# Solutions for Written Assignment 2 CSCI 2100A 2016 Spring

## Exercise 1.6

```
(1) for i = 1 to n;
        for j = 1 to n;
              x := x + 1;
```

**Solution:** $f(n) = n^2,\ g(n) = n^2$.

```
(3) for i = 1 to n;
        for j = i to n;
            for k = 1 to j;
                x := x + 1;
```

**Solution:** $f(n) = \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=1}^{j} 1 = \sum_{i=1}^{n}\sum_{j=i}^{n} j = \frac{1}{2}\sum_{i=1}^{n}(n+i)(n-i+1) = \frac{1}{2}\sum_{i=1}^{n}(n(n+1)+i-i^2) = \frac{n(n+1)(2n+1)}{6},\ g(n) = n^3$.

```
(5) for i = 1 to n;
        for j = i to n;
            for k = 1 to 1000;
                x := x + 1;
```

**Solution:** $f(n) = \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=1}^{1000} 1 = 1000\sum_{i=1}^{n}\sum_{j=i}^{n} 1 = 1000\sum_{i=1}^{n}(n-i+1) = 1000\sum_{i=1}^{n} i = 500n(n+1),\ g(n) = n^2$.

## Exercise 1.8

(3) What is the running time of this algorithm with the following assumptions? What is its big-O notation?

| Statement | Time Unit |
|---|---|
| assignment | 1 |
| + | 1.25 |
| * | 1.75 |
| for-next loop set-up | 2.3 |
| each loop | 1.5 |

**Solution**: First assignment consumes 1 unit. Setting up for-next loop consumes 2.3 units. For each loop, there are one $*$, one $+$, and one assignment. Thus, $(1.75+1.25+1+1.5)*(n+1) = 5.5n+5.5$ units will be consumed. In total, the running time is $5.5n+8.8$ units, which is $O(n)$ in big-O notation.

## Exercise 1.9

(2) Calculate the time and space complexity for $n = 10, 20, 30, 50, 70$, and 100 for each algorithm.

**Solution:** Just pay attention to different ranges.

(4) Come up with a strategy that you would use to minimize the time and space complexity individually?

**Solution:** $t(n) = min\{t_A(n),\ t_B(n)\}$, $s(n) = min\{s_A(n),\ s_B(n)\}$. Thus,

$$t(n) = \begin{cases} n & \text{if } 1 \leq n < 50 \\ n^2 & \text{if } 50 \leq n < 70 \\ n^3 & \text{if } 70 \leq n \leq 100 \end{cases}$$

$$s(n) = \begin{cases} n & \text{if } 1 \leq n < 20 \\ 1.5n & \text{if } 20 \leq n < 50 \\ 0.5n & \text{if } 50 \leq n \leq 100 \end{cases}$$