quadrature – a numerical integration technique – is adopted to approximate the multiple integrals by integrating the linear function that joins points of the function's graph (Burden and Faires [1]). For example, an integral $\int g(x)dx$ is approximated by $\sum_i w_i\, g(x_i)$ where the nodes $x_1, x_2, \ldots, x_n$ and coefficients $w_1, w_2, \ldots, w_n$ are chosen to minimize the expected error between the integral and the approximation. This technique can be modified in a straightforward manner for use in the approximation of multiple integrals. Hence, this Gaussian quadrature is implemented to calculate the multiple integrals in Equations (3), (5), (6) and (7).

Results and discussions

The analytical model for the uncertainty of 3D spatial features is applied to the example data of Shi and Cheung [14]. For a line segment, expected locations of its two nodes are assigned to be (0, 0, 0) and (1000, 0, 0) and the covariance matrix of x_1, y_1, z_1, x_2, y_2 and z_2 is

$$\begin{bmatrix} 61.993^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 121.506^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 91.749^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 18.598^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 48.354^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 55.794^2 \end{bmatrix}$$

The expected value of the discrepant area of this line segment is 62145.0.For a linear feature of three expected nodes (0, 0,

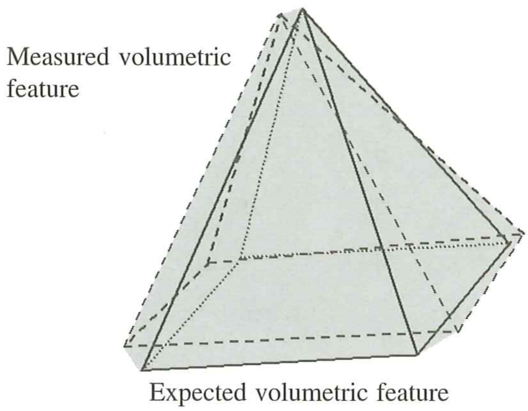

Figure 7. The discrepancy of a volumetric feature of five nodes

$$E(discrepancy) = \int f(x_1, y_1, z_1, \ldots, x_N, y_N, z_N) \times volume(x_1, y_1, z_1, \ldots, x_N, y_N, z_N)\, dz_N dy_N dx_N \cdots dz_1 dy_1 dx_1 \quad (6)$$

$$E(discrepancy) = \int f(x_1, y_1, z_1, \ldots, x_N, y_N, z_N) \times u\_volume(x_1, y_1, z_1, \ldots, x_N, y_N, z_N)\, dz_N dy_N dx_N \cdots dz_1 dy_1 dx_1 \quad (7)$$

0), (500, 500, 707.1) and (1500, 500, 707.1), it is considered that the covariance matrix is diagonal and the entities of the main diagonal are 61.993^2, 121.506^2, 91.749^2, 18.598^2, 48.354^2, 55.794^2, 61.993^2, 121.506^2 and 91.749^2. The expected value of the discrepant area of the linear feature is 85553.6. Using the three expected nodes of the linear feature for the areal feature results in an expected discrepant volume equal to 15209871.4. For the example of a volumetric feature, an additional node (to the existing three) is considered that is $(x_4, y_4, z_4) = (500, 707.1, 500)$. Variances of x_4, y_4 and z_4 are 18.598^2, 48.354^2 and 55.794^2 respectively. This addition now specifies a volumetric feature. The expected discrepant volume of the volumetric feature is 37688986.2.

Since the accuracy of the numerical result and the simulated result are uncertain, their results are compared to check their accuracy. In Table 1, the expected values of the discrepancy of the spatial features are listed for both the numerical integration and the simulation techniques. The ratio of the results from the numerical model to that from the simulation model is in the range of 0.819 to 1.047. In an ideal situation, this ratio should be one. A ratio varying from one is due to the approximation of the expected discrepancy for both techniques (numerical integration and simulation techniques).

In the simulation model, the accuracy of the result depends on the number of simulation. The larger the number of simulation, the higher the precision of the result. On the other hand, the accuracy of the approximation in the numerical model is related to the number of nodes chosen in the integral region. The oscillatory nature of the integral function affects the approximation. Here, it is impossible to determine which method provides a highly accurate result, since the true expected discrepancy is unknown. And the numerical and the simulated results are not identical. However, both of them are close to each other. Therefore, the error indicator (discrepancy) of the spatial features reflects the uncertainty to a certain extent.

Moreover, it is difficult to say which method dominates. The calculation for the numerical solution is much faster than that for the simulated solution for a line segment or linear feature of few nodes. For a 3D spatial feature of more nodes, the integral function becomes more complex so that much computation time is required to solve the analytical expression of the discrepancy. The simulation technique is our choice to estimate the expected discrepancy in this case. That is, which method we should use to estimate the discrepancy of a 3D spatial feature depends on the number of nodes of the spatial feature.

Table 1. The expected discrepant area of spatial features

Spatial feature	The expected discrepancy		Ratio =
	Numerical result	Simulated result	analytical result / simulated result
Line segment	62145	59344.1	1.047
Linear	85553.6	89036.1	0.961
Areal	15209871	18570274	0.819
Volumetric	37688986	44339984	0.849

The above four examples considered the discrepancy of the spatial features in a case where there is no correlation of nodal errors. This numerical model can also be implemented to estimate the discrepancy in a generic case where the covariance matrix of Equation (4) is non-diagonal.

V. CONCLUSIONS

A newly developed numerical model to measure the uncertainty of a spatial feature in 3D GIS was presented in this paper. The uncertainty is measured by the discrepancy of the spatial feature based on the 'true' location and the measured location of the spatial feature. The discrepancy was expressed as a mathematical function of which the measured location and the 'true' location of the spatial feature were variables. Given the measured location of the spatial feature, the discrepancy could be obtained. In general, the measured location may be in the vicinity of the 'true' location. Based on the assumption of error of a spatial feature, a number of possible measured locations were considered in our proposed model rather than one measured location of the spatial feature. Therefore, the expected discrepancy was in the form of a multiple integral. Since this multiple integral could be solved analytically, the Gaussian quadrature, a numerical integration, was implemented to provide an approximate solution for the analytical model. The estimated expected discrepancy was finally compared to the simulated solution.

In the previous uncertainty model for 3D spatial features, the uncertainty model of 3D spatial features was studied using the simulation technique. This simulation model was generated some possible measured locations of a spatial feature based on the same assumption of error of the spatial feature as stated in this paper, and computed the expected discrepancy. The simulation model only sampled a certain number of the possible measured locations of the spatial feature instead of all possible measured locations of the spatial feature. The accuracy of the expected discrepancy is questioned although more simulations can provide a more precise result. Moreover, the simulation model is quite time-consuming. Thus, we proposed the analytical model by taking all possible measured locations of the spatial feature into account, in order to provide the expected discrepancy with great accuracy in real time.

In this paper, an analytical model was provided to validate the simulation model. The numerical results obtained from the analytical model and the simulated results given in the previous study can approximate a similar value of the discrepancy. We can determine which method (numerical or simulation technique) of estimating the expected discrepancy depending on the number of nodes of a 3D spatial feature. The numerical integration technique derived in this study is considered the preferred approach in studying the uncertainty of 3D spatial features of few nodes (such as a line segment).

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