Kernelized Online Imbalanced Learning with Fixed Budgets

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Overview

1. Introduction
2. Related Work
3. The Proposed Method (KOIL)
4. Theoretical Analysis
5. Experiments
6. Conclusion

Online Learning

1. Definition of Online learning
    - Learn from the streaming data that keeps passing away
    - Update the model dynamically from the data stream
2. Properties
    - Process the data one by one
    - Update the model in each iteration
    - Approximate the learning performance of the batch-train methods

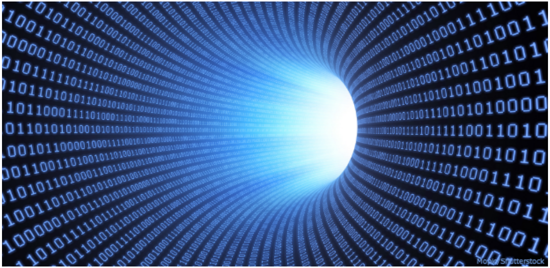


Figure: Rutrell Yasin, Amazon Kinesis does heavy-lifting on streaming, big data

Imbalanced Data & Cost-sensitive Learning

1. Properties:
   - Uneven data distribution
   - No. of samples in one class < No. of samples in the other class
2. Problems:
   - Accuracy: inappropriate
   - Misclassification costs for possitive and negative samples are not the same

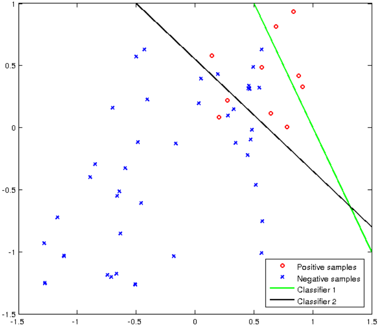


Figure: Imbalanced data

Support Vector Machine [Cortes 1995]

1. SVM maps the instance $\mathbf{x}$ to the Reproducing Kernel Hilbert Space

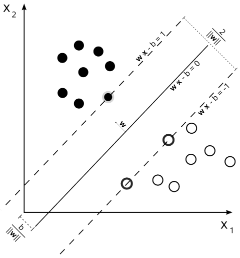
$$\phi : \mathbf{x} \longmapsto \phi(\mathbf{x})$$

.

2. In RKHS, dot product of two elements:

$$\langle \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) \rangle_{\mathcal{H}} = k(\mathbf{x}_i, \mathbf{x}_j)$$

3. The objective of SVM is to maximize the margins of the hyperplane in RKHS.



X_2
X_1
$\mathbf{w} \cdot \mathbf{x} - b = 1$
$\mathbf{w} \cdot \mathbf{x} - b = 0$
$\mathbf{w} \cdot \mathbf{x} - b = -1$
$\frac{2}{\|\mathbf{w}\|}$
$\frac{b}{\|\mathbf{w}\|}$
$\mathbf{w}$

Related Work

1. Online Learning with Kernels: minimize the hinge loss function

$$\min_f \ell_h(f, \mathbf{x}, y) := \max(0, 1 - yf(\mathbf{x})) \tag{1}$$

   - NORMA [Kivinen 2004]
   - Randomized Budget Perceptron [Cavallanti 2007]
   - Forgetron [Dekel 2008]
   - Projectron [Orabona 2008]

2. Online Linear AUC Maximization: minimize the AUC-based loss function
   - Online AUC Maximization (OAM) [Zhao 2011]

$$\min_w \ell_h(w, \mathbf{x}^+, \mathbf{x}^-) := \max(0, 1 - w \cdot (\mathbf{x}^+ - \mathbf{x}^-)) \tag{2}$$

   - One-Pass AUC Optimization (OPAUC) [Gao 2013]

$$\min_w \ell_h(w, \mathbf{x}^+, \mathbf{x}^-) := (1 - w \cdot (\mathbf{x}^+ - \mathbf{x}^-))^2 \tag{3}$$

