À Trous Wavelet Decomposition Applied to Image Edge Detection

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Abstract

Detecting image edge is considered as a key step in many complicated processing methods such as image segmentation, image recognition, and feature extraction. Many methods of detecting image edges have been developed, but almost every method has its restriction in application of image processing. In this paper, disadvantages and advantages of some classic methods for image edge detection are thoroughly discussed. Base on the analysis, a new À trous wavelet decomposition algorithm is applied to detecting image edge. From the experimental results, we can find that the edges detected by our À trous wavelet decomposition method are better than those processed by the classic Sobel, and Robert methods. In addition, when the original image is stained by noise, the new method almost is not disturbed, on the contrary, the classic algorithms are sensitive to noise. However, besides the advantages of the new method of detecting image edge, we also find out a shortcoming, the disadvantage need to further research for improving the result.

I. INTRODUCTION

Image edge can be defined as the difference of image features in a local region [2], [3]. Its appearance is the mutation of image gray or texture structure or color. Image edge is very important to both human being and machine vision, because they can describe the shape of a region, define local feature and transform most information of an image. Detecting image edges is considered as a key step in many complicated processing methods like image segmentation, image recognition and feature extraction [2], [3]. There are some classic operators which are used to detect image edges, like gradient, Laplace, LOG, Sobel, Prewitt and Robert operators etc. Gradient operator is like a high pass filter, it only can sharp image edges. Some experiments have proved that the methods based on difference are not effective to detect complicated image edges [5]; The Sobel method is a weighted average operator, it contributes weight to the center pixel in order to enhance the edges; Robert operator is sensitive to noise, so it is seldom used to detect image edges in a dense points region; The Laplace operator is invariable in different directions, that means you can get the same detected edges when you rotate the operator. The Laplace operator response not only to image edges but also to corner points, end points of a line and isolated points. To restrain noise the LOG operator smoothes an image first, then performs differential. Though the image noise is partly restrained and the detected result is influenced [5]. From above discussion, we can find the operators which are relate to direction are not effective to detecting edges when an image are complicated and there are abundant edges in the image. In addition when there is noise in an image the detected result is not perfect if the operators are directly used. So in this paper, we introduce a method based on wavelet decomposition to detect image edge and hope to get a better result.

In this paper, we present a new approach to effectively detect image edges. So that, we have more selective methods in applications

The remainder of this paper is organized as follows. Section 2 presents the theory of wavelet decomposition. The new approach based on À trous wavelet decomposition to image edge detection is presented in Section 3. Section 4 discusses the experiments and concluding remarks are offered in the section.

II. THE À TROUS WAVELET DECOMPOSITION

Wavelet decomposition [1]

Wavelet analysis has been successfully used in image processing field . Wavelet transform is a new method which can decompose an image into different resolution images .

If we suppose function $\psi(x) \in L^2(R)$, $L^2(R)$ is quadratically integrable space),and $\psi(x)$ satisfies:

$$C_\psi = 2\pi \int_{-\infty}^{+\infty} \frac{\left|\overline{\psi}(\omega)\right|^2}{|\omega|} d\omega < +\infty \quad (1)$$

or

$$\int_{-\infty}^{+\infty} \psi(x) dx = 0 \quad (2)$$

in which $\psi(\omega)$ is the Fourier transform result of $\psi(x)$.

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When $\psi(x)$ satisfies equation(1) or (2) and can rapidly converge, we call $\psi(x)$ basic wavelet.

$\psi(x)$ flexes a and shifts b then we have:

$$\psi_{a,b}(x)=|a|^{-\frac{1}{2}}\psi\left(\frac{x-b}{a}\right) \tag{3}$$

in which: $a, b\in R,\quad a\neq 0$ and $\psi_{a,b}(x)$ is called wavelet.

Wavelet transform of distribution function f(x) can be defined as following:

$$wf(a,b)=\int_{-\infty}^{+\infty} f(x)\cdot|a|^{\frac{1}{2}}\overline{\psi}\left[a^{-1}(x-b)\right]dx \tag{4}$$

in which f(x)$\in$ L2(R), b$\in$ R, $\overline{\psi}$ (x) is complex conjugate of $\psi(x)$.

