CSCI5070 Advanced Topics in Social Computing

Exercise 1

1. Figure 1 is a graph of 10 municipal regions that are served by a fire department. The city wants to locate a new fire station in the "middle" of these regions. Assume that the links in this graph represent equal transit times for a fire engine to reach each region from neighboring regions. The city wants to reduce transit time from the new station to all regions.
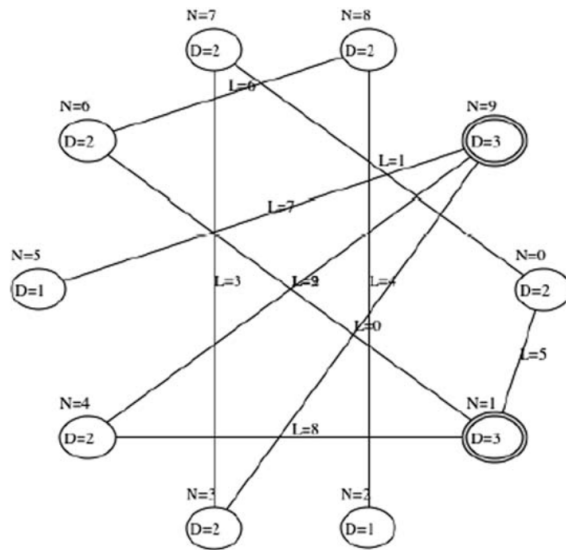

Figure 1: A graph with 10 nodes and 10 links

Questions:

(1) What is the diameter of the graph?

(2) What is the radius of the node v_1 and v_9 respectively?

(3) What is the center of the graph?

(4) Which region (node) is the best place to locate the fire station? Why?

(5) What is the adjacency matrix of the graph? What is the Laplacian matrix of the graph?

(6) What node(s) is (are) the farthest from the central node, and how far?

(7) What is the closest node in Fig. 1, according to the closeness property?

2. Graph G, $n = 5$, contains nodes v_1, v_2, v_3, v_4, v_5, with the following mapping function:

$$f = [e_1 : v_3 \sim v_4, e_2 : v_1 \sim v_2, e_3 : v_1 \sim v_3, e_4 : v_4 \sim v_5, e_5 : v_3 \sim v_5, e_6 : v_2 \sim v_5]$$

What is the cluster coefficient of node v_5? What's the cluster coefficient of the entire graph?

3. What is the density of a 5-regular graph with 20 nodes?

4. What is the average path length and the approximate link efficiency of a balanced binary tree network, for $n = 1023$?

5. Which network, binary tree, toroidal, or hypercube, has the shortest average path length for $4 \leq n \leq 9$?

6. How many links does a random network with $n = 200$ nodes need to guarantee an average closeness of 30?