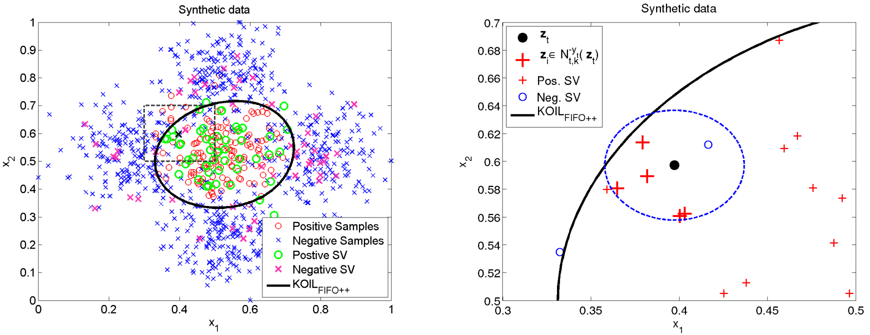
KOIL: Definitions & Notations

1. Non-linear decision function $f : \mathbb{R}^d \rightarrow \mathbb{R}$
2. A sequence of imbalanced feature-labeled pair instances $\{\mathbf{z}_t = (\mathbf{x}_t, y_t) \in \mathcal{Z}, t \in [T]\}$, where $\mathcal{Z} = \mathcal{X} \times \mathcal{Y}$, $\mathbf{x}_t \in \mathcal{X} \subseteq \mathbb{R}^d$, $y_t \in \mathcal{Y} = \{-1, +1\}$ and $[T] = \{1, \ldots, T\}$.
3. $f(\mathbf{x})$ can be calculated by

$$\langle f(\cdot), k(\mathbf{x}, \cdot)\rangle_{\mathcal{H}} = f(\mathbf{x}). \tag{4}$$

4. Assumption: positive class (minority) & negative class (majority)
5. $N_k^{\tilde{y}}(\mathbf{z})$: the set of the k-nearest neighbors of $\mathbf{z}$ and have the label of $\tilde{y}$.

KOIL: Intuition



(a)

(b)

1. Assign an initial weight to $\boldsymbol{z}_t$
2. Update the weight of SVs in the KNN of $\boldsymbol{z}_t$
3. Does not affect the weight of SVs in the whole budget

KOIL: Intuition – Update Kernel

Notation:

$\mathbf{z}_t^- := (\mathbf{x}_t, -1)$

$\mathbf{z}_t^+ := (\mathbf{x}_t, +1)$

Objective function:

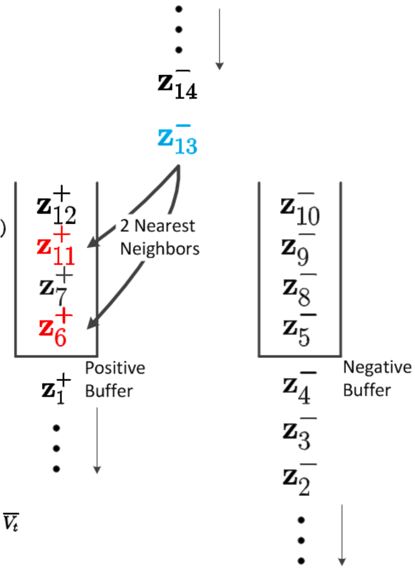
$$\hat{\mathcal{L}}(f, \mathbf{z}_t) = \frac{1}{2}\|f\|_{\mathcal{H}}^2 + C \sum_{\mathbf{z}_i \in N_k^{-y_t}(\mathbf{z}_t)} \ell_h(f, \mathbf{z}_t, \mathbf{z}_i)$$

Update Decision Function:

$$f_{t+1} := f_t - \eta \partial_f \hat{\mathcal{L}}(f, \mathbf{z}_t)|_{f=f_t}$$

Update Weight:

$$\alpha_{i,t} = \begin{cases} \eta C y_t |V_t|, & i = t \\ (1-\eta)\alpha_{i,t-1} - \eta C y_t, & \forall i \in V_t \\ (1-\eta)\alpha_{i,t-1}, & \forall i \in I_t^{y_t} \cup \overline{V_t} \end{cases}$$