The À Trous wavelet decomposition method [1], [4]

Continuous wavelet transform definition is introduced above. It can't be used in computation by program. To do it by computer, we must make the continuous wavelet transform discrete. There are many methods to perform discrete wavelet transform, such as pyramidal algorithm [8], [9] which make use of orthogonal basis to decompose an image (or a signal), but the dimension of the result image is changed, that is not advantageous in some applications like pattern recognition and image fusion etc.

To get the result image which is the same dimension as the original one, we adopt the algorithm: à trous algorithm. The discrete approach of the wavelet transform can be done with the special version of the so-called à trous algorithm (with holes). The algorithm can decompose an image (or a signal) into an approximate signal and a detail signal at a scale, the detail signal is called a wavelet plane, which is same as the original image in dimension.

One assumes that the sampled image data $C_0(k)$ are the scalar products at pixels k of the function f(x) with a scaling

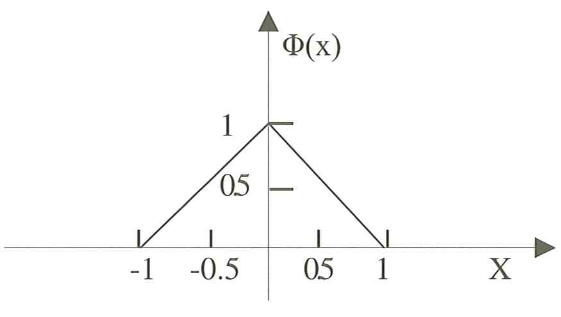

Figure 1. Linear interpolation

function ϕ (x) which corresponds to a low pass filter. The first filtering is then performed by a twice magnified scale leading to the $C_1(k)$ set. The image difference $C_0(k)-C_1(k)$ which is called first wavelet plane contains the information between those two scales and is the discrete set associated with the wavelet transform corresponding to ϕ (x). The associated wavelet is therefore $\psi(x)$.

$$\frac{1}{2}\psi\left(\frac{x}{2}\right)=\phi(x)-\frac{1}{2}\phi\left(\frac{x}{2}\right) \tag{5}$$

The distance between samples increasing by a factor 2 from the scale (i-1) (i>0) to the next one, $C_i(x)$ is given by

$$C_i(x)=\sum_l h(l)\cdot C_{i-1}\left(x+2^{i-1}l\right) \tag{6}$$

and the discrete wavelet transform $w_i(x)$ by:

$$w_i(x)=C_{i-1}(x)-C_i(x) \tag{7}$$

in which, $w_i(x)$ is the wavelet coefficient and $C_i(x)$ is approximate signal at the i scale, h(l) is a low pass filter.

The coefficients h(l) derive from the scaling function ϕ (x):

$$\frac{1}{2}\phi\left(\frac{x}{2}\right)=\sum_l h(l)\cdot\phi(x-l) \tag{8}$$

The algorithm allowing one to rebuild the data frame is evident: the last smoothed array C_{np} is added to all the differences w_i:

$$C_0(x)=C_{np}(x)+\sum_{j=1}^{np} w_j(x) \tag{9}$$

If we choose the linear interpolation for the şcaling function ϕ (x) (see Figure 1)

$$\phi(x)=1-|x| \qquad if \qquad x\in[-1,1]$$
$$\phi(x)=0 \qquad if \qquad x\notin[-1,1]$$

By computation, we can get h(-1) = 1/4, h(0) = 1/2, h(1) = 1/4. So we have:

$$\frac{1}{2}\phi\left(\frac{x}{2}\right)=\frac{1}{4}\phi(x+1)+\frac{1}{2}\phi(x)+\frac{1}{4}\phi(x-1)$$

$$C_{i+1}(x)=\frac{1}{4}C_i\left(x-2^i\right)+\frac{1}{2}C_i(x)+\frac{1}{4}C_i\left(x+2^i\right)$$

The Figure 2 shows the wavelet associated to the scaling function.

