

The Magic Money Machine

The wizards at Wall Street are up to it again. The Silverbags investment bank has invented the following machine.

The machine consists of 6 boxes numbered 1 to 6. When you first get the machine, it contains 6 tokens, one in each box. You have two buttons A,B on the machine and you can press them as many times as you like and in any order.

Button A Choose a number i from 1 to 5 and then take one token from box i and magically two tokens will be added to box $i + 1$.

Button B Choose a number i from 1 to 4 and then take one token from box i and then the contents of boxes $i + 1$ and $i + 2$ will be interchanged.

The machine sells for one trillion dollars. The contract says that you can take the machine back to the bank at any time and then the bank will give you one dollar for each token in the machine. Is the machine worth buying?

Solution: The machine would be worth buying, except that it would take you about 30,000 years at one button press per second to get the money. Here is a simple argument. First suppose that we only have 3 boxes and they contain $n, 0, 1$ respectively. We claim that we can generate the contents $0, 5 \cdot 2^{n-2}, 0$, for $n \geq 2$. We do this by the sequence

n	0	1	
$n - 1$	2	1	A
$n - 1$	0	5	AA
$n - 2$	5	0	B
$n - 2$	0	10	AAAAA
$n - 3$	10	0	B

and so on. In addition we need the moves

$$1\ 1 \xrightarrow{A} 0\ 3 \text{ and } 3\ 1\ 1 \xrightarrow{A} 2\ 3\ 1 \xrightarrow{AAA} 2\ 0\ 7 \xrightarrow{B} 1\ 7\ 0 \xrightarrow{A \dots A} 1\ 0\ 14 \xrightarrow{B} 0\ 14\ 0$$

With this in mind we can generate

1	1	1	1	1	1
0	3	1	1	1	1
0	0	14	0	1	1
0	0	0	20480	0	1
0	0	0	0	$5 \cdot 2^{20478}$	0

For more on this problem go to

http://michaelnielsen.org/polymath1/index.php?title=Imo_2010.

Thanks are due to the following people for sending in a solution:

Filip Barl, Ralf Brown, Dan Hirschberg, Sullivan Jung, YoungHoon Jung, Rashid Kaleem, Alan Keith, Reid McKenzie, Dennis Meng, Shortmanikos, Sushmita Swaminathan, Varun Mishra, Pratik Poddar, Michael Schuresko, Jake Trower.