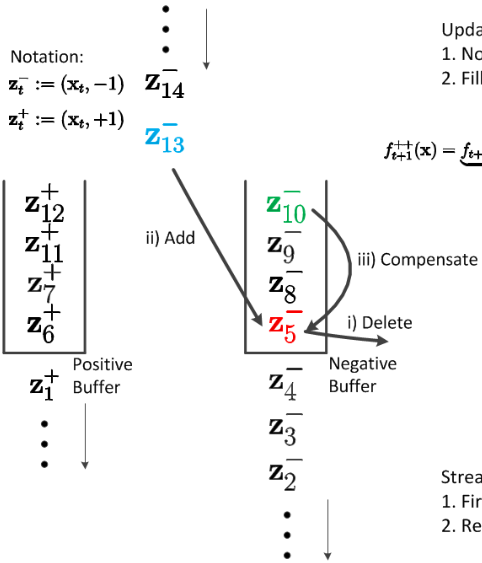


KOIL: Intuition – Update Buffers

Update Buffers:

1. Not filled: Add to buffer
2. Filled: i) Delete; ii) Add
   iii) Compensate

$$f_{t+1}^{++}(\mathbf{x}) = \underbrace{f_{t+1}(\mathbf{x}) - \alpha_r k(\mathbf{x}_r, \mathbf{x})}_{\text{Removal}} \underbrace{+\Delta\alpha_c \cdot k(\mathbf{x}_c, \mathbf{x})}_{\text{Compensation}}$$



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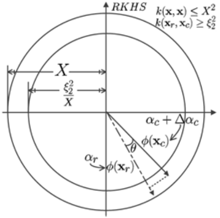


$RKHS$
$k(\mathbf{x},\mathbf{x}) \le X^2$
$k(\mathbf{x}_r,\mathbf{x}_c) \ge \xi_2^2$
X
$\frac{\xi_2^2}{X}$
$\alpha_c + \Delta\alpha_c$
$\phi(\mathbf{x}_c)$
θ
α_r
$\phi(\mathbf{x}_r)$

Stream oblivious policies:

1. First-In-First-Out (FIFO)
2. Reservoir Sampling (RS)

KOIL: Formulation

1. $\mathcal{K}^+$ and $\mathcal{K}^-$: the information of support vectors for two classes respectively, where $|B^+| = |B^-|$.

$$\mathcal{K}^+.\mathcal{A} := \{\alpha_i^+\}_{i=1}^{|B^+|}, \quad \mathcal{K}^+.\mathcal{B} := \{\mathbf{z}_i \,|\, y_i = +1\}_{i=1}^{|B^+|} \tag{5}$$

$$\mathcal{K}^-.\mathcal{A} := \{\alpha_i^-\}_{i=1}^{|B^-|}, \quad \mathcal{K}^-.\mathcal{B} := \{\mathbf{z}_i \,|\, y_i = -1\}_{i=1}^{|B^-|}. \tag{6}$$

2. Goal: to seek a decision function f in Eq. (7).

$$f(\mathbf{x}) = \sum_{\substack{\alpha_i^+ \in \mathcal{K}^+.\mathcal{A} \\ \mathbf{x}_i^+ \in \mathcal{K}^+.\mathcal{B}}} \alpha_i^+ k(\mathbf{x}_i^+, \mathbf{x}) + \sum_{\substack{\alpha_j^- \in \mathcal{K}^-.\mathcal{A} \\ \mathbf{x}_j^- \in \mathcal{K}^-.\mathcal{B}}} \alpha_j^- k(\mathbf{x}_j^-, \mathbf{x}), \tag{7}$$

KOIL: AUC Optimization

1. Given the positive dataset $D^+ = \{\mathbf{z}_i | y_i = +1\}$ and the negative dataset $D^- = \{\mathbf{z}_j | y_j = -1\}$, the AUC is measured as:

$$\begin{aligned} AUC(f) &= \frac{\sum_{i=1}^{|D^+|} \sum_{j=1}^{|D^-|} \mathbb{I}[f(\mathbf{x}_i^+) - f(\mathbf{x}_j^-) > 0]}{|D^+||D^-|} \\ &= 1 - \frac{\sum_{i=1}^{|D^+|} \sum_{j=1}^{|D^-|} \mathbb{I}[f(\mathbf{x}_i^+) - f(\mathbf{x}_j^-) \leq 0]}{|D^+||D^-|} \end{aligned} \tag{8}$$

   where $\mathbb{I}[\pi]$ is the indicator function.