The above à trous algorithm is easily extensible to the two dimensions space. This leads to convolution with a mask of 3×3 pixels for the wavelet connected to linear interpolation. The coefficients of the mask are:

$$\begin{bmatrix} \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \end{bmatrix}$$

At each scale j, we obtain a set of $w_i(x)$ (we also call it wavelet plan) which has the same number of pixels as the image. If we choose a B_3-spline for the scaling function, the coefficients of convolution mask in one dimension are (1/16,1/4,3/8,1/4,1/16), and in two dimensions:

$$\begin{bmatrix} \frac{1}{256} & \frac{1}{64} & \frac{3}{128} & \frac{1}{64} & \frac{1}{256} \\ \frac{1}{64} & \frac{1}{16} & \frac{3}{32} & \frac{1}{16} & \frac{1}{64} \\ \frac{3}{128} & \frac{3}{32} & \frac{9}{64} & \frac{3}{32} & \frac{3}{128} \\ \frac{1}{64} & \frac{1}{16} & \frac{3}{32} & \frac{1}{16} & \frac{1}{64} \\ \frac{1}{256} & \frac{1}{64} & \frac{3}{128} & \frac{1}{64} & \frac{1}{256} \end{bmatrix}$$

III. DETECTING EDGES USING THE ÀTROUS ALGORITHM

From above analysis, we can find that an image can be decompose into several wavelet planes at different scales, some high frequency information is included in them. So we can detect edges of a remote sensing image using the following method:

a) Initialize i = 0, and input an original image $f_i(x,y)$;
b) Convolute the image with the low pass filter h(x,y): $f_{i+1}(x,y) = f_i(x,y) * h(x,y)$
c) Get a wavelet plan: $W_{i+1}(x,y) = f_i(x,y) - f_{i+1}(x,y)$
d) If i < n (the n is defined the decomposition number) then i = i+1, and return to the 2) step;
e) Repeat the steps 2), 3), 4) until i = n.

To process the image borders, we adapt the mirror symmetry method, namely:
In the row direction:
$f(-i, j) = f(i, j)$
$f(i+k, j) = f(i-k, j)$
In which $i \leq N$, k = 1, 2, … , N is the total rows of the image.
In the column direction:
$f(i, -j) = f(i, j)$
$f(i, j+k) = f(i, j-k)$

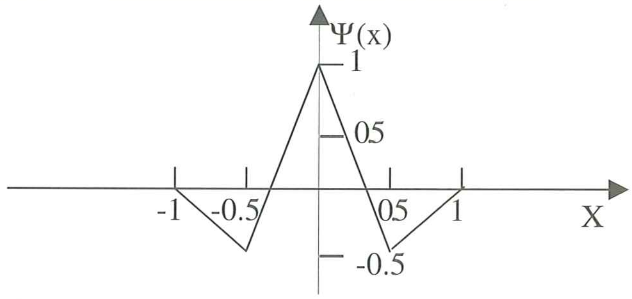

Figure 2. Wwavelet $\psi(x)$

In which: $j \leq N$, k = 1, 2, …, N is the total columns of the image.

In the experiment, the n is three, we add the three wavelet planes w1, w2 and w3 to get an result image which contains abundant high frequency information and little low frequency information.

IV. DISCUSSING THE EXPERIMENTS AND CONCLUSIONS

The Figure 3(a) is a SPOT panchromatic image covering Wuhan, P.R.China, in which there are two obvious roads and some fields. The Figure 4(a) is the same image, but now it is stained by random noise. The Figure 3(b), 3(c) and 3(d) are the results which are respectly detected by the classic Sobel, Robert operators and the à trous wavelet decomposition method. In the condition of noise, we use the methods and get the results as the Figure 4(b), 4(c), 4(d) show. Comparing the Figure 3, we can find the main edges of the Figure 3(d) like the two roads are finer than those of the Figure 3(b) and 3(c). The tiny field edges are obvious in the Figure 3(d), but those do not exist in the Figure 3(b) and Figure 3(c). From the results of Figure 3, we think the edges detected by the proposed method are finer and it can detect the tiny edges.

Looking at the Figure 4(b) and 4(c), we find the random noise effect is apparent, but almost can't find the noise effect in the Figure 4(d). So the classic Soble and Robert methods can not filter noise, they are sensitive to noise. The à trous wavelet decomposition method can detect edges filtering random noise and get satisfactory result as the Figure 4(d) shows. Why can the proposed method overcome random noise? We think the big features of the image almost don't verify at the different scales of wavelet transform, but the random noise rapidly attenuates with the increasing scales. So selecting proper scales, we can detect the edges and overcome random noise. In some applications we have used the results detected by the à trous wavelet decomposition method and the results are satisfactory.

However, besides the advantages of the new method of image edge detection, we also find out the contrast of result image is weaker, so that the detected image edges is not more evident.