2. Maximizing AUC equals to minimizing $\sum_{i=1}^{|D^+|} \sum_{j=1}^{|D^-|} \mathbb{I}[f(\mathbf{x}_i^+) - f(\mathbf{x}_j^-) \leq 0]$

3. Replace the discrete indicator function $\mathbb{I}[\pi]$ in Eq. (8) by the surrogate convex loss function in Eq. (9)

$$\ell_h(f, \mathbf{z}, \mathbf{z}') := \frac{|y - y'|}{2} \left[1 - \frac{1}{2}(y - y')(f(\mathbf{x}) - f(\mathbf{x}'))\right]_+ \tag{9}$$

KOIL: Update Kernel

1. Minimize the *instantaneous regularized risk of AUC*.

$$\min_f \mathcal{L}(f_t, \mathbf{z}_t) = \frac{1}{2}\|f_t\|^2_{\mathcal{H}} + C\sum_{i=1}^{t-1} \ell_h(f_t, \mathbf{z}_t, \mathbf{z}_i) \tag{10}$$

2. Minimize the *localized instantaneous regularized risk of AUC* (Reduce the effect of outliers):

$$\min_f \hat{\mathcal{L}}(f_t, \mathbf{z}_t) = \frac{1}{2}\|f_t\|^2_{\mathcal{H}} + C\sum_{\mathbf{z}_i \in N_k^{-y_t}(\mathbf{z}_t)} \ell_h(f_t, \mathbf{z}_t, \mathbf{z}_i) \tag{11}$$

3. Stochastic Gradient Descent: update f_t in each iteration

$$f_{t+1} := f_t - \eta \partial_f \hat{\mathcal{L}}(f, \mathbf{z}_t)|_{f=f_t} \tag{12}$$

4. Updating rule for the kernel weights:

$$\alpha_i = \begin{cases} \eta C y_t \sum_{\mathbf{z}_j \in N_k^{-y_t}(\mathbf{z}_t)} \mathbb{I}[\phi(\mathbf{z}_t, \mathbf{z}_j) < 1 \wedge y_t \neq y_j], & i = t \\ (1-\eta)\alpha_i - \eta C y_t, & \forall i, \mathbf{z}_i \in N_k^{-y_t}(\mathbf{z}_t) \\ (1-\eta)\alpha_i, & \text{otherwise} \end{cases} \tag{13}$$

KOIL: Update Budget

1. Remove SV via Reservoir Samping (RS) or FIFO:

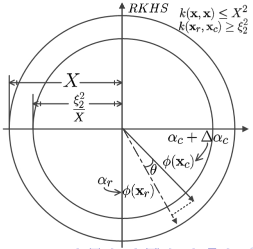
$$\hat{f}_{t+1}(\mathbf{x}) = f_{t+1}(\mathbf{x}) - \alpha_r k(\mathbf{x}_r, \mathbf{x}) \tag{14}$$

2. Compensate the loss by adding $\Delta\alpha_c$:

$$\begin{aligned} f_{t+1}^{++}(\mathbf{x}) &= \hat{f}_{t+1}(\mathbf{x}) + \Delta\alpha_c \cdot k(\mathbf{x}_c, \mathbf{x}) \\ &= \underbrace{f_{t+1}(\mathbf{x}) - \alpha_r k(\mathbf{x}_r, \mathbf{x})}_{\text{Removal}} \underbrace{+\Delta\alpha_c \cdot k(\mathbf{x}_c, \mathbf{x})}_{\text{Compensation}} \end{aligned} \tag{15}$$

3. By Eq. (15), we have

$$\Delta\alpha_c = \alpha_r \frac{k(\mathbf{x}_r, \mathbf{x})}{k(\mathbf{x}_c, \mathbf{x})} \approx \alpha_r. \tag{16}$$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X
$\frac{\xi_2^2}{X}$
$\alpha_c + \Delta\alpha_c$
$\phi(\mathbf{x}_c)$
θ
α_r
$\phi(\mathbf{x}_r)$

Theoretical Analysis

Lemma 1 (Norm of f)

Suppose for all $\mathbf{x} \in \mathbb{R}^d$, $k(\mathbf{x}, \mathbf{x}) \leq X^2$, where $X > 0$. Let ξ_1 be in $[0, X]$, such that $k(\mathbf{x}_t, \mathbf{x}_i) \geq \xi_1^2$, $\forall\, \mathbf{z}_i = (\mathbf{x}_i, y_i) \in N_t^{-y_t}(\mathbf{z}_t)$. With $f_1 = 0$, we have

$$\|f_{t+1}\|_{\mathcal{H}} \leq Ck\sqrt{2X^2 - 2\xi_1^2}. \tag{17}$$

Lemma 2 (pair-wise hinge loss bound)

With the same assumption in Lemma 1 and the pair-wise hinge loss function $\ell : \mathcal{H} \times \mathcal{Z} \times \mathcal{Z} \to [0, U]$ defined by Eq. (9), we can determine the bound by

$$U = 1 + 2Ck(X^2 - \xi_1^2). \tag{18}$$

Theoretical Analysis

Theorem (Regret bound of KOIL)

Suppose for all $\mathbf{x} \in \mathbb{R}^d$, $k(\mathbf{x}, \mathbf{x}) \le X^2$, *where* $X > 0$. *Let* ξ_1 *be in* $[0, X]$, *such that* $k(\mathbf{x}_t, \mathbf{x}_i) \ge \xi_1^2$, $\forall\, \mathbf{z}_i = (\mathbf{x}_i, y_i) \in N_t^{-y_t}(\mathbf{z}_t)$. *Given* $k > 0, C > 0, \eta > 0$ *and a bounded convex loss function* $\ell : \mathcal{H} \times \mathcal{Z} \times \mathcal{Z} \to [0, U]$ *for* f_t *updated by Eq. (12), with* $f_1 = 0$, *we have*

$$R_T \le \frac{\|f^*\|_{\mathcal{H}}^2}{2\eta} + \eta C k \sum_{t=1}^{T} \Big((U-1) + (k+1)C(X^2 - \xi_1^2)\Big). \tag{19}$$

Moreover, assume that $\forall i \in I_t^+ \cup I_t^-$, $|\alpha_{i,t}| \in [0, \gamma\eta]$ *and* $k(\mathbf{x}_r, \mathbf{x}_c) \ge \xi_2^2$ *with* $0 < \xi_2 \le X$ *for any replaced support vector* $\mathbf{x}_r$ *and compensated support vector* $\mathbf{x}_c$ *at any trial. With* $f_1^{++} = 0$ *and* f_t^{++} *updated by Eq. (15), we have*

$$R_T^{++} \le R_T + T\Big(4\gamma C k \sqrt{(X^2 - \xi_2^2)(X^2 - \xi_1^2)} + 2\gamma^2 (X^2 - \xi_2^2)\Big). \tag{20}$$

Set η to be $O(\frac{1}{\sqrt{T}})$, $R_T \sim O(\sqrt{T})$, as tight as the standard regret bound.

Experiment Setup

1. All algorithms adopt the same setup.
2. the learning rate: $\eta = 0.01$
3. A 5-fold cross validation on the training data is applied to find the penalty cost $C \in 2^{[-10:10]}$.
4. For kernel-based methods, we use the Gaussian kernel and tune its parameter $\sigma \in 2^{[-10:10]}$ by a 5-fold cross validation on the training data.