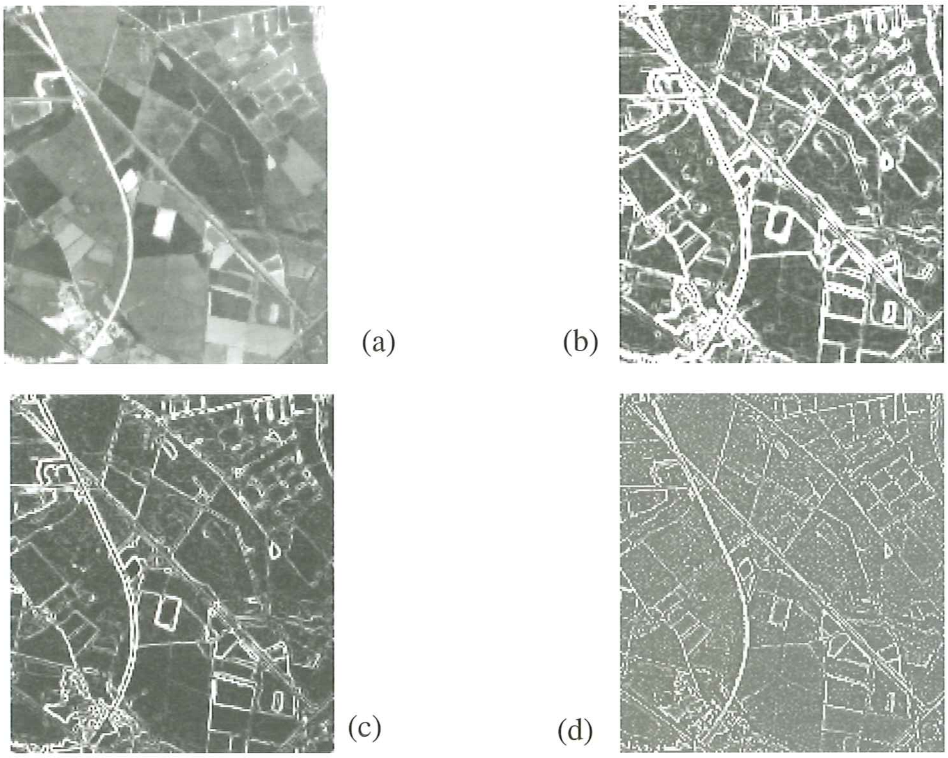

Figure 3. SPOT image and detected results. (a) Original SPOT image; (b) Sobel; (c) Robert; and (d) À trous wavelet

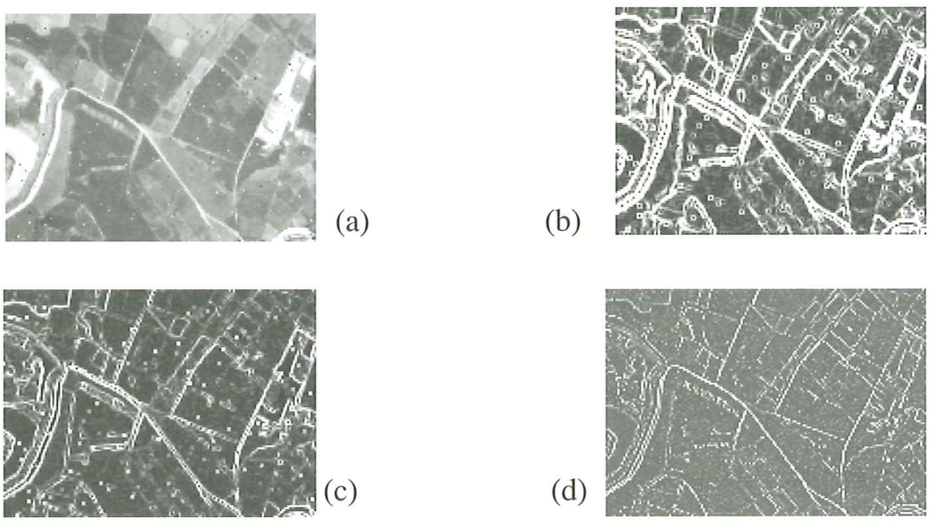

Figure 4. SPOT image stained by noise and detected results. (a) Original SPOT stained by noise; (b) Sobel; (c) Robert; and (d) À trous wavelet decomposition.

The disadvantage need to further research for improving the result

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