Methods in Comparison

- "Perceptron": the classical perceptron algorithm [Rosenblatt 1958];
- "OAM_{seq}": an online linear AUC maximization algorithm [Zhao 2011];
- "OPAUC": One-pass AUC maximization [Gao 2013];
- "NORMA": online learning with kernels [Kivinen 2004]
- "RBP": Randomized budget perceptron [Cavallanti 2007];
- "Forgetron": a kernel-based perceptron on a fixed budget [Dekel 2008];
- "Projectron/Projectron++": a bounded kernel-based perceptron [Orabona 2008];
- "$\text{KOIL}_{\text{RS}++}$": our proposed kernelized online imbalanced learning algorithm with fixed budgets updated by RS++.
- "$\text{KOIL}_{\text{FIFO}++}$": our proposed kernelized online imbalanced learning algorithm with fixed budgets updated by FIFO++.

Benchmark Datasets

Table: Summary of the benchmark datasets.

Dataset	Samples	Dimensions	T^-/T^+
sonar	208	60	1.144
australian	690	14	1.248
heart	270	13	1.250
ionosphere	351	34	1.786
diabetes	768	8	1.866
glass	214	9	2.057
german	1000	24	2.333
svmguide2	391	20	2.342
segment	2310	19	6.000
satimage	4435	36	9.687
vowel	528	10	10.000
letter	15000	16	26.881
poker	25010	10	47.752
shuttle	43500	9	328.546

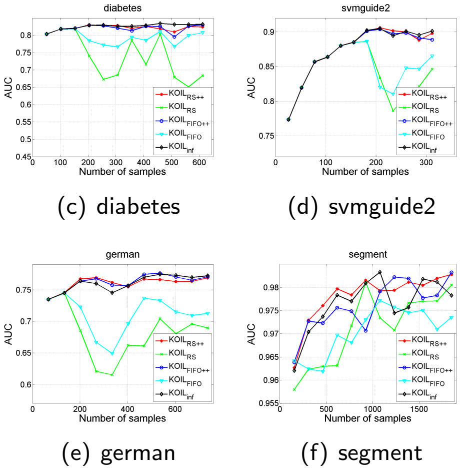
AUC Measure on Benchmark Dataset

Table: Average AUC performance (mean±std) on the benchmark datasets, ●/○ (-) indicates that both/one of KOIL$_{RS++}$ and KOIL$_{FIFO++}$ are/is significantly better (worse) than the corresponding method (pairwise *t*-tests at 95% significance level).

Data	KOIL$_{RS++}$	KOIL$_{FIFO++}$	Perceptron	OAM$_{seq}$	OPAUC	NORMA	RBP	Forgetron	Projectron	Projectron++
sonar	**.955**±.028	**.955**±.028	.803±.083●	.843±.056●	.844±.077●	.925±.044●	.913±.032●	.896±.054●	.896±.049●	.896±.049●
australian	.923±.023	.922±.026	.869±.035●	**.925**±.024	.923±.025	.919±.023	.911±.017●	.912±.026●	.923±.024	.923±.024
heart	.908±.040	.910±.040	.876±.066●	**.912**±.040	.901±.043○	.890±.051●	.865±.043●	.900±.053	.902±.038	.905±.042
ionosphere	**.985**±.015	**.985**±.015	.851±.056●	.905±.041●	.888±.046●	.961±.016●	.960±.030●	.945±.031●	.964±.025●	.963±.027●
diabetes	.826±.036	.830±.030	.726±.059●	.827±.033	.805±.035●	.792±.032●	.828±.034	.820±.027○	.832±.033	**.833**±.033
glass	**.887**±.053	.884±.054	.810±.065●	.827±.064●	.800±.074●	.811±.077●	.811±.071●	.813±.075●	.811±.070●	.781±.076●
german	.769±.032	.778±.031	.748±.033●	.777±.027	**.787**±.026-	.766±.032○	.699±.038●	.712±.054●	.769±.028○	.770±.024
svmguide2	**.897**±.040	.885±.043	.860±.037●	.886±.045○	.859±.050●	.865±.046●	.890±.038	.864±.045●	.886±.044○	.886±.045○
segment	.983±.008	**.985**±.012	.875±.020●	.919±.020●	.882±.019●	.910±.042●	.969±.017●	.943±.038●	.979±.013●	.978±.016●
satimage	**.924**±.012	.923±.015	.700±.015●	.755±.018●	.724±.016●	.914±.025●	.899±.018●	.892±.032●	.910±.015●	.904±.011●
vowel	**1.000**±.000	**1.000**±.001	.848±.070●	.905±.024●	.885±.034●	.996±.005●	.968±.017●	.987±.027●	.982±.013●	.994±.019●
letter	.933±.021	**.942**±.017	.767±.029●	.827±.021●	.823±.018●	.910±.027●	.928±.011○	.815±.102●	.926±.016●	.926±.015●
poker	.681±.031	**.693**±.032	.514±.030●	.503±.024●	.509±.031●	.577±.040●	.501±.031●	.572±.029●	.675±.027●	.675±.027●
shuttle	.950±.040	.956±.021	.520±.134●	**.999**±.000-	.754±.043●	.725±.053●	.844±.041●	.839±.060●	.873±.063●	.795±.063●
win/tie/loss			14/0/0	9/4/1	12/1/1	13/1/0	12/2/0	13/1/0	11/3/0	10/4/0

KOIL: RS++/FIFO++ vs RS/FIFO

1. RS/FIFO $\downarrow$ when the budget is full
2. RS++/FIFO++ approximate KOIL without removing SVs.



(c) diabetes

(d) svmguide2

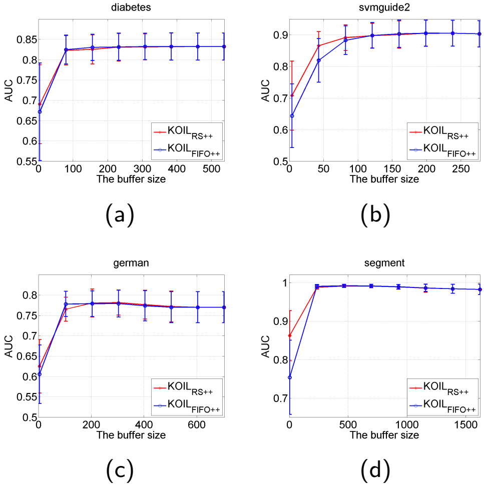
(e) german

(f) segment

Figure: Average AUC performance of KOIL.

Experiment: Effect of Buffer Size

1. Stay unchange when buffer size is large enough.
2. KOIL cannot learn well when buffer size is extremely small.



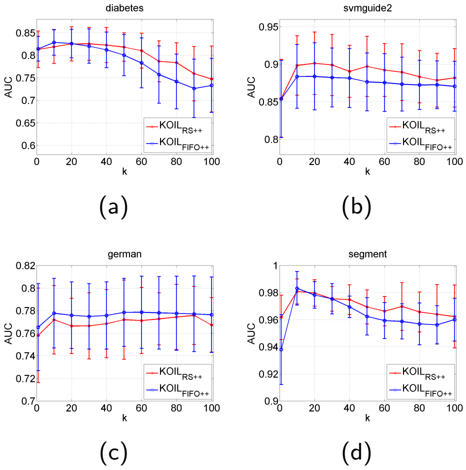
(a) (b)

(c) (d)

Figure: Average AUC of KOIL for buffer sizes.

Experiment: Effect of k

1. For noisy dataset, set k small to avoid global effect
2. k extremely small, KOIL cannot learn enough knowledge.



(a) (b)

(c) (d)

Figure: Average AUC of KOIL with different k.

Conclusion

In this talk, I introduced the KOIL algorithm, which has the following properties :

1. AUC maximization for streaming data
2. Two fixed-size budgets
3. *k*-nearest neighbors to reduce the effect of outliers
4. loss compensation for support vector replacement
5. Regret bound for KOIL and two lemmas
6. Experiments on benchmark and synthetic datasets

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Thanks!